Name & ID ______________________________

University of Alabama
Department of Physics and Astronomy

PH 125 / LeClair   Fall 2014

Exam I

Instructions

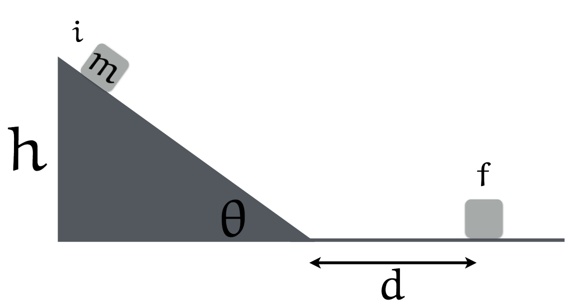
1. **Solve 3 of 5 problems below.** All problems have equal weight.
2. Do your work on separate sheets rather than on the exam.
3. Do not hesitate to ask if you are unsure what a problem is asking for.
4. Show your work for full credit. Significant partial credit will be given.
5. **Symbolic** solutions give more partial credit than purely numerical ones.
6. You are allowed 2 sides of a standard 8.5 x 11 in paper and a calculator.

1. Two trains are 100 km apart on the same track, headed on a collision course towards each other. Both are traveling 50 km per hour. A very speedy bird takes off from the first train and flies at 75 km per hour toward the second train. The bird then immediately turns around and flies back to the first train. Then he flies back to the second train, and repeats the process over and over as the distance between the trains diminishes, always flying at 75 km/h and only stopping instantaneously. How far will he have flown before the trains collide? Neglect air resistance.

2. Prove that a person who can throw a stone to a maximum distance of 64 m over level ground can also throw it so as to clear a wall 24 m high at a distance of 32 m. Neglect air resistance.

3. A particle sliding down a frictionless ramp of angle θ is to attain a given *horizontal* displacement Δx in a minimum amount of time. **(a)** What is the best angle for the ramp? *Hint: find the time in terms of the angle and minimize it. See the formula sheet.* **(b)** What is the minimum time?

4. Starting from rest, a block of mass m slides down an inclined plane of angle $\theta = 45°$ and height $h = 0.25$ m as shown in the figure below. On all surfaces (ramp included), the block experiences a coefficient of kinetic friction $\mu_k = 0.1$. **(a)** What is the speed of the block when it reaches the bottom of the inclined plane? **(b)** For what values of μ_k, in terms of θ, would the block *not* reach the bottom? *Hint: imaginary velocities are unphysical.* **(c)** How far from the end of the ramp d does the mass go before it stops?



5. You're driving your car on the highway at 75 mph, and you notice a sign that says you are 75 miles from your destination. If you continue driving at that speed, you'd be there in an hour. But, you're not going to do that, because then it wouldn't be an interesting problem. When you have driven one mile and you are now 74 miles from your destination, you drop your speed down to 74 mph.

So, you drive that first mile at 75 mph; when you are 74 miles from your destination, you drop your speed down to 74 mph; and then 73 mph, 72 mph ... and so on. Until, finally, you get down to 1 mile from your destination and you're driving at one mile per hour.

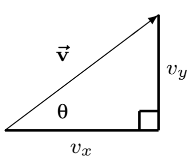
And the question is, if you do this, how long is it going to take you to travel the entire 75 miles? You may write your answer in terms of a summation, a (tedious) numerical evaluation is not necessary.

Formula sheet

$$g = 9.81\,\mathrm{m/s^2}$$
$$0 = ax^2 + bx^2 + c \Longrightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$1\,\mathrm{N} = 1\,\mathrm{kg}\cdot\mathrm{m/s^2}$$

Vectors:

$$\vec{\mathbf{a}} = a_x\,\hat{\boldsymbol{\imath}} + a_y\,\hat{\boldsymbol{\jmath}} + a_z\,\hat{\mathbf{k}}$$
$$\vec{\mathbf{b}} = b_x\,\hat{\boldsymbol{\imath}} + b_y\,\hat{\boldsymbol{\jmath}} + b_z\,\hat{\mathbf{k}}$$
$$|\vec{\mathbf{a}}| = \sqrt{a_x^2 + a_y^2}$$
$$\tan\theta = \frac{a_y}{a_x}$$
$$\vec{\mathbf{a}} + \vec{\mathbf{b}} = (a_x + b_x)\,\hat{\boldsymbol{\imath}} + (a_y + b_y)\,\hat{\boldsymbol{\jmath}} + (a_z + b_z)\,\hat{\mathbf{k}}$$
$$\vec{\mathbf{a}}\cdot\vec{\mathbf{b}} = a_x b_x + a_y b_y + a_z b_z$$
$$|\vec{\mathbf{a}}\cdot\vec{\mathbf{b}}| = |\vec{\mathbf{a}}||\vec{\mathbf{b}}|\cos\varphi$$



$$v_y = |\vec{\mathbf{v}}|\sin\theta$$
$$v_x = |\vec{\mathbf{v}}|\cos\theta$$
$$\tan\theta = \frac{v_y}{v_x}$$

1-D motion:

$$v(t) = \frac{d}{dt}x(t)$$
$$a(t) = \frac{d}{dt}v(t) = \frac{d^2}{dt^2}x(t)$$

const. acc. ↓

$$x_f = x_i + v_{xi}t + \frac{1}{2}a_x t^2$$
$$v_f^2 = v_i^2 + 2a_x\Delta x$$
$$v_f = v_i + at$$

Projectile motion:

$$v_x(t) = v_{ix} = |\vec{\mathbf{v}}_i|\cos\theta$$
$$v_y(t) = |\vec{\mathbf{v}}_i|\sin\theta - gt = v_{iy}\sin\theta - gt$$
$$x(t) = x_i + v_{ix}t$$
$$y(t) = y_i + v_{iy}t - \frac{1}{2}gt^2$$
$$y(x) = x\tan\theta - \frac{gx^2}{2|\vec{\mathbf{v}}_i|^2\cos^2\theta}$$

over level ground:

$$\text{max height } = H = \frac{v_i^2\sin^2\theta_i}{2g}$$
$$\text{Range } = R = \frac{v_i^2\sin 2\theta_i}{g}$$

Force:

$$\sum\vec{\mathbf{F}} = \vec{\mathbf{F}}_{\text{net}} = m\vec{\mathbf{a}}$$
$$\sum F_x = ma_x \qquad \sum F_y = ma_y$$
$$F_{\text{grav}} = mg = \text{weight}$$
$$\vec{\mathbf{F}}_{12} = -\vec{\mathbf{F}}_{21}$$
$$f_s \le \mu_s n$$
$$f_{s,\max} = \mu_s n$$
$$f_k = \mu_k n$$
$$\vec{\mathbf{F}}_{\text{centr.}} = -\frac{mv^2}{r}\,\hat{\mathbf{r}} \quad \text{circular}$$

2-D motion:

$$\vec{\mathbf{r}} = x(t)\,\hat{\boldsymbol{\imath}} + y(t)\,\hat{\boldsymbol{\jmath}}$$
$$x(t) = x_i + v_{ix}t + \frac{1}{2}a_x t^2$$
$$y(t) = y_i + v_{iy}t + \frac{1}{2}a_y t^2$$
$$\vec{\mathbf{v}} = v_x(t)\,\hat{\boldsymbol{\imath}} + v_y(t)\,\hat{\boldsymbol{\jmath}}$$
$$v_x(t) = \frac{dx}{dt} = v_{xi} + a_x t$$
$$v_y(t) = \frac{dy}{dt} = v_{yi} + a_y t$$
$$\vec{\mathbf{a}} = a_x(t)\,\hat{\boldsymbol{\imath}} + a_y(t)\,\hat{\boldsymbol{\jmath}}$$
$$a_x(t) = \frac{dv_x}{dt} = \frac{d^2x}{dt^2}$$
$$\vec{\mathbf{a}}_c = -\frac{v^2}{r}\,\hat{\mathbf{r}} \quad \text{circ.}$$
$$T = \frac{2\pi r}{v} \quad \text{circ.}$$

Math:

$$ax^2 + bx^2 + c = 0 \Longrightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$\sin\alpha \pm \sin\beta = 2\sin\frac{1}{2}(\alpha \pm \beta)\cos\frac{1}{2}(\alpha \mp \beta)$$
$$\cos\alpha \pm \cos\beta = 2\cos\frac{1}{2}(\alpha + \beta)\cos\frac{1}{2}(\alpha - \beta)$$
$$c^2 = a^2 + b^2 - 2ab\cos\theta_{ab}$$
$$\frac{d}{dx}\sin^n(ax) = an\cos(ax)\sin^{n-1}(ax)$$
$$\frac{d}{dx}\cos^n(ax) = -an\sin(ax)\cos^{n-1}((ax))$$
$$\frac{d}{dx}\frac{1}{\sqrt{\sin(ax)}} = -\frac{a\cos(ax)}{2\sin^{3/2}(ax)} \quad \text{oddly specific information}$$

Power	Prefix	Abbreviation
10^{-12}	pico	p
10^{-9}	nano	n
10^{-6}	micro	µ
10^{-3}	milli	m
10^{-2}	centi	c
10^{3}	kilo	k
10^{6}	mega	M
10^{9}	giga	G
10^{12}	tera	T