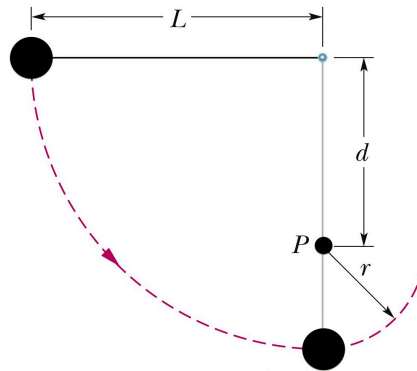


Exam II

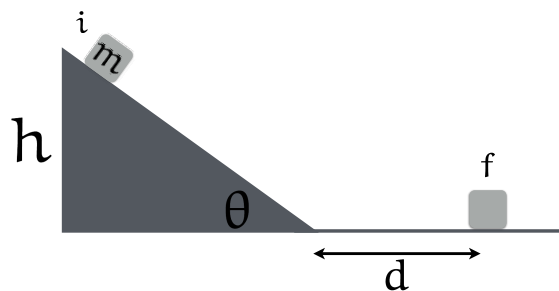
Instructions

1. Solve 3 of 4 problems below. All problems have equal weight.
2. You must answer all parts of multi-part questions for full credit.
3. Show your work for full credit. Significant partial credit will be given.
4. You are allowed a calculator and 2 sides of 8.5 x 11in paper with notes.
5. You have 50 min. You will not need the calculator.

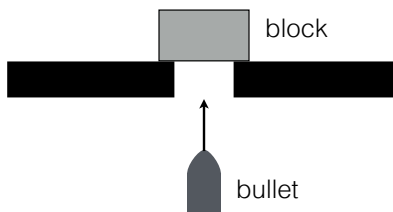
1. The string in the figure below, of length L , has a ball of mass m attached to one end, and is fixed at the other end. The distance from the fixed end to a fixed peg at point P is d . When the initially stationary ball is released with the string horizontal as shown, it will swing along the dashed arc. What is its speed when it reaches (a) its lowest point and (b) its highest point *after* the string catches on the peg?



2. Starting from rest, a block of mass m slides down an inclined plane of angle θ and height h as shown in the figure below. While the ramp surface is frictionless, the block experiences a coefficient of kinetic friction μ_k on the flat surface. (a) What is the speed of the block when it reaches the bottom of the inclined plane? (b) How far from the end of the ramp d does the mass go before it stops?



3. In the figure below, a bullet of mass m moving directly upward at velocity v_i strikes and passes through the center of mass of a block of mass M initially at rest. The bullet emerges from the block moving directly upward, but now with velocity $\frac{1}{2}v_i$. To what maximum height does the block then rise above its initial position? Assume that the block does not lose any mass.



4. During spring semester at MIT, residents of the parallel buildings of the East Campus dorms battle one another with large catapults that are made with surgical hose mounted on a window frame. A balloon filled with dyed water is placed in a pouch attached to the hose, which is then stretched through the width of the room. Assume that the stretching of the hose obeys Hooke's law (i.e., $F = kx$) with a spring constant of k . If the hose is stretched by a distance d and then released, how much work does the force from the hose do on the balloon in the pouch by the time the hose reaches its relaxed length?

Formula sheet

$$g = 9.81 \text{ m/s}^2$$

$$0 = ax^2 + bx + c \implies x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$1 \text{ N} = 1 \text{ kg} \cdot \text{m/s}^2$$

$$1 \text{ J} = 1 \text{ kg} \cdot \text{m}^2/\text{s}^2 = 1 \text{ N} \cdot \text{m}$$

1-D motion:

$$v(t) = \frac{d}{dt}x(t)$$

$$a(t) = \frac{d}{dt}v(t) = \frac{d^2}{dt^2}x(t)$$

const. acc. ↓

$$x_f = x_i + v_{xi}t + \frac{1}{2}a_x t^2$$

$$v_f^2 = v_i^2 + 2a_x \Delta x$$

$$v_f = v_i + at$$

Projectile motion:

$$v_x(t) = v_{ix} = |\vec{v}_i| \cos \theta$$

$$v_y(t) = |\vec{v}_i| \sin \theta - gt = v_{iy} \sin \theta - gt$$

$$x(t) = x_i + v_{ix}t$$

$$y(t) = y_i + v_{iy}t - \frac{1}{2}gt^2$$

over level ground:

$$\text{max height} = H = \frac{v_i^2 \sin^2 \theta_i}{2g}$$

$$\text{Range} = R = \frac{v_i^2 \sin 2\theta_i}{g}$$

Force:

$$\sum \vec{F} = \vec{F}_{\text{net}} = m\vec{a} = \frac{d\vec{p}}{dt}$$

$$\sum F_i = ma_i \quad \text{by component}$$

$$\vec{F}_c = \sum F_r = -\frac{mv^2}{r} \hat{r}$$

$$f_k = \mu_k n$$

$$F_s = -kx$$

$$F_g = -mg$$

2-D motion:

$$\vec{r} = x(t)\hat{i} + y(t)\hat{j}$$

$$x(t) = x_i + v_{ix}t + \frac{1}{2}a_x t^2$$

$$y(t) = y_i + v_{iy}t + \frac{1}{2}a_y t^2$$

$$\vec{a}_c = -\frac{v^2}{r} \hat{r} \quad \text{circ.}$$

$$T = \frac{2\pi r}{v} \quad \text{circ.}$$

Work-Energy:

$$K = \frac{1}{2}mv^2 = \frac{p^2}{2m}$$

$$\Delta K = K_f - K_i = W$$

$$W = \int F(x) dx = -\Delta U$$

$$U_g(y) = mgy$$

$$U_s(x) = \frac{1}{2}kx^2$$

$$F = -\frac{dU(x)}{dx}$$

$$K_i + U_i = K_f + U_f + W_{\text{ext}} = K_f + U_f + \int F_{\text{ext}} dx$$

Momentum, etc.:

$$x_{\text{com}} = \frac{1}{M_{\text{tot}}} \sum_{i=1}^n m_i x_i = \frac{m_1 x_1 + m_2 x_2 + \dots + m_n x_n}{m_1 + m_2 + \dots + m_n}$$

$$v_{\text{com}} = \frac{1}{M_{\text{tot}}} \sum_{i=1}^n m_i v_i = \frac{m_1 v_1 + m_2 v_2 + \dots + m_n v_n}{m_1 + m_2 + \dots + m_n}$$

$$F_{\text{net}} = M_{\text{tot}} a_{\text{com}} = \frac{dp}{dt} \quad p_{\text{tot}} = M_{\text{tot}} v_{\text{com}}$$

$$\vec{p} = m\vec{v} \quad \Delta p = p_f - p_i = F_{\text{avg}} \Delta t \quad (\Delta p = 0 \text{ for isolated system})$$

$$v_{1f} = \left(\frac{m_1 - m_2}{m_1 + m_2} \right) v_{1i} + \left(\frac{2m_2}{m_1 + m_2} \right) v_{2i} \quad \text{1D elastic}$$

$$v_{2f} = \left(\frac{2m_1}{m_1 + m_2} \right) v_{1i} + \left(\frac{m_2 - m_1}{m_1 + m_2} \right) v_{2i} \quad \text{1D elastic}$$

$$v_f = \frac{m_1 v_{1i} + m_2 v_{2i}}{m_1 + m_2} \quad \text{1D inelastic}$$

Power	Prefix	Abbreviation
10 ⁻¹²	pico	p
10 ⁻⁹	nano	n
10 ⁻⁶	micro	μ
10 ⁻³	milli	m
10 ⁻²	centi	c
10 ³	kilo	k
10 ⁶	mega	M
10 ⁹	giga	G
10 ¹²	tera	T