PHI25 lab I

uncertainty analysis & statistics

Sciencing

- So you have an idea.
- This idea must be testable ... or it is not science
- So we test it.

- How good is our test? How well did it work?
 - a measure of the result & accuracy
 - does it make any sense? predict something else ...



SCIENTIFIC METHOD

- Make an Observation "What is happening?"
 An Observation is when you notice something in the world around you and decide you want to find out more about it.
- 2. Define the Question "Why is this happening?"
 <u>Defining the Question</u> creates an idea that can be tested using a series of Experiments.
- **3. Form a Hypothesis** "I think this happens because..."

 A <u>Hypothesis</u> is a statement that uses a few observations, without any experimental evidence, to define why something happens.
- 4. Perform Experiments "Let's test my Hypothesis..." An <u>Experiment</u> is a series of tests to see if your Hypothesis is correct or incorrect. For each test, record the data you discover.
- 5. Analyze the Data "Was my Hypothesis right?" <u>Analyzing data</u> takes what you found in your experiments and compares it to your Hypothesis. If needed, perform another Experiment to gather better data.
- 6. Conclusion "Experiments show my hypothesis was..."
 Forming a <u>Conclusion</u> presents the Experimental Data and explains how it proves or disproves your Hypothesis. Often, Scientists will take this Conclusion and perform other Experiments on it to discover new things.

Example

- Does a heavier or lighter object fall faster?
- can't do just one experiment
 - by "chance" one will land first
- do it many times
 - what is the variability?
 - how much is too much?
 - at what point are they basically the same?

Measurements

- we don't do just one
- make a series of measurements, average them
- this should improve accuracy, right?
- how much? when to stop?

need to quantify degree of uncertainty

Today's lab: counting cards

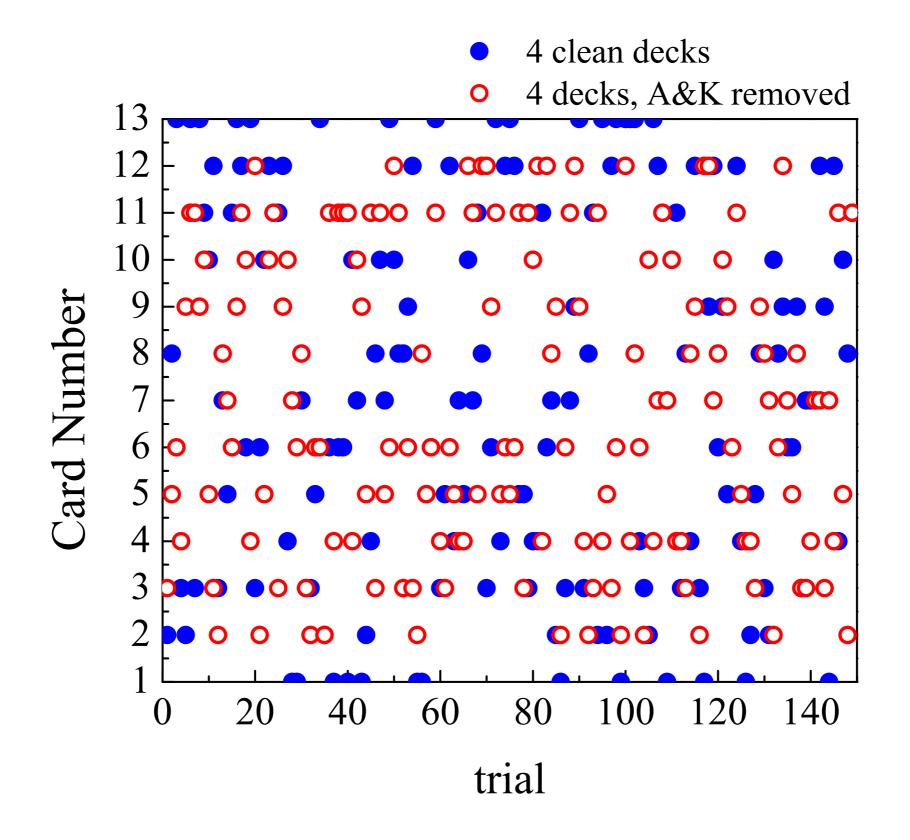
- one measurement vs. many
- how does accuracy improve?
- how to measure accuracy?
- statistical measures of uncertainty & dispersion

 if you don't see the whole deck at once, what can you still say?

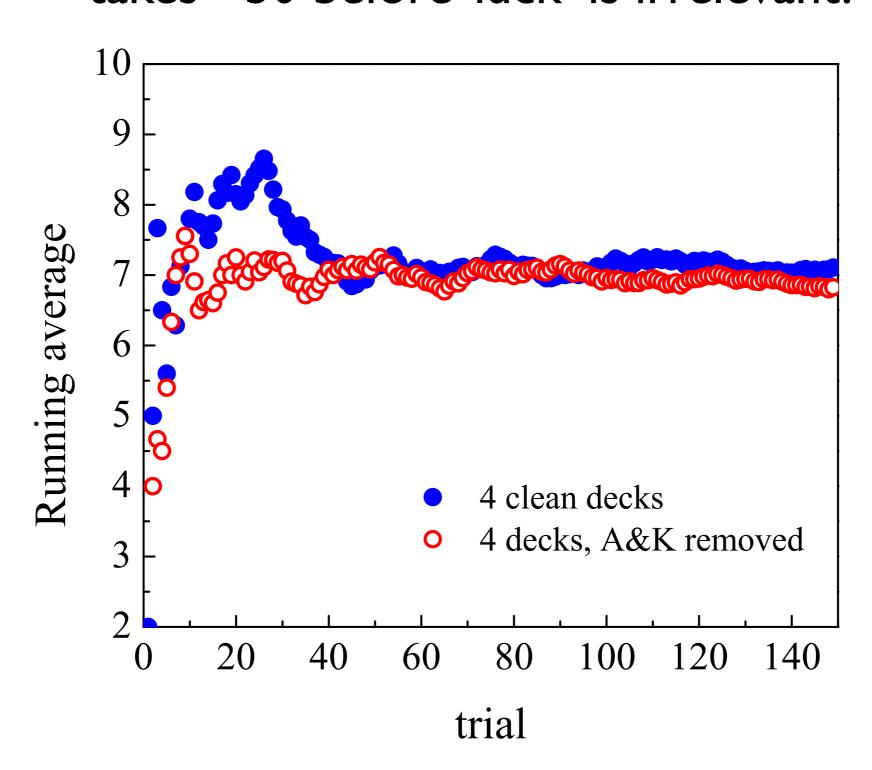
The experiment: picking cards

- give each one a number
- Ace = 1, 2 = 2 ... Jack = 11 ... King = 13
- what is the average card?
 - we expect it must be 7 ...
- what is the spread? how to define this?

150 trials ...



draw I card, record, shuffle, repeat cards have values I-I3, equal number of each average must be 7, if one chooses enough cards takes ~50 before 'luck' is irrelevant!

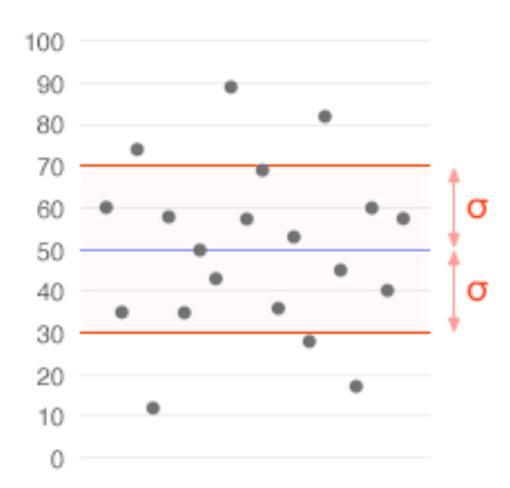


The mean isn't enough. how about the dispersion?

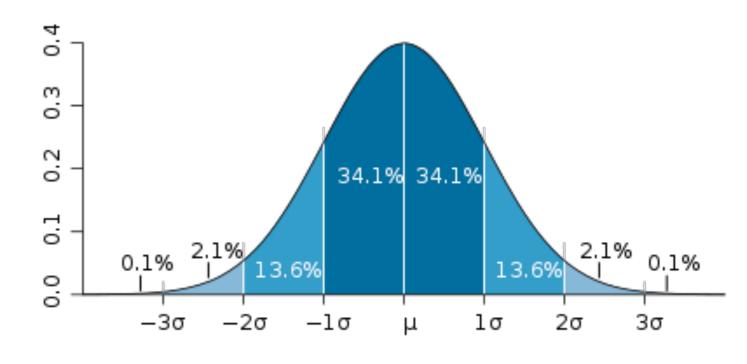
standard deviation is a measure of the variability dispersion in a population or data set

low standard deviation: data tends to lie close to the average (mean)

high standard deviation: data spread over a large range



data set: data clustered about average



many trials: follow a distribution

~68% within +/- 1 standard deviation ~95% within +/- 2 standard deviations ~99.7% within +/- 3 ...

so what?

- knowing the standard deviation tells you
 - if subsequent measurements are outliers
 - what to expect next, on average
 - accuracy of a set of data
 - variability in a large batch
- "six sigma" quality control
 - means one out of 500 million!

so what?

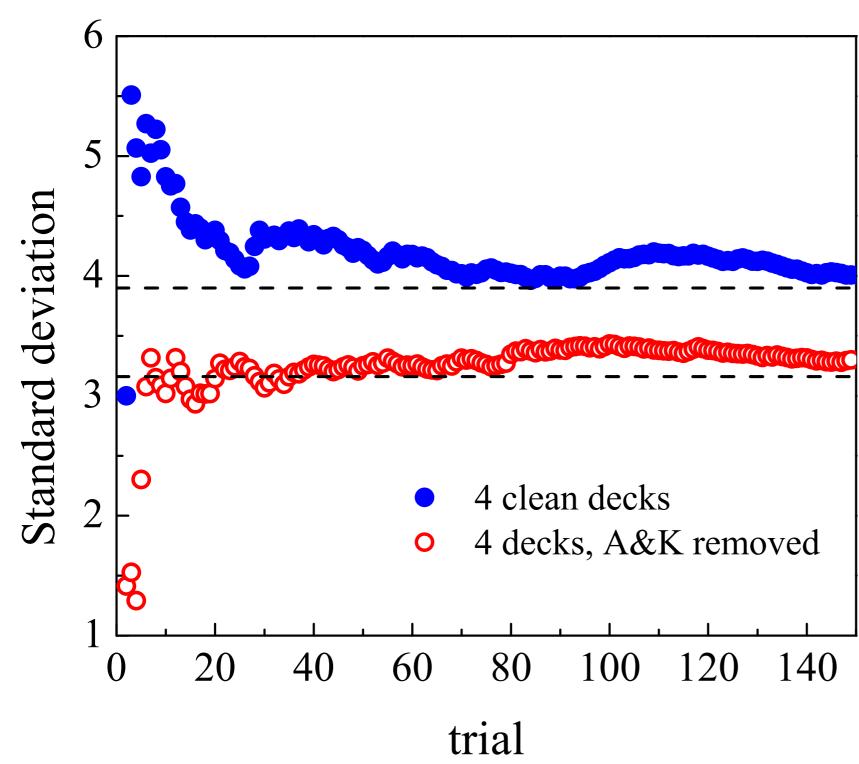
if the mean of the measurements is too far away from the prediction compared to the standard deviation, then the theory being tested probably needs to be revised!

particle physics: 5-sigma standard typical I out of 1.7M chance of false positive

larger signal than that ... probably a new effect!

for the cards?

take out highest and lowest cards, data is more tightly distributed lower standard deviation!



wait, there's more

expect 75% of cards within 2 standard deviations of average

or, 75% are within about 4 cards from the average after 100 trials

or, 75% of cards should be between 3 and Jack (inclusive)

It works!

flip side: we could estimate the distribution of cards without prior knowledge from a run of cards, if long enough, could say what's missing or in excess could take many samples of the deck though (e.g., removing all 7s)

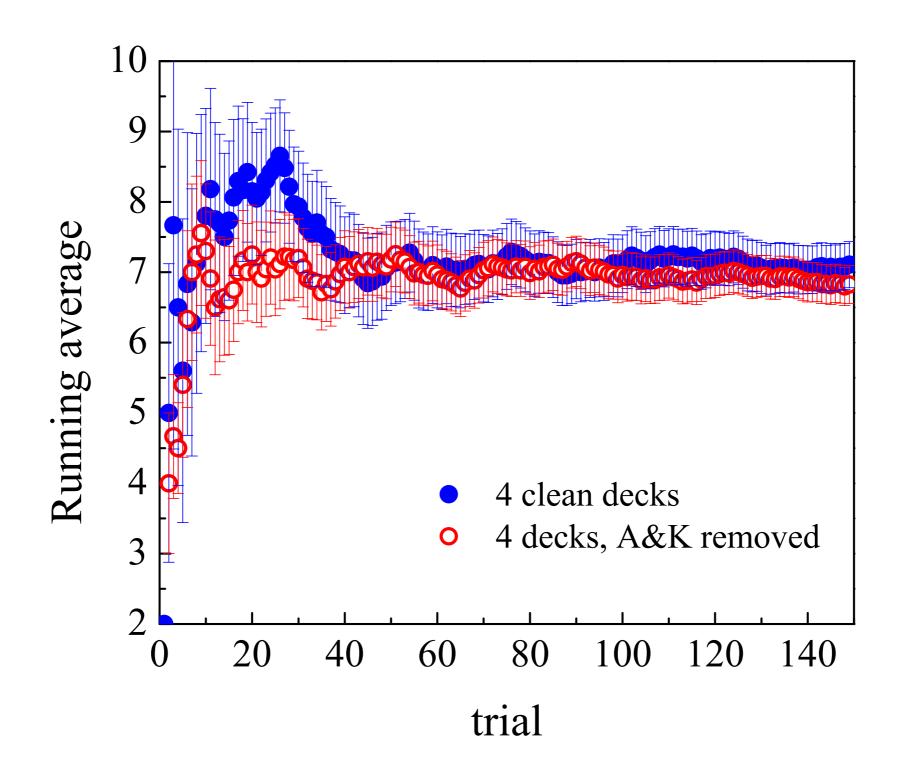
what else?

- standard deviation gives accuracy of averages
- if you do n measurements, average is more accurate for higher n. makes sense!
- uncertainty of the average is standard deviation over the root of the number of measurements

$$\sigma_{\overline{\chi}} = \frac{\sigma}{\sqrt{n}}$$

(best value of
$$x$$
) = $\overline{x} \pm \sigma_{\overline{x}}$

Now we can add error bars to our running averages error bars are large where N is small more data - more accurate average initial differences are not significant!



detailed explanation & examples in lab procedure ... so read it first. really. excel example included too

	draw	card	running average	running standard deviation	
	1	2	2.00		
	2	8	5.00	4.24	
	3	13	7.67	5.51	
	4	3	6.50	5.07	
	5	2	5.60	4.83	
:::	A	В	С	D	E
1	draw	card	running average	running standard deviation	
2	1	2	2.00		
3	2	8	=AVERAGE(\$B\$2:B3)	=STDEV(\$B\$2:B3)	
4	3	13	^ drag down	^ drag down	
5	4	3	6.50	5.07	
6	5	2	5.60	4.83	