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Laboratory 8: rotational inertia

Hypothesis: An object moving predominantly under the influence of gravity (falling, rolling) increases its speed as it falls or rolls, and does so at a constant rate. However, this rate is shape-dependent when an object is smoothly rolling. For example spheres and cylinders should not roll at the same rate, everything else being the same.

Goals: Understanding how the rotational inertia of an object affects rolling motion

1 Equipment

- $\sim 1\,\mathrm{m}$ long rail
- ruler
- objects with 4 shapes (solid sphere, hollow sphere, solid cylinder, hollow cylinder)
- stopwatch (i.e., your phone)
- mechanism to record and plot data

2 Introduction

We have already discussed motion under the influence of gravity alone, which worked fine for sliding and falling. However, when an object is rolling, it behaves as though it has a larger inertia than expected. This is due to the *rotational inertia* of the object. For sliding or falling, the inertia (mass) is a measure of how hard it is to change the straight-line motion of the object. For rotation, the rotational inertia is a measure of how hard it is to change the *rotational* motion of the object. In the case of a rolling object, the resulting motion depends on both inertia and rotational inertia, since a rolling object is both rotating and moving in a straight line. Put another way, compared to an object sliding without friction, for a rolling object work must also be done to rotate the object. If the amount of work that can be done is fixed, a rolling object will have slower straight line motion compared to one sliding without friction, because some of the work must be done to get the rolling object rotating.

In the present case, we will look at differently shaped objects rolling down a ramp of fixed height and angle. If all shapes start from the same height, they all have the same work done on them by gravity by the time they reach the end of the ramp. The larger the rotational inertia, the less work available to accelerate the object down the ramp, and the longer it takes to reach the bottom of the ramp.

3 Rotational Inertia

The rotational inertia (I) of an object depends on both its mass and its characteristic dimension(s) (e.g., radius, length). For instance, for a solid sphere the rotational inertia is $I = \frac{2}{5}mR^2$ where m is the mass of the sphere and R its radius. The expressions for rotational inertia of a few common shapes are tabulated below in Table 1, and generally follow the form $I = k \cdot (\text{mass}) \cdot (\text{dimension})^2$, where k is just a constant called the *shape factor*.

As it turns out, in most cases of interest the dependence on dimension and mass end up canceling out upon completing the calculation, in the same way that the mass of an object doesn't end up mattering when figuring out how long it takes to fall a given distance. Typically, only the shape factor k matters. For instance, in calculating the acceleration of an object sliding without friction down a ramp or rolling down the same ramp, only k is involved. This indicates two important points:

1. Since mass and dimension don't matter, all objects of the same shape accelerate the same way when rolling, *regardless of mass or size.* E.g., all spheres behave the same way.
2. The shape factor tells us how much additional work is required for rolling beyond what is already necessary for falling or sliding

For acceleration down a ramp making an angle θ with respect to the horizontal, we have the following results for frictionless sliding compared to rolling:

$$a_{\text{sliding}} = g \sin\theta \tag{1}$$

$$a_{\text{rolling}} = \frac{a_{\text{sliding}}}{1+k} = \frac{g \sin\theta}{1+k} \tag{2}$$

Don't worry about the $\sin\theta$ part - it is at most equal to 1 for a completely vertical ramp, and zero for a horizontal ramp. More importantly, for a given ramp it is constant. Since the denominator for a_{rolling} is always greater than 1, it means the acceleration for the rolling object is always smaller than that of a sliding object, and the larger the shape factor the lower the acceleration. For a given ramp, $g \sin\theta$ is just a constant factor, so we can compare different shapes. If the object rolls down the ramp through a distance d, since we know that $d = \frac{1}{2}2at^2$ we can find the time it takes to roll to the end of the ramp:

$$a_{\text{rolling}} = \frac{a_{\text{sliding}}}{1+k} = \frac{g\sin\theta}{1+k} \tag{3}$$

$$d = \frac{1}{2}a_{\text{rolling}}t^2 \qquad \text{algebra ensues} \ldots \tag{4}$$

$$\implies \qquad t = \sqrt{\frac{2d}{g\sin\theta}\left(1+k\right)} \tag{5}$$

This looks a bit scary, but keep in mind that the $\frac{2d}{g\sin\theta}$ part is just a constant provided you have the same ramp and the same distance along the ramp for every comparison. Further, if we take the ratio of the time to the bottom for two different shapes, we find

$$\frac{t_{\text{shape 1}}}{t_{\text{shape 2}}} = \sqrt{\frac{1+k_{\text{shape 1}}}{1+k_{\text{shape 2}}}} \tag{6}$$

For instance, the ratio of times for a solid sphere to a solid cylinder should be about 0.966, whereas the ratio of a hollow cylinder to a solid cylinder should be about 1.15. In short, the time it takes an object to roll down a ramp is a measure of its rotational inertia (longer time = larger rotational inertia). Everything else being the same, the ratio of the times for two differently shaped objects on the same ramp should be determined by their shape factors alone.

Table 1: Rotational inertia of various shapes (of mass m and radius R)

Shape	**Rotational Inertia I**	**Shape Factor k**
solid sphere	$\frac{2}{5}mR^2$	$\frac{2}{5}$
hollow sphere	$\frac{2}{3}mR^2$	$\frac{2}{3}$
solid cylinder	$\frac{1}{2}mR^2$	$\frac{1}{2}$
hollow cylinder/hoop	mR^2	1

4 Preliminary questions

Question 1.

A solid sphere, a hollow cylinder, and a solid cylinder roll down a ramp. Rank them in order of fastest to slowest.

Question 2.

If you wanted to use a rapidly rotating object to *store* energy, which shape would store the most energy? (Hint: look up "flywheel".)

Question 3.

Given the difference in shape factors between solid and hollow cylinders, why are car and bicycle wheels/rims completely solid? What other constraints are there aside from rotational inertia?

5 Procedure

The basic idea is to make a ramp of fixed angle, and let the object roll down the ramp through a fixed distance, e.g., from 100 cm to 0 cm. The angle and the distance covered should be the same for every portion of the experiment. Keep in mind that the more variables you can control, and the better you can reproduce your measurements, the better your experiment will be. Be systematic!

Here is a basic outline of a *suggested* procedure, which is certainly lacking in details you will need to fill in to make accurate measurements.

1. Construct a ramp using the rail and items in the classroom. Record the angle of the ramp either by making measurements and using geometry, or by using an app on your phone. The ramp should be very shallow to maximize the time an object needs to roll to the bottom.
2. Put something at the end of the ramp to stop the object.
3. Place your object at the top of the ramp (say, at the 100cm mark) and time how long it takes to roll to the bottom (say, the 0cm mark). Do this at least 5 times and compute the average time for this shape. This average time is you primary data.
4. Repeat this for at least 3 shapes, ideally 4 (hollow cylinder, solid cylinder, hollow sphere, solid sphere).
5. Compare the ratio of the times for different combinations of shapes and compare to the prediction above (Equation 6).

You will need to turn in your raw data and ramp measurements (especially the angle) from this section. This can all be in an Excel/Google Sheets document if you like.

6 Data Analysis

As noted above, compare the ratio of the times for different combinations of shapes and compare to the prediction above (Equation 6). What is the percent difference?

Additionally, given the ramp angle you measured and the distance along the ramp, can you make a concrete prediction for your shapes based on Equation 5? That is given measured d and θ and a shape factor k, what is your predicted time to the bottom for each shape? What is the percentage difference from the average of the measured times? If you are not familiar with how to calculate $\sin\theta$ that is fine, it is straightforward once you know how - please ask one of the instructors for help.

For this part, we would be looking for two tables something like this:

Table 1: (ratio of times for shape 1 / shape 2), (predicted ratio)

Table 2: (shape), (time measured), (time predicted), (percent difference).

7 Things to (minimally) include in your Report

- Raw data and ramp measurements
- Answers to questions in boxes above
- Data and tables mentioned in the Data Analysis section
- Include your conclusion as to whether the hypothesis is supported or not.

8 Format of Report

Your report need not be formal, the format is largely up to you. A single Excel/Google Sheets file is fine if well-organized and labeled, as is e.g., a Word document with graphs/sheets included. Handwritten reports and graphs are also fine. Whatever you are most comfortable with is fine, so long as all the requested information is present and easy to find.