

### Problem Set 4: Mostly Capacitance

1. **20 points.** Two long, cylindrical conductors of radius  $a_1$  and  $a_2$  are parallel and separated by a distance  $d$  which is large compared with either radius. Find the capacitance per unit length of the two conductors.
  
2. **20 points.** Find the equivalent capacitance for *both* combinations shown below. Be sure to consider the symmetry involved and the relative electric potential at different points in the circuits.

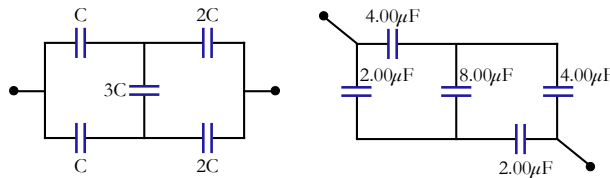


Figure 1: Capacitor combinations.

3. **20 points.** A capacitor is constructed from two square plates of sides  $l$  and separation  $d$ . A material of dielectric constant  $\kappa$  is inserted a distance  $x$  into the capacitor, as shown below. **(a)** Find the equivalent capacitance of this device as a function of  $x$ . **(b)** Calculate the energy stored in the capacitor, letting  $\Delta V$  represent the potential difference. **(c)** Find the direction and magnitude of the force exerted on the dielectric, assuming a constant potential difference  $\Delta V$ . Ignore friction.

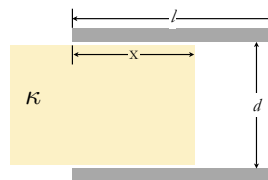


Figure 2: Capacitor combinations.

4. **20 points.** Using the same figure as the previous question, imagine now that the block being inserted is metal, rather than dielectric. Assume that  $d \ll l$ , and that the plates carries charges  $+Q_o$  and  $-Q_o$ . **(a)** Calculate the stored energy as a function of  $x$ . **(b)** Find the direction and magnitude of the force acting on the metallic block. *Hint: a metal can be considered a perfect dielectric,  $\kappa \rightarrow \infty$ , which allows no electric field to penetrate it.*

5. **10 points.** A charge  $Q$  is located  $h$  meters above a conducting plane. How much work is required to bring this charge out to an infinite distance above the plane? *Hint: Consider the method of images.*

6. **10 points.** Two capacitors, one charged and the other uncharged, are connected in parallel. (a) Prove that when equilibrium is reached, each carries a fraction of the initial charge equal to the ratio of its capacitance to the sum of the two capacitances. (b) Show that the final energy is less than the initial energy, and derive a formula for the difference in terms of the initial charge and the two capacitances.