

Quiz 4: Solution

1. Capacitors connected in parallel *must always* have the same:

- Charge
- Potential difference
- Energy stored
- None of the above

Because capacitors in parallel are connected to their voltage source at the same points, they must have the same potential difference.

2. An ideal parallel plate capacitor is completely charged up, and then disconnected from a battery. The plates are then pulled a small distance apart. What happens to the capacitance, C , and charge stored, Q , respectively?

- decreases; increases
- increases; decreases
- decreases; stays the same
- stays the same; decreases

Since the capacitor is disconnected from the battery, the amount of charge stored cannot change - there is nowhere for the charges to go! However, the capacitance depends on the spacing of the plates d - $C = \epsilon_0 A/d$, where A is the area of the plates. If we slightly increase d , the capacitance must decrease.

3. An isolated conductor has a surface electric potential of 10 Volts. An electron on the surface is moved by 0.1 m. How much work must be done to move the charge? (e is the electron charge.)

- $1e$ Joules
- $0.1e$ Joules
- $10e$ Joules
- 0

Remember, the electric potential is constant on an isolated conductor, and therefore two points on the surface have zero potential difference. Thus the work required, $W = -q\Delta V = 0$. Charges can move freely without change in energy on the surface of an isolated ideal conductor.

4. A parallel plate capacitor is shrunk by a factor of two in every dimension - the separation between the plates, as well as the plates' length and width are all two times smaller. If the original capacitance is C_0 , what is the capacitance after all dimensions are shrunk?

- $2C_0$
- $\frac{1}{2}C_0$
- $4C_0$
- $\frac{1}{4}C_0$

Let us say the original capacitor has plates of area A a distance d apart. Then

$$C_0 = \frac{\epsilon_o A}{d}$$

If we halve every linear dimension, the spacing d becomes $d/2$, but the area A becomes $A/4$. If we had circular plates and halved the radius, the area decreases by four times; if we had square plates and halved the length of each side, area also decreases by four times, and so it is for any reasonable two-dimensional set of plates. Thus, the new capacitance is

$$C = \frac{\epsilon_o A/4}{d/2} = \frac{\epsilon_o A}{d} \frac{1}{2} = \frac{1}{2} C_0$$

5. A capacitor with air between its plates is charged to 120 V and then disconnected from the battery. When a piece of glass is placed between the plates, the voltage across the capacitor drops to 30 V. What is the dielectric constant of the glass? (Assume the glass completely fills the space between the plates.)

- 4
- 2
- 1/4
- 1/2

Without the piece of glass, our capacitor has a value we'll call C_o . The charge stored on the capacitor is $Q = C_o \Delta V_o$ when the initial potential difference is $\Delta V_o = 120$ V. The piece of glass acts as a dielectric, whose dielectric constant we will call κ .

Since the battery was disconnected, after inserting the piece of glass the total amount of charge Q stays the same - there is no source for additional charge to enter the capacitor, nor anywhere for charges to leave. Thus, since the potential difference ΔV has decreased, and the product of capacitance and potential difference must be constant (Q), the capacitance must be greater after we insert the glass piece. From our discussion of dielectrics, we know that the capacitance is greater by a factor κ after inserting the glass piece (κ is always greater than 1). We can set the initial amount of charge before inserting the glass equal to the final charge after inserting the glass, and solve for κ :

$$\begin{aligned} Q &= C_o \Delta V_o = \kappa C_o \Delta V \\ \implies \Delta V_o &= \kappa \Delta V \\ \implies \kappa &= \frac{\Delta V_o}{\Delta V} = \frac{120}{30} = 4 \end{aligned}$$