Name & ID

PH125 Exam 4

Instructions

1. **Solve any 3 problems below.** All problems have equal weight.
2. You must answer all parts of multi-part questions for full credit.
3. Show your work for full credit. Significant partial credit will be given.
4. You are allowed a calculator and 2 sides of 8.5 x 11in paper with notes.
5. You have 50 min.

1. *Harmonic motion.* **(a)** A mass hanging from the end of a spring has a period of 1.0 s. If the mass is doubled, what is the new period? **(b)** The position of a particle moving in simple harmonic motion is given by $x(t) = 2\cos(50t)$ where x is in meters and t is in seconds. What is maximum velocity of the particle in m/s?

2. An aircraft door closes by pushing it inside the airplane first. We will assume $P = 0$ outside the aircraft, and $P = 0.9$ atm inside during flight. If the sealing surface of the door is 5 cm wide all around the door, and the door's outer dimensions are 2 m by 0.7 m, what is the approximate total force required to open the door while in flight? One atmosphere is 1.01×10^5 Pa.

3. Suppose that you release a ball from rest at a depth of 0.6 m below the surface of a pool of water. If the density of the ball is 0.3 times that of water and if the drag force on the ball from the water is negligible, how high above the water surface will the ball shoot as it emerges from the water? (Neglect any transfer of energy to the splashing and waves produced by the emerging ball.)

4. The fastest possible rate of rotation of a planet is that for which the gravitational force on material at the equator just barely provides the centripetal force needed for the rotation. Show that the corresponding shortest period of rotation is

$$T = \sqrt{\frac{3\pi}{G\rho}}$$

where ρ is the uniform density (mass per unit volume) of the spherical planet. The volume of a sphere is $\frac{4}{3}\pi r^3$, where r is the radius of the sphere.

5. The period of the earth's rotation about the sun is 365.256 days. It would be more convenient to have a period of exactly 365 days. How should the mean distance from the sun be changed to correct this anomaly?

Numbers & math:

$$g = 9.81\,\mathrm{m/s^2} \quad G = 6.67\times10^{-11}\,\mathrm{N\,m^2/kg^2}$$

$$M_e = 5.972\times10^{24}\,\mathrm{kg} \quad R_e = 6.371\times10^{6}\,\mathrm{m} \quad \leftarrow \text{earth}$$

$$M_s = 1.9891\times10^{30}\,\mathrm{kg} \quad \leftarrow \text{sun}$$

$$\text{sphere} \quad V = \frac{4}{3}\pi r^3 \qquad M = \rho V$$

$$ax^2 + bx^2 + c = 0 \Longrightarrow x = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$\sin\alpha \pm \sin\beta = 2\sin\frac{1}{2}(\alpha\pm\beta)\cos\frac{1}{2}(\alpha\mp\beta)$$

$$\cos\alpha \pm \cos\beta = 2\cos\frac{1}{2}(\alpha+\beta)\cos\frac{1}{2}(\alpha-\beta)$$

$$c^2 = a^2 + b^2 - 2ab\cos\theta_{ab}$$

$$\frac{d}{dx}\sin ax = a\cos ax \qquad \frac{d}{dx}\cos ax = -a\sin ax$$

$$\int \cos ax\,\mathrm{dx} = \frac{1}{a}\sin ax \qquad \int \sin ax\,\mathrm{dx} = -\frac{1}{a}\cos ax$$

$$\sin\theta\approx\theta \qquad \cos\theta \approx 1-\frac{1}{2}\theta^2 \qquad \text{small } \theta$$

Vectors:

$$|\vec{\mathbf{F}}| = \sqrt{F_x^2+F_y^2} \quad \text{magnitude}$$

$$\theta = \tan^{-1}\left[\frac{F_y}{F_x}\right] \quad \text{direction}$$

$$\vec{\mathbf{a}}\cdot\vec{\mathbf{b}} = a_xb_x + a_yb_y + a_zb_z = \sum_{i=1}^{n} a_ib_i = |\vec{\mathbf{a}}||\vec{\mathbf{b}}|\cos\theta$$

1-D motion:

$$v(t) = \frac{\mathrm{d}}{\mathrm{d}t}x(t) \qquad a(t) = \frac{\mathrm{d}}{\mathrm{d}t}v(t) = \frac{\mathrm{d}^2}{\mathrm{d}t^2}x(t)$$

const. acc.

$$x_f = x_i + v_{xi}t + \frac{1}{2}a_xt^2$$

$$v_{xf}^2 = v_{xi}^2 + 2a_x\Delta x$$

$$v_f = v_i + at$$

Force:

$$\Sigma\vec{\mathbf{F}} = \vec{\mathbf{F}}_{\mathrm{net}} = m\vec{\mathbf{a}} \qquad \Sigma F_x = ma_x \qquad \Sigma F_y = ma_y$$

$$F_{\mathrm{grav}} = mg = \text{weight}$$

$$\vec{\mathbf{F}}_{12} = -\vec{\mathbf{F}}_{21}$$

$$\vec{\mathbf{F}} = \frac{d\vec{\mathbf{p}}}{dt} \approx \frac{\Delta(m\vec{\mathbf{v}})}{\Delta t} \quad \text{direction} \ldots$$

$$f_s \le \mu_s n \qquad f_k = \mu_k n$$

$$\vec{\mathbf{F}}_c = -\frac{mv^2}{r}\,\hat{\mathbf{r}}$$

Work-Energy:

$$K = \frac{1}{2}mv^2 = \frac{p^2}{2m}$$

$$\Delta K = K_f - K_i = W$$

$$W = \int F(x)\,dx = -\Delta U$$

$$U_g(y) = mgy$$

$$U_s(x) = \frac{1}{2}kx^2$$

$$F = -\frac{dU(x)}{dx}$$

$$K_i + U_i = K_f + U_f + W_{\mathrm{ext}} = K_f + U_f + \int F_{\mathrm{ext}}\,dx$$

Gravitation:

$$\vec{\mathbf{F}}_{12} = -\frac{Gm_1m_2}{r^2}\,\hat{\mathbf{r}}_{12} = -\frac{dU_g}{dr}\,\hat{\mathbf{r}}$$

$$g = \frac{GM_e}{R_e^2}$$

$$U_g(r) = -\int F(r)\,dr = \frac{GMm}{r}$$

$$K + U_g \ge 0 \quad \text{escape} \qquad K + U_g < 0 \quad \text{bound}$$

$$\frac{dA}{dt} = \frac{1}{2}r^2\omega = \frac{L}{2m} \qquad T^2 = \left(\frac{4\pi^2}{GM}\right)r^3$$

$$E_{\mathrm{orbit}} = \frac{-GMm}{2a} \quad \text{elliptical; } a\to r \text{ for circular}$$

Oscillations:

$$T = \frac{1}{f} \quad \omega = \frac{2\pi}{T} = 2\pi f$$

$$x(t) = x_m\cos(\omega t + \varphi)$$

$$a = \frac{d^2x}{dt^2} = -\omega^2 x$$

$$\omega = \sqrt{k/m} \quad \text{linear osc./spring}$$

$$T = \begin{cases} 2\pi\sqrt{I/\kappa} & \text{torsion pendulum} \\ 2\pi\sqrt{L/g} & \text{simple pendulum} \\ 2\pi\sqrt{I/mgh} & \text{physical pendulum} \end{cases}$$

$$U = -\frac{1}{2}kx^2 \quad U = -\frac{1}{2}\kappa\theta^2 \quad F = -\frac{dU}{dx} = ma \quad \text{SHM}$$

fluids:

$$P = F/A$$

$$P(h) = P_{\mathrm{above}} + \rho gh \qquad \rho = M/V$$

$$\frac{F_1}{A_1} = \frac{F_2}{A_2} \quad F_1x_1 = F_2x_2 \qquad \text{hydraulics}$$

$$B = \text{buoyant force} = \text{weight of water displaced}$$