Name & ID ______________________________

University of Alabama
Department of Physics and Astronomy

PH 125 / LeClair Fall 2017

Exam III Solution

1. A child throws a ball with an initial speed of 8.00 m/s at an angle of 40.0° above the horizontal. The ball leaves her hand 1.00 m above the ground and experience negligible air resistance. (a) How far from where the child is standing does the ball hit the ground? (b) How long is the ball in flight before it hits the ground?

Solution: Let the ball's starting height be y_o, the initial velocity be v_i, the launch angle be θ, and the horizontal distance traveled be d. If we put the origin at ground level where the child stands, such that the ball starts at coordinates $(0, y_o)$, we can use the $y(x)$ equation to find the distance traveled by finding the x coordinate at which $y=0$.

$$y(x) = 0 = y_o + x\tan\theta - \frac{gx^2}{2v_i^2\cos^2\theta} \tag{1}$$

$$0 = -gx^2 + 2v_i^2\cos^2\theta\tan\theta x + 2v_i^2 y_o\cos^2\theta \tag{2}$$

$$0 = gx^2 - 2v_i^2\sin\theta\cos\theta x - 2v_i^2 y_o\cos^2\theta \tag{3}$$

Solving the quadratic and simplifying,

$$x = \frac{1}{2g}\left(2v_i^2\sin\theta\cos\theta \pm \sqrt{4v_i^2\sin^2\theta\cos^2\theta + 8v_i^2 y_o g\cos^2\theta}\right) \tag{4}$$

$$= \frac{v_i^2}{g}\left(\sin\theta\cos\theta \pm \cos\theta\sqrt{\sin^2\theta + \frac{2y_o g}{v_i^2}}\right) \tag{5}$$

$$= \frac{v_i^2\cos\theta}{g}\left(\sin\theta \pm \sqrt{\sin^2\theta + \frac{2y_o g}{v_i^2}}\right) \approx 7.46, -1.03\,\mathrm{m} \tag{6}$$

Clearly the positive solution is the one we seek, so $d=7.46\,\mathrm{m}$. The time spent in the air is then easily found from the x component of the launch velocity:

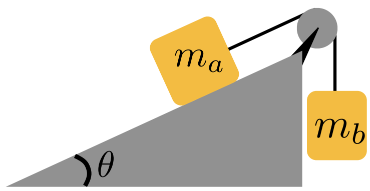
$$d = v_x t = v_i\cos\theta t \quad\Longrightarrow\quad t = \frac{d}{v_i\cos\theta} \approx 1.22\,\mathrm{s} \tag{7}$$

2. A uniform solid sphere of mass M and radius R rotates with an angular speed ω about an axis through its center. A uniform solid cylinder of mass M, radius R, and length $2R$ rotates through an axis running through the central axis of the cylinder. What must be the angular speed of the cylinder so it will have the same rotational kinetic energy as the sphere?

Solution: First: we don't need the length of the cylinder at all. All we need to do is equate the rotational kinetic energy of the two, with the sphere rotating with ω_s and the cylinder at ω_c.

$$K_{r,\text{cylinder}} = K_{r,\text{sphere}} \tag{8}$$
$$\frac{1}{2}I_c\omega_c^2 = \frac{1}{2}I_s\omega_s^2 \tag{9}$$
$$\omega_c = \sqrt{\frac{I_s}{I_c}}\omega_s = \sqrt{\frac{\frac{2}{5}}{\frac{1}{2}}}\omega_s = \frac{2}{\sqrt{5}}\omega_s \tag{10}$$

3. Two blocks are connected by a string that goes over an ideal pulley as shown in the figure. Block A has a mass of 3.00 kg and can slide over a rough plane inclined 30.0° to the horizontal. The coefficient of kinetic friction between block A and the plane is 0.400. Block B has a mass of 2.77 kg. What is the acceleration of the blocks?



Solution: We need free-body diagrams for each mass. Let the x axis run up the ramp for mass A. We will assume that mass B falls and pulls mass A up the ramp, meaning the acceleration is in the x direction for mass A and downward for mass B. For mass A do we need to consider a friction force. Since the rope is presumably taut the entire time of interest, the acceleration is the same for both blocks. For the same reason, the tension applied to both blocks is the same. Newton's second law and geometry will suffice to find the acceleration if we neglect the pulley's rotational inertia. Along the y direction for either mass, the forces must sum to zero, while along the x direction, the forces must give the acceleration for each mass.

$$\sum F_y = 0$$
$$\sum F_x = ma_x$$

First consider mass A. The free body diagram above yields the following, noting that the acceleration will be purely along the $-x$ direction:

$$\sum F_y = n - m_a g\cos\theta = 0$$
$$\sum F_x = T - f - m_a g\sin\theta = m_a a_x$$

From the first equation, we see $n = m_a g\cos\theta$, so $f = \mu_k m_a g\cos\theta$. For mass B,

$$\sum F_y = T - m_b g = -m_b a \qquad \Longrightarrow \qquad T = m_b(g-a)$$

Combining,

$$T - f - m_a g \sin\theta = m_b g - m_b a - \mu_k m_a g \cos\theta - m_a g \sin\theta = m_a a \tag{11}$$

$$(m_b + m_a)a = g\left(m_b - \mu_k m_a \cos\theta - m_a \sin\theta\right) \tag{12}$$

$$a = \left(\frac{m_b - \mu_k m_a \cos\theta - m_a \sin\theta}{m_a + m_b}\right) \approx 0.39\,\mathrm{m/s^2} \tag{13}$$

4. A record is dropped vertically onto a freely rotating (undriven) turntable. Frictional forces act to bring the record and turntable to a common angular speed. If the rotational inertia of the record is 0.54 times that of the turntable, what percentage of the initial kinetic energy is lost?

Solution: The record starts at rest with the turntable rotating with velocity ω_i. Afterwards, both rotate together with velocity ω_f. Dropping the record on the turntable is essentially an inelastic rotational collision, so conservation of angular momentum will relate the two velocities.

$$L_i = L_f \tag{14}$$

$$I_t \omega_i = \left(I_d + I_t\right)\omega_f \tag{15}$$

$$\omega_f = \left(\frac{I_t}{I_t + I_d}\right)\omega_i \tag{16}$$

The ratio of final to initial kinetic energy is then readily found:

$$\frac{K_f}{K_i} = \frac{\frac{1}{2}\left(I_t + I_d\right)\omega_f^2}{\frac{1}{2} I_t \omega_i^2} = \frac{I_t}{I_t + I_d} \tag{17}$$

The fraction lost is then

$$\frac{K_f - K_i}{K_i} = \frac{K_f}{K_i} - 1 = \frac{-I_d}{I_t + I_d} = \frac{-0.54}{1 + 0.54} \approx -0.35 \tag{18}$$

Approximately 35% of the initial kinetic energy is lost.

5. A string is wrapped around a pulley with a radius of 2.0 cm and no appreciable friction in its axle. The pulley is initially not turning. A constant force of 50 N is applied to the string, which does not slip, causing the pulley to rotate and the string to unwind. If the string unwinds 1.2 m in 4.9 s, what is the moment of inertia of the pulley?

Solution: The distance the string travels Δx in a time t implies an acceleration a:

$$\Delta x = \frac{1}{2} a t^2 \tag{19}$$

Since the string does not slip, the rotational acceleration of the pulley must match the linear acceleration of the string divided by the radius of the pulley, $\alpha = a/R$. A torque balance relates the acceleration to the force present:

$$\sum \tau = RF = I\alpha = I\frac{a}{R} \qquad \Longrightarrow \qquad a = \frac{R^2F}{I} \tag{20}$$

$$\Delta x = \frac{1}{2}at^2 = \frac{R^2F}{2I}t^2 \tag{21}$$

$$I = \frac{R^2Ft^2}{2\Delta x} \approx 0.20\,\mathrm{kg}\cdot\mathrm{m}^2 \tag{22}$$

Formula sheet

basics

$$g = |\vec{\mathbf{a}}_{\text{free fall}}| = 9.81\,\text{m/s}^2 \quad \text{near earth's surface}$$

$$\text{sphere} \quad V = \frac{4}{3}\pi r^3$$

$$ax^2 + bx^2 + c = 0 \Longrightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

1D & 2D motion

$$\text{speed} = v = |\vec{\mathbf{v}}| \quad \vec{\mathbf{v}}_{av} \equiv \frac{\Delta\vec{\mathbf{r}}}{\Delta t} \quad \vec{\mathbf{v}} = \lim_{\Delta t\to 0}\frac{\Delta\vec{\mathbf{r}}}{\Delta t} \equiv \frac{d\vec{\mathbf{r}}}{dt}$$

$$a_x = \lim_{\Delta t\to 0}\frac{\Delta v_x}{\Delta t} \equiv \frac{dv_x}{dt} = \frac{d}{dt}\left(\frac{dx}{dt}\right) = \frac{d^2x}{dt^2}$$

$$v_x(t) = v_{x,i} + a_x t$$

$$x(t) = x_i + v_{x,i}t + \frac{1}{2}a_x t^2$$

$$v_{x,f}^2 = v_{x,i}^2 + 2a_x\Delta x$$

$\downarrow$ launched from origin, level ground

$$y(x) = (\tan\theta_o)\,x - \frac{gx^2}{2v_o^2\cos^2\theta_o}$$

$$\text{max height } = H = \frac{v_i^2\sin^2\theta_i}{2g}$$

$$\text{Range } = R = \frac{v_i^2\sin 2\theta_i}{g}$$

interactions

$$\Delta U^G = mg\Delta x \qquad \frac{a_{1x}}{a_{2x}} = -\frac{m_2}{m_1}$$

$$E_{\text{mech}} = K + U \quad K = \frac{1}{2}mv^2$$

$$\Delta E_{\text{mech}} = \Delta K + \Delta U = 0 \quad \text{non-dissipative, closed}$$

$$\Delta E = W$$

$$\Delta U_{\text{spring}} = \frac{1}{2}k\,(x - x_o)^2$$

$$P = \frac{dE}{dt}\text{general} \qquad P = F_{\text{ext,x}}\,v_x \quad \text{1D const force}$$

Rotation: we use radians

$$s = \theta r \quad \leftarrow \text{arclength}$$

$$\omega = \frac{d\theta}{dt} = \frac{v_t}{r} \qquad \alpha = \frac{d\omega}{dt}$$

$$a_t = \alpha r \quad \text{tangential} \qquad a_r = -\frac{v^2}{r} = -\omega^2 r \quad \text{radial}$$

$$v_t = r\omega \qquad v_r = 0$$

$$\Delta\theta = \omega_i t + \frac{1}{2}\alpha t^2 \quad \text{const } \alpha$$

$$\omega_f = \omega_i + \alpha t \quad \text{const } \alpha$$

$$\Delta x = r\theta \quad v = r\omega \quad a = r\alpha \quad \text{no slipping}$$

$$I = \sum_i m_i r_i^2 \Rightarrow \int r^2\,dm = cmr^2 \qquad I = mr^2 \quad \text{point particle}$$

$$I_z = I_{com} + md^2 \quad \text{axis } z \text{ parallel, dist } d$$

$$\vec{\mathbf{L}} = \vec{\mathbf{r}} \times \vec{\mathbf{p}} = I\vec{\boldsymbol{\omega}} \qquad L = I\omega = mvr_\perp$$

$$K = \frac{1}{2}I\omega^2 = L^2/2I$$

$$\Delta K = \frac{1}{2}I\omega_f^2 - \frac{1}{2}I\omega_i^2 = W = \int \tau\,d\theta$$

$$P = \frac{dW}{dt} = \tau\omega$$

$$\tau = rF\sin\theta_{rF} = r_\perp F = rF_\perp$$

$$\tau_{net} = \sum \vec{\boldsymbol{\tau}} = I\vec{\boldsymbol{\alpha}} = \frac{d\vec{\mathbf{L}}}{dt}$$

$$K_{\text{tot}} = K_{cm} + K_{rot} = \frac{1}{2}mv_{cm}^2 + \frac{1}{2}I\omega^2$$

work

$$\Delta E_{\text{mech}} = \Delta K + \Delta U = W \quad \leftarrow \text{not closed} \qquad \Delta U_{\text{spring}} = \frac{1}{2}k\,(x - x_o)^2$$

$$P = \frac{dE}{dt} \quad P = F_{\text{ext,x}}\,v_x \quad \text{one dimension}$$

$$W = \left(\sum \vec{\mathbf{F}}\right)\Delta x_F \quad \text{constant foce 1D}$$

$$W = \sum_n \left(F_{\text{ext,x}}\,\Delta x_{Fn}\right) \quad \text{const nondiss., many particles, 1D}$$

$$W = \int_{x_i}^{x_f} F_x(x)\,dx \quad \text{nondiss. force, 1D}$$

$$(F_{12}^s)_{\max} = \mu_s F_{12}^n \quad \text{static} \qquad F_{12}^k = \mu_k F_{12}^n \quad \text{kinetic}$$

$$W = \vec{\mathbf{F}} \cdot \Delta\vec{\mathbf{r}}_F \quad \text{const non-diss force}$$

$$W = \int_{\vec{\mathbf{r}}_i}^{\vec{\mathbf{r}}_f} \vec{\mathbf{F}}(\vec{\mathbf{r}}) \cdot d\vec{\mathbf{r}} \quad \text{variable nondiss force}$$

Moments of inertia of things of mass M

Object	axis	**dimension**	**I**
solid sphere	central axis	radius R	$\frac{2}{5}MR^2$
hollow sphere	central axis	radius R	$\frac{2}{3}MR^2$
solid disc/cylinder	central axis	radius R	$\frac{1}{2}MR^2$
hoop	central axis	radius R	MR^2
point particle	pivot point	distance R to pivot	MR^2
rod	center	length L	$\frac{1}{12}ML^2$
rod	end	length L	$\frac{1}{3}ML^2$
solid regular octahedron	through vertices	side a	$\frac{1}{10}ma^2$

Derived unit	**Symbol**	**equivalent to**
newton	N	kg·m/s^2
joule	J	kg·m^2/s^2 =N·m
watt	W	J/s=m^2·kg/s^3