Name & ID

UNIVERSITY OF ALABAMA
Department of Physics and Astronomy

PH 106-4 / LeClair | Fall 2008

Exam II

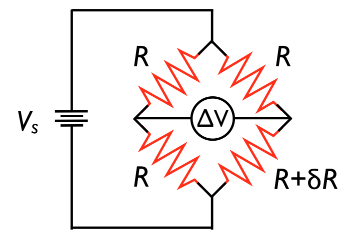
Instructions

1. **Solve 3 of the 7 problems below.** All have equal weight. This gives you about 15 minutes per problem!
2. Clearly mark the problems you choose using the tick boxes.
3. **There will be partial credit given.** Show as much work as possible; you must show work for full credit.
4. You are allowed 2 sides of a standard 8.5x11 inch piece of paper with notes/formulas, and a calculator.

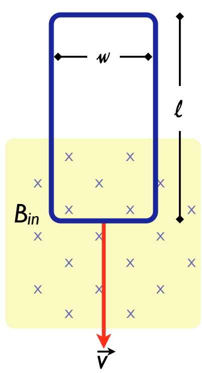
□ **1.** The circuit at right is known as a *Wheatstone Bridge*, and it is a useful circuit for measuring small changes in resistance. Perhaps you can figure out why. Three of the four branches on our bridge have identical resistance R, but the fourth has a slightly different resistance, by an amount δR such that its total resistance is $R + \delta R$.

In terms of the source voltage V_s, base resistance R and change in resistance δR, what is the reading on the voltmeter, ΔV? You may assume the voltmeter and voltage source are perfect (drawing no current and having no internal resistance, respectively).

Bonus: Simplify your expression for $\delta R \ll R$, i.e., the change in resistance is small. Show that this results in $\Delta V \propto \delta R$. (Worth +20% credit on this question.)



Problem 1: Wheatstone Bridge



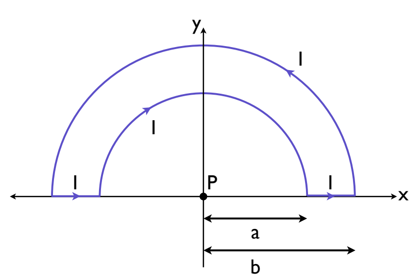
Problem 2: Falling loop in a magnetic field

□ **2.** A conducting rectangular loop of mass M, resistance R, and dimensions w by l falls from rest into a magnetic field $\vec{\mathbf{B}}$, as shown at left. At some point before the top edge of the loop reaches the magnetic field, the loop attains a constant terminal velocity v_T. Show that the terminal velocity is:

$$v_T = \frac{MgR}{B^2 w^2}$$

Hint: what is true at terminal velocity?

□ **3.** Find the magnetic field at point P due to the current distribution shown below. *Hint: Break the loop into segments, and use superposition.*



Problem 3: A current loop

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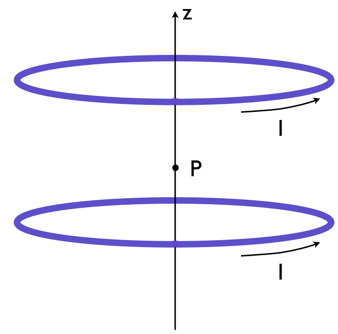
□ **4.** If the voltage at the terminals of an automobile battery drops from ΔV_1 to ΔV_2 ($\Delta V_1 > \Delta V_2$) when a resistor R is connected across the battery, what is the internal resistance r of the battery?

□ **5.** A Helmholtz coil, useful for producing a region of uniform field, consists of two identical circular coils separated by a distance equal to their radius R, as shown at right. Show that dB_z/dz vanishes at the midpoint P, indicating a homogeneous field.

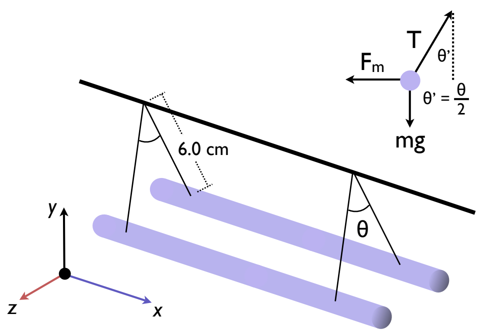
Hint: The field from a single current loop of radius R a distance z from the center of the loop along the coil axis is

$$B = \frac{\mu_o I}{2} \frac{R^2}{(z^2 + R^2)^{3/2}} \quad \text{(single loop)}$$

Bonus: show that $d^2 B_z/dz^2$ is also zero at P. (Worth +20% credit on this question.)



Problem 5: A Helmholtz coil



Problem 6: Parallel current-carrying wires

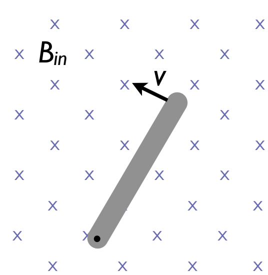
□ **6.** Two long parallel wires, each with a mass per unit length of λ, are supported in a horizontal plane by strings of length d, as shown at left. Each wire carries the same current I, causing the wires to repel one another, which causes the supporting strings to make an angle θ with one another.

Find the magnitude of the current.

Hint: consider the free-body diagram in the upper right.

□ **7.** A conducting bar of length l rotates with a constant angular speed ω about a pivot at one end. A uniform magnetic field $\vec{\mathbf{B}}$ is directed perpendicular to the plane of rotation, as shown in the figure below. Find the motional voltage induced between the ends of the bar.

Hint: recall $v = r\omega$ at a distance r from the pivot point.



Problem 7: A conducting rod rotates about one end in a magnetic field.

BONUS (worth 20% of a normal question): During an in-class demonstration, we dropped a magnet and a non-magnet of equal weight and size through a copper tube. The non-magnet fell through the tube at the expected rate, but the magnet took many times longer to fall out, due to eddy current braking.

Is it possible to have a magnet strong enough (or a tube conductive enough, *etc*) that it would actually *stop* inside the tube? Explain.

Constants:

$$
\begin{aligned}
k_e &\equiv 1/4\pi\epsilon_o = 8.98755 \times 10^{9}\ \mathrm{N \cdot m^2 \cdot C^{-2}} \\
\epsilon_o &= 8.85 \times 10^{-12}\ \mathrm{C^2/N \cdot m^2} \\
\mu_o &\equiv 4\pi \times 10^{-7}\ \mathrm{T \cdot m/A} \\
c^2 &= 1/\mu_o\epsilon_o \\
e &= 1.60218 \times 10^{-19}\ \mathrm{C} \\
m_{e^-} &= 9.10938 \times 10^{-31}\ \mathrm{kg} \\
m_{p^+} &= 1.67262 \times 10^{-27}\ \mathrm{kg} \\
1\ \mathrm{u} &= 931.494\ \mathrm{MeV}/c^2 \\
g &= 9.81\ \mathrm{m/s^2}
\end{aligned}
$$

Basic Equations:

$$
\begin{aligned}
0 &= ax^2 + bx^2 + c \Longrightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\
\vec{\mathbf{F}}_{\text{centr}} &= -\frac{mv^2}{r}\hat{\mathbf{r}} \ \text{ Centripetal}
\end{aligned}
$$

Electric Force & Field:

$$
\begin{aligned}
\vec{\mathbf{F}}_{12} &= k_e \frac{q_1 q_2}{r^2}\,\hat{\mathbf{r}} = q_2 \vec{\mathbf{E}}_1 \\
\vec{\mathbf{E}}_1 &= \vec{\mathbf{F}}_{12}/q_2 = k_e \frac{q_1}{r^2}\,\hat{\mathbf{r}} \\
\vec{\mathbf{E}} &= k_e \sum_i \frac{q_i}{r_i^2}\,\hat{\mathbf{r}}_i \rightarrow k_e \int \frac{dq}{r^2}\,\hat{\mathbf{r}} = k_e \int \frac{\rho \hat{\mathbf{r}}}{r^2}\, dV_{ol}
\end{aligned}
$$

Electric Potential (static case!):

$$
\begin{aligned}
\Delta V = V_B - V_A &= \frac{\Delta U}{q} = -\int_A^B \vec{\mathbf{E}} \cdot d\vec{\mathbf{l}} \\
V_{\text{point}} &= k_e \frac{q}{r} \\
V &= k_e \int \frac{dq}{r} \quad \text{continuous} \\
E_x &= -\frac{dV}{dx} \rightarrow \vec{\mathbf{E}} = -\vec{\boldsymbol{\nabla}} V \\
Q &= C\Delta V \quad \text{capacitor}
\end{aligned}
$$

Current & Resistance:

$$
\begin{aligned}
I &= \int_S \vec{\mathbf{J}} \cdot d\vec{\mathbf{A}} \xrightarrow{\text{uniform J}} I = \frac{dQ}{dt} = nqAv_d \\
J &= \frac{I}{A} = nqv_d \quad \text{uniform J} \\
\int_S \vec{\mathbf{J}} \cdot d\vec{\mathbf{A}} &= -\frac{d}{dt}\int_V \rho\, dV_{ol} \\
R &= \frac{\varrho l}{A} \quad \rho = 1/\sigma \\
\mathscr{P} &= U \cdot \Delta t = I\Delta V \quad \text{power} \\
R_{\text{eq}} &= R_1 + R_2 + \ldots \quad \text{series} \\
1/R_{\text{eq}} &= 1/R_1 + 1/R_2 + \ldots \quad \text{parallel} \\
\sum I_{\text{in}} &= \sum I_{\text{out}} \quad \text{junction} \\
\sum_{\text{closed path}} \Delta V &= 0 \quad \text{loop}
\end{aligned}
$$

Magnetism:

$$
\begin{aligned}
\vec{\mathbf{F}}_B &= q\vec{\mathbf{v}} \times \vec{\mathbf{B}} \\
d\vec{\mathbf{F}}_B &= I\,d\vec{\mathbf{l}} \times \vec{\mathbf{B}} \quad \text{I-carrying wire} \\
d\vec{\mathbf{B}} &= \frac{\mu_o}{4\pi}\frac{I\,d\vec{\mathbf{l}} \times \hat{\mathbf{r}}}{r^2} \quad \text{wire} \\
B &= \frac{\mu_o N I}{l} = \mu_o n I \quad \text{solenoid} \\
\frac{F_B}{l} &= \frac{\mu_o I_1 I_2}{2\pi d} \quad \text{2 wires} \\
\Phi_B &= \oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{A}} \\
\vec{\boldsymbol{\tau}} &= \vec{\boldsymbol{\mu}} \times \vec{\mathbf{B}} \quad \text{with} \quad \vec{\boldsymbol{\mu}} = I\vec{\mathbf{A}} \quad \text{I loop} \\
U &= -\vec{\boldsymbol{\mu}} \times \vec{\mathbf{B}}
\end{aligned}
$$

Induction & Maxwell

$$
\begin{aligned}
\Delta V &= \oint \vec{\mathbf{E}} \cdot\, d\vec{\mathbf{l}} = -\frac{d\Phi_B}{dt} \\
\Delta V &= Blv \quad \text{motional} \\
\Phi_E &= \oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = 4\pi k_e q_{\text{encl}} = \frac{q_{\text{encl}}}{\epsilon_o} \\
\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{A}} &= 0 \\
\oint \vec{\mathbf{B}} \cdot\, d\vec{\mathbf{l}} &= \mu_o I + \frac{1}{c^2}\frac{d\Phi_E}{dt}
\end{aligned}
$$

Vectors:

$$
\begin{aligned}
|\vec{\mathbf{F}}| &= \sqrt{F_x^2 + F_y^2} \quad \text{magnitude} \\
\theta &= \tan^{-1}\left[\frac{F_y}{F_x}\right] \quad \text{direction} \\
\hat{\mathbf{r}} &= \vec{\mathbf{r}}/|\vec{\mathbf{r}}| \\
d\vec{\mathbf{l}} &= dx\,\hat{\mathbf{x}} + dy\,\hat{\mathbf{y}} + dz\,\hat{\mathbf{z}}
\end{aligned}
$$

$$
\text{let} \quad \vec{\mathbf{a}} = a_x\,\hat{\mathbf{x}} + a_y\,\hat{\mathbf{y}} + a_z\,\hat{\mathbf{z}} \quad \text{and} \quad \vec{\mathbf{b}} = b_x\,\hat{\mathbf{x}} + b_y\,\hat{\mathbf{y}} + b_z\,\hat{\mathbf{z}}
$$

$$
\begin{aligned}
\vec{\mathbf{a}} \cdot \vec{\mathbf{b}} &= a_x b_x + a_y b_y + a_z b_z = \sum_{i=1}^{n} a_i b_i = |\vec{\mathbf{a}}||\vec{\mathbf{b}}|\cos\theta \\
|\vec{\mathbf{a}} \times \vec{\mathbf{b}}| &= |\vec{\mathbf{a}}||\vec{\mathbf{b}}|\sin\theta \\
\vec{\mathbf{a}} \times \vec{\mathbf{b}} &= \begin{vmatrix} \hat{\mathbf{x}} & \hat{\mathbf{y}} & \hat{\mathbf{z}} \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix} = (a_y b_z - a_z b_y)\,\hat{\mathbf{x}} + (a_z b_x - a_x b_z)\,\hat{\mathbf{y}} + (a_x b_y - a_y b_x)\,\hat{\mathbf{z}}
\end{aligned}
$$

Derived unit	Symbol	equivalent to
newton	N	$\mathrm{kg \cdot m/s^2}$
joule	J	$\mathrm{kg \cdot m^2/s^2} = \mathrm{N \cdot m}$
watt	W	$\mathrm{J/s = m^2 \cdot kg/s^3}$
coulomb	C	$\mathrm{A \cdot s}$
V	$\mathrm{W/A = m^2 \cdot kg/ \cdot s^3 \cdot A}$	
farad	F	$\mathrm{C/V = A^2 \cdot s^4/m^2 \cdot kg}$
ohm	Ω	$\mathrm{V/A = m^2 \cdot kg/s^3 \cdot A^2}$
tesla	T	$\mathrm{Wb/m^2} = \mathrm{kg/s^2 \cdot A}$
electron volt	eV	$1.6 \times 10^{-19}\ \mathrm{J}$
-	$1\ \mathrm{T \cdot m/A}$	$1\ \mathrm{N/A^2}$
-	$1\ \mathrm{T \cdot m^2}$	$1\ \mathrm{V \cdot s}$
-	$1\ \mathrm{N/C}$	$1\ \mathrm{V/m}$

Right-hand rule #1

1. Point the fingers of your right hand along the direction of $\vec{\mathbf{v}}$.
2. Point your thumb in the direction of $\vec{\mathbf{B}}$.
3. The magnetic force on a $+$ charge points out from the back of your hand.

Right-hand rule #2:
Point your right thumb along the wire in the direction of the current. Your fingers curl around the direction of the magnetic field, which circulates around the wire.