Back of the envelope: estimation & orders of magnitude

how to solve problems?

- write down the answer (or something close)
- break the problem into estimable chunks
- actually solve it, if accuracy is required

writing down the answer

- i.e., come up with a reasonably close solution
- do you need more than that? often not!

example

- it is about 200mi to Atlanta
- how long will it take to drive?

- 50-60mph average, few stops ... ~3-4 hours
- probably good enough for planning!

ballpark ...

- too high, too low, or just right?

- e.g., buying a new game -
  - think ~$100 is reasonable
  - see it for $30: buy right now
  - see it for $300: no way
  - see it for $100: now have to think harder ...
- only the power of 10 really matters!

break up the problem

- too hard to guess the answer within 10x?
- break it up into estimable pieces
  - upper and lower bounds on the answer
  - e.g., people in this room: more than 10, less than 100.
  - *what is a good guess between these two?*

Average power of ten

- *geometric mean* is a good ‘average’ guess
    - better than mean of 55
    - nth root of the product of n numbers
    - same factor away from upper & lower bounds
- powers of 10: average exponents. $10^{1/2} \sim 3$
    - $10^{1+2} = 10^{1.5} \sim (3)(10) \sim 30$
- otherwise, average prefactors & exponents
    - e.g., 3 & 100 = (3×10^0) & (1×10^2) ~ 15

example: stack of lottery tickets

- chances are 1 in 100 million
- how tall is a stack of 100 million tickets?
  - tall building, 100m?
  - small mountain, 1000m?
  - Everest, 10^4m?
  - atmosphere, 10^5m?
  - NYC-Chicago, 10^6m?

what do we know?

- 10^8 possible tickets
- thickness? hard to reliably estimate ...
- ream of paper: 500 sheets ~ 3cm
- stack of cards: 52 cards ~ 1cm
  - this is probably closer ...

$$t = \frac{1\,\mathrm{cm}}{50\,\mathrm{tickets}} = 2 \times 10^{-4}\,\mathrm{m/ticket}$$

total ...

$$T = 2 \times 10^{-4}\,\frac{\mathrm{m}}{\mathrm{ticket}} \times 10^{8}\,\mathrm{tickets} \sim 2 \times 10^{4}\,\mathrm{m}$$

- twice as high as Everest or commercial jet
- pick one out of this stack ...
- say we guessed thickness of paper?
  - no big deal
  - factor of 2 or 3 doesn't change the 'character' of the answer

a bit harder ...

- how many airplane flights do Americans take per year?
- solution 1:
  - (number of americans)(flights per year per person)

- ~3×10^8 americans (> 10 million, < 1 billion)
- most: 1 trip (2 flights) per year
- some small % take many more
- # flights should be between 2-4; estimate 3
- N ~ 9×10^8 passengers/year

solution 2

- how many airports? say, 3 per state ~ 150
- airport is open about 16 hours (8am-midnight)
- at most 2 flights per min, 30/hr, ~500 per 16hrs

- airplane holds 50-250 passengers; geom mean ~100

- N ~ (airports)(flights/day)(pass/flight)(days/year)

  ~ (150)(100)(100)(365) ~ 5×10^8 passengers/yr

- factor of 2 agreement! (2005 data: 6.6×10^8)

Classic problem ...

- Fermi: piano tuners in Chicago ...

- many estimations!
  - errors tend to ‘cancel’ (partly psychological?)
  - large leeway for ‘reasonable-sounding’ answers
  - you can ‘rig’ this one before-hand ...

going wrong ...

- you have to at least be able to estimate all quantities!
- e.g., "Drake equation"
- number of extraterrestrial civilizations in our galaxy with which we might come into contact

$$N = R \times f_p \times n_e \times f_l \times f_i \times f_c \times L$$

$$N = R \times f_p \times n_e \times f_l \times f_i \times f_c \times L$$

- R = rate of star formation per year
- f_p = fraction with planets
- n_e = number possibly supporting life
- f_l = fraction developing life at some point
- f_i = fraction developing intelligent life
- f_c = fraction developing tech for comm.
- L = length of time civ releases signals

problem?

- each fraction is 0-1, so the result is also 0-1
  - reasonable looking probability guaranteed!
  - R is big ... almost guaranteed integer result
- other numbers (not R) - wild variation
- none of the prob. terms are known, or can be!
- (nonsense)(nonsense) = ?

- still ... stimulates discussion

Accuracy

- significant figures ...
  - do not imply more precision than you have
  - do not loose information
- These sorts of problems: one significant digit
- apocryphal anecdote: dinosaur skeleton
  - 75 million and 3 years old ...

Specific cases

- 3.14 is always enough for π; 3 is often fine!
- unit conversions ... *rough* non-metric
- sphere's volume and area
  - 1/2 the volume and area of box it came in! (5%)
  - e.g., ball diameter 10cm,

$$V \sim (0.5)(10)(10)(10) \sim 500\text{cm}^2$$

exact is 523cm^2 ... 4% error

units: have them in your head

- 1 meter ~ 3ft
- 1km ~ 0.6mi
- 1cm ~ 0.4in
- 1L ~ 1 quart
- 1kg ~ 2.2lb mass
- 1000kg ~ ton
- 1m/s ~ 2.2mph
- 1 year ~ $\pi \times 10^7$sec

- 1mi ~ 1.6km
- 1in ~ 2.5cm
- 1L = 1000 cm^3
- 1m^3 = 1000L

unit conversion

- or, dimensional analysis

$$1\text{yr} = \left(\frac{365\,\text{days}}{1\,\text{year}}\right)\left(\frac{24\,\text{hr}}{1\,\text{day}}\right)\left(\frac{60\,\text{min}}{1\,\text{hr}}\right)\left(\frac{60\,\text{s}}{1\,\text{min}}\right) = 3.15 \times 10^7\,\text{s}$$

$$1\text{m/s} = \left(\frac{1\,\text{m}}{\text{s}}\right)\left(\frac{1\,\text{km}}{10^3\,\text{m}}\right)\left(\frac{0.6\,\text{mi}}{1\,\text{km}}\right)\left(\frac{60\,\text{min}}{1\,\text{hr}}\right)\left(\frac{60\,\text{s}}{1\,\text{min}}\right) = 2.2\,\text{mph}$$

problem 1

- How many golf balls would it take to circle the Earth at the equator?

hint 1

- What's the diameter of a golf ball, roughly?
- 1 inch = 2.5 cm

hint 2

- What is the circumference of the earth?
- $C = 2\pi R$ (if you remember R)
- 24 time zones (if you know one)

hint 3

- 3 time zones from NY - LA, 24 total
- NY - LA ~ 3000 mi ... good reference

problem 2

- what is the surface area of a towel?
- include the fibers!

hint 1

- how many fibers per square centimeter?

hint 2

- area of a towel? total fibers?

hint 3

- length and thickness of a fiber?

problem 3

- what is the mass of a mole of cats?
- Note $M_{moon}=7x10^{22}kg$, $M_{earth}=6x10^{24}kg$

more problems ...

Guesstimation: Solving the World's Problems on the Back of a Cocktail Napkin (Paperback)

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