Name & ID

PH125 Exam 4 Solutions

1. *Harmonic motion.* **(a)** A mass hanging from the end of a spring has a period of 1.0 s. If the mass is doubled, what is the new period? **(b)** The position of a particle moving in simple harmonic motion is given by $x(t) = 2\cos(50t)$ where x is in meters and t is in seconds. What is maximum velocity of the particle in m/s?

Solution: The period of the first is

$$T_1 = \frac{2\pi}{\omega_1} = 2\pi\sqrt{\frac{m}{k}} = 1.0\,\mathrm{s} \tag{1}$$

The second is

$$T_2 = 2\pi\sqrt{\frac{2m}{k}} = 2\pi\cdot\sqrt{2}\sqrt{\frac{m}{k}} = \sqrt{2}T_1 \approx 1.4\,\mathrm{s} \tag{2}$$

2. An aircraft door closes by pushing it inside the airplane first. We will assume $P = 0$ outside the aircraft, and $P = 0.9\,\mathrm{atm}$ inside during flight. If the sealing surface of the door is 5 cm wide all around the door, and the door's outer dimensions are 2 m by 0.7 m, what is the approximate total force required to open the door while in flight? One atmosphere is 1.01×10^5 Pa.

Solution: The force will be the surface area of the door *exposed to the outside pressure* times the pressure difference. That means the area of the door minus the area of the seal. Why not the area of the seal?

The force to overcome is due to the pressure difference between inside and out, and that pressure difference only exists over the part of the door that isn't in contact with the seal. If you imagined the door as a rubber sheet, think about where it would bulge out - only where there isn't a seal, because that's the only place the door 'feels' the pressure difference.

The area of the door minus the sealing surface is 1.9 by 0.6 m, or $A = 1.14\,\mathrm{m}^2$. The pressure difference is 0.9 atm, or $\Delta P = 1.01 \times 10^5$ Pa. The net force is then $F = A\Delta P \approx 1.15 \times 10^5$ N. This is a truly ridiculous force, and it explains why of all the things you're instructed to do and not do on an airplane, they never mention "don't open the door" during the safety talk. They don't because you couldn't if you tried. For comparison, note that the gravitational force required to lift a 1000 lb weight is only about 4.4×10^3 N. The force for the door is over *twenty five* times larger, so there is no way you're going to open that door at altitude while the cabin is pressurized.

Semi-related tangent: the seal area part is very important for figuring out the stress put on the fuselage. The force we calculate above is over almost the whole door. The same force must be exerted by the seal to keep the door in place, but that force is over a much smaller area, making the stress (the solid analog of pressure, force/area) very high around the seal. That's why the door has no 'corners' but is always rounded - stress is higher at sharp points, and with such a high stress already, one can't take any chances. This is something we figured out the hard way: planes with square windows had an ugly tendency to crash. `http://en.wikipedia.org/wiki/De_Havilland_Comet`

3. Suppose that you release a ball from rest at a depth of 0.6 m below the surface of a pool of water. If the density of the ball is 0.3 times that of water and if the drag force on the ball from the water is negligible, how high above the water surface will the ball shoot as it emerges from the water? (Neglect any transfer of energy to the splashing and waves produced by the emerging ball.)

Solution: Don't overthink this one. Use the forces to get the acceleration while underwater (it is not just g). Use the acceleration to get the velocity at the surface, and use that to get the final height since the acceleration above the surface *is* just g.

The forces are the buoyant force (weight of the displaced water) and the object's weight. Let the water's density be ρ_w and the ball's density be ρ_s. Noting that the volume of a sphere is $\frac{4}{3}\pi r^3$ and mass is density times volume,

$$\sum F = B - W = \frac{4}{3}\pi r^3 \rho_w g - \frac{4}{3}\pi r^3 \rho_s g = ma = \frac{4}{3}\pi r^3 \rho_s a \tag{3}$$

$$a = \frac{\rho_w - \rho_s}{\rho_s} g = \frac{7}{3} g \tag{4}$$

That's the acceleration while underwater. We can use work-energy or kinematics to get the velocity at the top. Either way, if the ball is a distance h below the surface,

$$v_{\text{surf}}^2 = 2ah = 2gh\left(\frac{7}{3}\right) \tag{5}$$

Finally, above the surface we have an acceleration of g, and we can use conservation of energy (or kinematics) to get the maximum height.

$$mgh = \frac{1}{2} m v_{\text{surf}}^2 \tag{6}$$

$$h = \frac{v_{\text{surf}}^2}{2g} = \frac{7}{3} h \approx 1.4\,\text{m} \tag{7}$$

4. The fastest possible rate of rotation of a planet is that for which the gravitational force on material at the equator just barely provides the centripetal force needed for the rotation. Show that the corresponding shortest period of rotation is

$$T = \sqrt{\frac{3\pi}{G\rho}}$$

where ρ is the uniform density (mass per unit volume) of the spherical planet. The volume of a sphere is $\frac{4}{3}\pi r^3$, where r is the radius of the sphere.

Solution: There are a few ways to go about this. Perhaps the shortest is just to use Kepler's law, which we derived from the gravitational and centripetal forces in the first place, along with the fact that the mass is $M = \frac{4}{3}\pi r^3 \rho$

$$T^2 = \frac{4\pi^2}{GM} r^3 = \frac{4\pi^2}{G\frac{4}{3}\pi r^3 \rho} r^3 = \frac{3\pi}{G\rho} \tag{8}$$

$$T = \sqrt{\frac{3\pi}{G\rho}} \tag{9}$$

If you didn't think to use Kepler's law, you'd first start with $T = 2\pi r/v$ and add the centripetal force balance $mv^2/r = GMm/r^2$ (which you'd solve for v and plug in the equation for T). That will bring you to Kepler's law, at which point you proceed as above.

5. The period of the earth's rotation about the sun is 365.256 days. It would be more convenient to have a period of exactly 365 days. How should the mean distance from the sun be changed to correct this anomaly?

Solution: What you don't want to do is complicate this one with numbers right away, or it will become messy. Symbolic solution first. Start with Kepler's law, which relates period and orbital distance. Consider the present case period T_1 and orbital distance r_1, and the hypothetical 365 day year is T_2 with orbital distance r_2. Kepler's law states

$$T^2 = \frac{4\pi^2}{GM} r^3 \tag{10}$$

That means $T^2 \propto r^3$. Taking the ratio of T_1 to T_2 is perhaps the easiest thing to do.

$$\frac{T_1^2}{T_2^2} = \frac{r_1^3}{r_2^3} \tag{11}$$

$$r_2^3 = \left(\frac{T_2}{T_1}\right)^2 r_1^3 \tag{12}$$

$$r_2 = r_1 \sqrt[3]{\frac{T_2^2}{T_1^2}} \approx 0.9995 r_1 \tag{13}$$

Moving the earth closer to the sun by about 0.05% will do the job. But wait, you say, we don't know what r_1 is, so we don't know how much the distance has to change by! Perhaps not, but you can find r_1 from the known period $T_1 = 365.256$ days and the mass of the sun (given on the formula sheet). Solving Kepler's law for r_1,

$$r_1 = \sqrt[3]{\frac{GM_s T_1^2}{4\pi^2}} \approx 1.496 \times 10^{11}\,\mathrm{m} \tag{14}$$

Armed with this, you should find that the sun needs to move closer to the sun by about 7×10^7 m

Numbers & math:

$$g = 9.81\,\mathrm{m/s^2} \quad G = 6.67\times 10^{-11}\,\mathrm{N\,m^2/kg^2}$$
$$M_e = 5.972\times 10^{24}\,\mathrm{kg} \quad R_e = 6.371\times 10^{6}\,\mathrm{m} \quad \leftarrow \text{earth}$$
$$M_s = 1.9891\times 10^{30}\,\mathrm{kg} \quad \leftarrow \text{sun}$$
$$\text{sphere} \quad V = \frac{4}{3}\pi r^3 \qquad M = \rho V$$
$$ax^2 + bx^2 + c = 0 \Longrightarrow x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$
$$\sin\alpha \pm \sin\beta = 2\sin\frac{1}{2}(\alpha\pm\beta)\cos\frac{1}{2}(\alpha\mp\beta)$$
$$\cos\alpha \pm \cos\beta = 2\cos\frac{1}{2}(\alpha+\beta)\cos\frac{1}{2}(\alpha-\beta)$$
$$c^2 = a^2 + b^2 - 2ab\cos\theta_{ab}$$
$$\frac{d}{dx}\sin ax = a\cos ax \qquad \frac{d}{dx}\cos ax = -a\sin ax$$
$$\int \cos ax\,\mathrm{dx} = \frac{1}{a}\sin ax \qquad \int \sin ax\,\mathrm{dx} = -\frac{1}{a}\cos ax$$
$$\sin\theta \approx \theta \qquad \cos\theta \approx 1 - \frac{1}{2}\theta^2 \qquad \text{small } \theta$$

Vectors:

$$|\vec{\mathbf{F}}| = \sqrt{F_x^2 + F_y^2} \quad \text{magnitude}$$
$$\theta = \tan^{-1}\left[\frac{F_y}{F_x}\right] \quad \text{direction}$$
$$\vec{\mathbf{a}}\cdot\vec{\mathbf{b}} = a_x b_x + a_y b_y + a_z b_z = \sum_{i=1}^{n} a_i b_i = |\vec{\mathbf{a}}||\vec{\mathbf{b}}|\cos\theta$$

1-D motion:

$$v(t) = \frac{\mathrm{d}}{\mathrm{d}t}x(t) \qquad a(t) = \frac{\mathrm{d}}{\mathrm{d}t}v(t) = \frac{\mathrm{d}^2}{\mathrm{d}t^2}x(t)$$

const. acc.

$$x_f = x_i + v_{xi}t + \frac{1}{2}a_x t^2$$
$$v_{xf}^2 = v_{xi}^2 + 2a_x\Delta x$$
$$v_f = v_i + at$$

Force:

$$\Sigma\vec{\mathbf{F}} = \vec{\mathbf{F}}_{\mathrm{net}} = m\vec{\mathbf{a}} \qquad \Sigma F_x = ma_x \qquad \Sigma F_y = ma_y$$
$$F_{\mathrm{grav}} = mg = \text{weight}$$
$$\vec{\mathbf{F}}_{12} = -\vec{\mathbf{F}}_{21}$$
$$\vec{\mathbf{F}} = \frac{d\vec{\mathbf{p}}}{dt} \approx \frac{\Delta(m\vec{\mathbf{v}})}{\Delta t} \quad \text{direction} \ldots$$
$$f_s \le \mu_s n \qquad f_k = \mu_k n$$
$$\vec{\mathbf{F}}_c = -\frac{mv^2}{r}\,\hat{\mathbf{r}}$$

Work-Energy:

$$K = \frac{1}{2}mv^2 = \frac{p^2}{2m}$$
$$\Delta K = K_f - K_i = W$$
$$W = \int F(x)\,dx = -\Delta U$$
$$U_g(y) = mgy$$
$$U_s(x) = \frac{1}{2}kx^2$$
$$F = -\frac{dU(x)}{dx}$$
$$K_i + U_i = K_f + U_f + W_{\mathrm{ext}} = K_f + U_f + \int F_{\mathrm{ext}}\,dx$$

Gravitation:

$$\vec{\mathbf{F}}_{12} = -\frac{Gm_1m_2}{r^2}\,\hat{\mathbf{r}}_{12} = -\frac{dU_g}{dr}\,\hat{\mathbf{r}}$$
$$g = \frac{GM_e}{R_e^2}$$
$$U_g(r) = -\int F(r)\,dr = \frac{GMm}{r}$$
$$K + U_g \ge 0 \quad \text{escape} \qquad K + U_g < 0 \quad \text{bound}$$
$$\frac{dA}{dt} = \frac{1}{2}r^2\omega = \frac{L}{2m} \qquad T^2 = \left(\frac{4\pi^2}{GM}\right)r^3$$
$$E_{\mathrm{orbit}} = \frac{-GMm}{2a} \quad \text{elliptical; } a \to r \text{ for circular}$$

Oscillations:

$$T = \frac{1}{f} \quad \omega = \frac{2\pi}{T} = 2\pi f$$
$$x(t) = x_m\cos(\omega t + \varphi)$$
$$a = \frac{d^2x}{dt^2} = -\omega^2 x$$
$$\omega = \sqrt{k/m} \quad \text{linear osc./spring}$$
$$T = \begin{cases} 2\pi\sqrt{I/\kappa} & \text{torsion pendulum} \\ 2\pi\sqrt{L/g} & \text{simple pendulum} \\ 2\pi\sqrt{I/mgh} & \text{physical pendulum} \end{cases}$$
$$U = -\frac{1}{2}kx^2 \quad U = -\frac{1}{2}\kappa\theta^2 \quad F = -\frac{dU}{dx} = ma \quad \text{SHM}$$

fluids:

$$P = F/A$$
$$P(h) = P_{\mathrm{above}} + \rho g h \qquad \rho = M/V$$
$$\frac{F_1}{A_1} = \frac{F_2}{A_2} \quad F_1x_1 = F_2x_2 \qquad \text{hydraulics}$$
$$B = \text{buoyant force} = \text{weight of water displaced}$$