Name & ID ____

University of Alabama
Department of Physics and Astronomy

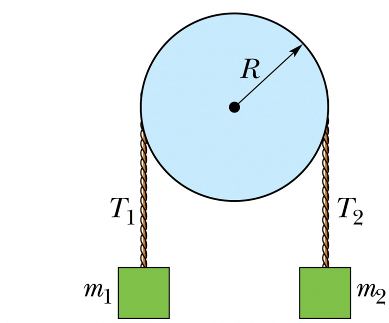
PH 125 / LeClair   Fall 2014

Exam III

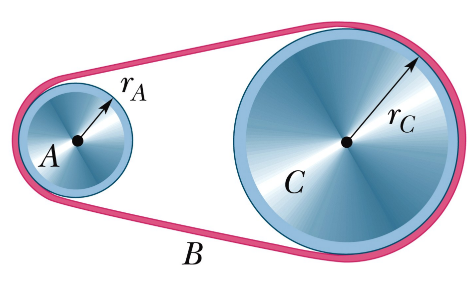
Instructions

1. **Solve 3 of 4 problems below.** All problems have equal weight.
2. You must answer all parts of multi-part questions for full credit.
3. Show your work for full credit. Significant partial credit will be given.
4. You are allowed a calculator and 2 sides of 8.5 x 11in paper with notes.
5. You have 50 min.

1. In the figure below, block 1 has mass m_1, block 2 has mass m_2 (with $m_2 > m_1$), and the pulley (a solid disc), which is mounted on a horizontal axle with negligible friction, has radius R and mass M. When released from rest, block 2 falls a distance d in t seconds without the cord slipping on the pulley. **(a)** What are the magnitude of the accelerations of the blocks? **(b)** What is T_1? **(c)** What is T_2? **(d)** What is the pulley's angular acceleration? The moment of inertia of a solid disc is $I = \frac{1}{2}MR^2$.



2. A flywheel rotating freely on a shaft is suddenly coupled by means of a drive belt to a second flywheel sitting on a parallel shaft (see figure below). The initial angular velocity of the first flywheel is ω, that of the second is zero. The flywheels are uniform discs of masses M_a and M_c with radii R_a and R_c respectively. The moment of inertia of a solid disc is $I = \frac{1}{2}MR^2$. The drive belt is massless and the shafts are frictionless. **(a)** Calculate the final angular velocity of each flywheel. **(b)** Calculate the kinetic energy lost during the coupling process. *Hint: if the belt does not slip, the* **linear** *speeds of the two rims must be equal.*



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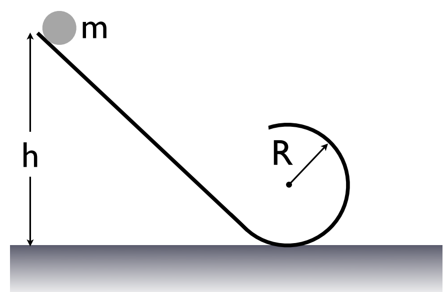
3. A solid sphere, a solid cylinder, and a thin-walled pipe, all of mass m, roll smoothly along identical loop-the-loop tracks when released from rest along the straight section (see figure below). The circular loop has radius R, and the sphere, cylinder, and pipe have radius $r \ll R$ (i.e., the size of the objects may be neglected when compared to the other distances involved). If $h = 2.8R$, which of the objects will make it to the top of the loop? Justify your answer with an explicit calculation. The moments of inertia for the objects are listed below.

$$I = \begin{cases} \frac{2}{5}mr^2 & \text{sphere} \\ \frac{1}{2}mr^2 & \text{cylinder} \\ mr^2 & \text{pipe} \end{cases} \tag{1}$$

Hint: consider a single object with $I = kmr^2$ to solve the general problem, and evaluate these three special cases only at the end.



4. The rotational inertia (moment of inertia) of a collapsing spinning star drops to $\frac{1}{3}$ its initial value. What is the ratio of the new rotational kinetic energy to the initial rotational kinetic energy?

Formula sheet

$$g = 9.81\,\mathrm{m/s^2}$$
$$1\,\mathrm{N} = 1\,\mathrm{kg}\cdot\mathrm{m/s^2}$$
$$1\,\mathrm{J} = 1\,\mathrm{kg}\cdot\mathrm{m^2/s^2} = 1\,\mathrm{N}\cdot\mathrm{m}$$

Math:

$$ax^2 + bx^2 + c = 0 \Longrightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$\sin\alpha \pm \sin\beta = 2\sin\frac{1}{2}(\alpha \pm \beta)\cos\frac{1}{2}(\alpha \mp \beta)$$
$$\cos\alpha \pm \cos\beta = 2\cos\frac{1}{2}(\alpha + \beta)\cos\frac{1}{2}(\alpha - \beta)$$
$$c^2 = a^2 + b^2 - 2ab\cos\theta_{ab}$$
$$\frac{d}{dx}\sin ax = a\cos ax \qquad \frac{d}{dx}\cos ax = -a\sin ax$$
$$\int \cos ax\,\mathrm{dx} = \frac{1}{a}\sin ax \qquad \int \sin ax\,\mathrm{dx} = -\frac{1}{a}\cos ax$$
$$\sin\theta \approx \theta \qquad \text{small } \theta \qquad \cos\theta \approx 1 - \frac{1}{2}\theta^2$$

1-D motion:

$$v(t) = \frac{d}{dt}x(t)$$
$$a(t) = \frac{d}{dt}v(t) = \frac{d^2}{dt^2}x(t)$$

const. acc. ↓

$$x_f = x_i + v_{xi}t + \frac{1}{2}a_x t^2$$
$$v_f^2 = v_i^2 + 2a_x\Delta x$$
$$v_f = v_i + at$$

Projectile motion:

$$v_x(t) = v_{ix} = |\vec{\mathbf{v}}_i|\cos\theta$$
$$v_y(t) = |\vec{\mathbf{v}}_i|\sin\theta - gt = v_{iy}\sin\theta - gt$$
$$x(t) = x_i + v_{ix}t$$
$$y(t) = y_i + v_{iy}t - \frac{1}{2}gt^2$$

over level ground:

$$\text{max height } = H = \frac{v_i^2\sin^2\theta_i}{2g}$$
$$\text{Range } = R = \frac{v_i^2\sin 2\theta_i}{g}$$

Force:

$$\sum \vec{\mathbf{F}} = \vec{\mathbf{F}}_{\text{net}} = m\vec{\mathbf{a}} = \frac{d\vec{\mathbf{p}}}{dt}$$
$$\sum F_i = ma_i \quad \text{by component}$$
$$\vec{\mathbf{F}}_c = \sum F_{\mathrm{r}} = -\frac{mv^2}{r}\,\hat{\mathbf{r}}$$
$$f_k = \mu_k n$$
$$F_s = -kx$$
$$F_g = -mg$$

Work-Energy:

$$K = \frac{1}{2}mv^2 = \frac{p^2}{2m}$$
$$\Delta K = K_f - K_i = W$$
$$W = \int F(x)\,dx = -\Delta U$$
$$U_g(y) = mgy$$
$$U_s(x) = \frac{1}{2}kx^2$$
$$F = -\frac{dU(x)}{dx}$$
$$K_i + U_i = K_f + U_f + W_{\text{ext}} = K_f + U_f + \int F_{\text{ext}}\,dx$$

Momentum, etc.:

$$x_{\text{com}} = \frac{1}{M_{\text{tot}}}\sum_{i=1}^{n} m_i x_i = \frac{m_1x_1 + m_2x_2 + \dots m_nx_n}{m_1 + m_2 + \dots m_n}$$
$$v_{\text{com}} = \frac{1}{M_{\text{tot}}}\sum_{i=1}^{n} m_i v_i = \frac{m_1v_1 + m_2v_2 + \dots m_nv_n}{m_1 + m_2 + \dots m_n}$$
$$F_{\text{net}} = M_{\text{tot}}a_{\text{com}} = \frac{dp}{dt} \qquad p_{\text{tot}} = M_{\text{tot}}v_{\text{com}}$$
$$\vec{\mathbf{p}} = m\vec{\mathbf{v}} \qquad \Delta p = p_f - p_i = F_{\text{avg}}\Delta t \qquad (\Delta p = 0 \text{ for isolated system})$$

Rotation: we use radians

$$s = \theta r \quad \leftarrow \text{arclength}$$
$$\omega = \frac{d\theta}{dt} = \frac{v}{r} \qquad \alpha = \frac{d\omega}{dt}$$
$$a_t = \alpha r \quad \text{tangential} \qquad a_r = \frac{v^2}{r} = \omega^2 r \quad \text{radial}$$
$$I = \sum_i m_i r_i^2 \Rightarrow \int r^2\,dm = kmr^2$$
$$I_z = I_{com} + md^2 \quad \text{axis } z \text{ parallel, dist } d$$
$$\tau_{net} = \sum \vec{\boldsymbol{\tau}} = I\vec{\boldsymbol{\alpha}} = \frac{d\vec{\mathbf{L}}}{dt}$$
$$\vec{\boldsymbol{\tau}} = \vec{\mathbf{r}} \times \vec{\mathbf{F}} \qquad |\vec{\boldsymbol{\tau}}| = rF\sin\theta_{rF}$$
$$\vec{\mathbf{L}} = \vec{\mathbf{r}} \times \vec{\mathbf{p}} = I\vec{\boldsymbol{\omega}}$$
$$K = \frac{1}{2}I\omega^2 = L^2/2I$$
$$\Delta K = \frac{1}{2}I\omega_f^2 - \frac{1}{2}I\omega_i^2 = W = \int \tau\,d\theta$$
$$P = \frac{dW}{dt} = \tau\omega$$

Vectors:

$$|\vec{\mathbf{F}}| = \sqrt{F_x^2 + F_y^2} \quad \text{magnitude}$$
$$\theta = \tan^{-1}\left[\frac{F_y}{F_x}\right] \quad \text{direction}$$
$$\vec{\mathbf{a}}\cdot\vec{\mathbf{b}} = a_xb_x + a_yb_y + a_zb_z = \sum_{i=1}^{n} a_ib_i = |\vec{\mathbf{a}}||\vec{\mathbf{b}}|\cos\theta$$
$$\vec{\mathbf{a}}\times\vec{\mathbf{b}} = \begin{vmatrix} \hat{\mathbf{x}} & \hat{\mathbf{y}} & \hat{\mathbf{z}} \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix} \qquad |\vec{\mathbf{a}}\times\vec{\mathbf{b}}| = |\vec{\mathbf{a}}||\vec{\mathbf{b}}|\sin\theta$$