UNIVERSITY OF ALABAMA
Department of Physics and Astronomy

PH 106 Fall 2008

Mutual Inductance Laboratory

For this lab, you will need:

1 function generator
1 handheld multimeter
1 pair of coaxial solenoids (pictured below)
4 banana cables
1 male BNC to banana plug adaptor
1 ruler or calipers

Introduction

Consider two coaxial solenoids, one inside the other, a situation we discussed recently in lecture:

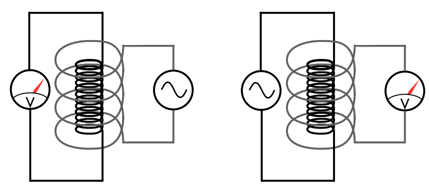


Figure 1: *Schematic of your coaxial solenoids*

In the arrangement shown at left, the outer coil is powered with an ac (time-varying) voltage. The resulting time-varying current creates a time varying magnetic field, which penetrates the inner coil uniformly. Thus, there is a time-varying magnetic flux in the inner coil, which induces a voltage on the inner coil. This voltage will also be time-varying, with the same frequency the voltage driving the outer coil.

We derived an expression relating the rate at which the current in the outer coil changes with time to the magnitude of the voltage induced across the inner coil:

$$|V_{\text{outer}}| = -M\frac{dI_{\text{inner}}}{dt} \tag{1}$$

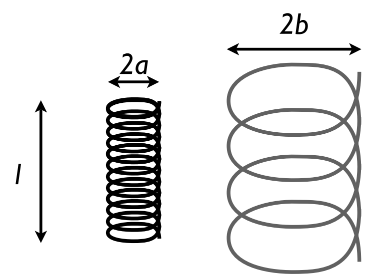
The constant of proportionality M is the mutual inductance of the two coils. If the current in the inner coil varies sinusoidally with frequency f, this expression becomes:

$$|V_{\text{outer}}| = -2\pi f M I_{\text{inner}} \tag{2}$$

Remarkably, the situation is exactly the same if we instead power the inner coil and measure the voltage on the outer coil, as shown at right in the figure, even though the field produced by the shorter inner solenoid is terribly nonuniform. The relation between voltage in one coil and current in the other remains exactly the same, no matter which is powered and which is measured. Therefore: we choose the easier system to actually measure, namely, the right-hand one.

Question: Why do we say the situation on the right in Figure 1 is easier to measure?

For two solenoids, the mutual inductance M may be calculated exactly. Let the inner solenoid have length l and diameter $2a$, the outer solenoid diameter $2b$, as shown below



$$M = \mu_0 \pi a^2 n_{\text{inner}} n_{\text{outer}} l$$

Figure 2: *Mutual inductance of two solenoids*

In general, mutual inductance depends only on the geometry and number of turns in each coil, as in this example. If the outer coil completely encloses the inner, we can make one more simplification, as we did when discussing transformers: we know that the flux produced by the inner coil is completely captured by the outer coil, so the product of the number of turns and current is the same for both coils

$$n_{\text{inner}} I_{\text{inner}} = n_{\text{outer}} I_{\text{outer}} \tag{3}$$

Using this relationship and the equations above, we can write the voltage on the outer coil as a function of the frequency driving the inner coil:

$$|V_{\text{outer}}| = -\left[2\mu_o \pi^2 a^2 l I_{\text{outer}}\right] n_{\text{outer}}^2 f \tag{4}$$

The quantity in brackets depends only on constants, the geometry of the inner coil, and the current in the outer coil, which you can determine from the measured voltage and resistance.[i] Thus, a plot of V versus f should allow us to measure the number of turns in the outer coil.

Preliminaries

- □ Measure the diameter of both coils, and the length of the inner coil. Record these numbers below. Your coils should look something like this:

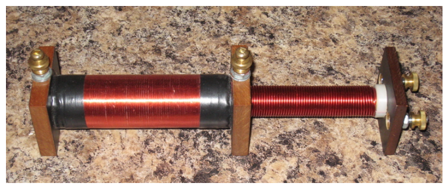


Figure 3: *Your coaxial solenoids should look something like this.*

- □ Count the number of turns on your inner coil, and record this number below.
- □ Measure the resistance of the outer coil using the hand-held voltmeter. Record this number below. It should be $50-100\,\Omega$ in most cases.

Procedure

- □ Turn on your function generator, and connect its output[ii] to the handheld voltmeter.
- □ Set the function generator's frequency to $f = 1\,\text{kHz}$, its waveform to a sine wave, and maximize the amplitude. With the voltmeter in "ac volts" mode (symbolized by $V\sim$), measure the output of the function generator and record it.
- □ Now connect the function generator output to the inner coil instead of the voltmeter. This will power the inner solenoid.
- □ Connect the outer coil's terminals to the voltmeter, which should still be in "ac volts" mode (symbolized by $V\sim$)
- □ Place the inner solenoid completely inside the outer one, and note the voltage measured on the outer coil.

> Question: What happens when you move the inner coil slowly out of the outer coil? Why?

- □ Replace the inner coil such that is completely enclosed by the outer coil.

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[i] Since $n_{\text{inner}}I_{\text{inner}} = n_{\text{outer}}I_{\text{outer}}$, we can determine the current at a single frequency, and presume it constant over the course of the measurement. The voltage on the outer coil will change, but its current will be roughly the same over the frequency range of interest.

[ii] Use the "$50\,\Omega$" output, not the "TTL" output.

- □ In steps of $\Delta f = 1\,\text{kHz}$, record the voltage on the outer coil versus the frequency of the voltage applied to the inner coil from $1-10\,\text{kHz}$ using the table below. Do not change the amplitude of the function generator output applied to the inner coil.
- □ Plot the voltage on the outer coil versus frequency of the inner coil's signal in excel, as shown in the example below. Using a linear trendline, determine the slope of the curve and record it below.?

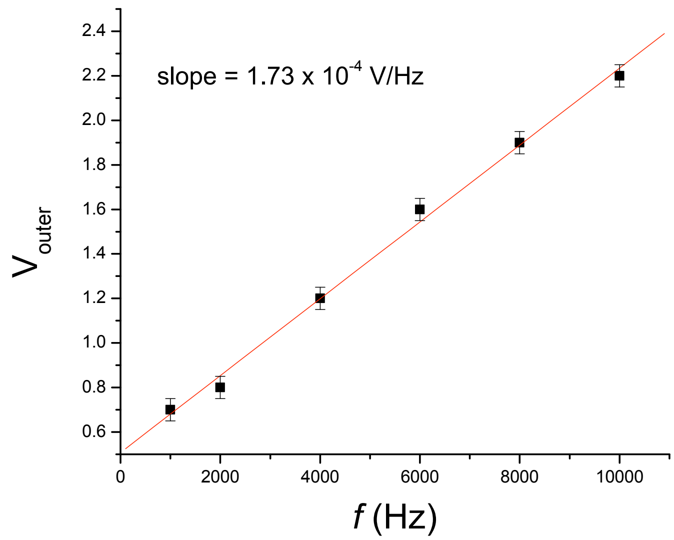

Figure 4: *Example plot of the outer coil's voltage versus the frequency of the voltage applied to the inner coil.*

- □ Using the relation below, determine the number of turns in the outer coil, and record this below. Determine I_{outer} from the measured resistance, and the measured voltage at $1\,\text{kHz}$. Since we measure only the magnitude of the voltage, the minus sign is irrelevant here.

$$V_{\text{outer}} = -\left[2\mu_o \pi^2 a^2 l I_{\text{outer}}\right] n_{\text{outer}}^2 f \qquad \Longrightarrow \qquad n_{\text{outer}}^2 = \frac{\text{slope}}{\left[2\mu_o \pi^2 a^2 l I_{\text{outer}}\right]} \tag{5}$$

- □ Is your value reasonable? Geometrically estimate (roughly) the number of turns per layer and the number of layers in your outer coil to verify.

When you are finished

- □ Power off the function generator and multimeter
- □ Straighten up your components and wires, return them to the cart
- □ Turn in a hard copy of your report and plots

Data

f (kHz)	outer coil voltage (V)
1.0	
2.0	
3.0	
4.0	
5.0	
6.0	
7.0	
8.0	
9.0	
10.0	

function generator output V_o ________________ V

frequency ________________ Hz

outer coil resistance R_o ________________ Ω

outer coil current $I_o = V_o / R_o$ ________________ A

turns on inner coil n_{inner} ________________

inner coil diameter $2a$ ________________ m

inner coil length l ________________ m

outer coil diameter $2b$ ________________ m