Name & ID ____

University of Alabama
Department of Physics and Astronomy

PH 125 / LeClair Fall 2017

Exam I

Instructions

1. **Solve 3 of 5 problems below.** All problems have equal weight.
2. Do your work on separate sheets rather than on the exam.
3. Do not hesitate to ask if you are unsure what a problem is asking for.
4. Show your work for full credit. Significant partial credit will be given.
5. **Symbolic** solutions give more partial credit than purely numerical ones.
6. You are allowed 2 sides of a standard 8.5 x 11 in paper and a calculator.

1. A particle sliding down a frictionless ramp of angle θ is to attain a given displacement Δx along the ramp in a minimum amount of time. **(a)** What is the best angle for the ramp? *Hint: find the time in terms of the angle and minimize it. See the formula sheet.* **(b)** What is the minimum time?

2. The position of a particle in meters can be described by $x = 10t - 2.5t^2$, where t is in seconds. **(a)** What is the position of the particle when it changes direction? **(b)** For the particle in the question above, what is its velocity when it returns to its original $t = 0$ position?

3. A hockey puck on a frozen pond is given an initial speed of 20.0 m/s. The puck always remains on the ice and slides 115 m before coming to rest. Determine the acceleration due to friction between the puck and the ice.

4. A bullet of mass m is fired at velocity v_i. It strikes a wooden block of mass M, resulting in a completely inelastic collision in which the bullet ends up embedded in the block. After the collision, block plus bullet move smoothly along a surface with coefficient of kinetic friction μ, giving rise to an acceleration of μg. **(a)** What is the velocity of the block plus bullet immediately after the collision? **(b)** How far does the block slide before coming to a stop?

5. A 70 kg person standing on a frictionless surface fires a 0.025 kg arrow horizontally at 100 m/s. With what velocity does the archer move backwards across the ice after firing the arrow?

Formula sheet

$$g = |\vec{\mathbf{a}}_{\text{free fall}}| = 9.81\,\text{m/s}^2 \quad \text{near earth's surface}$$
$$\Delta\vec{\mathbf{r}} = \vec{\mathbf{r}}_f - \vec{\mathbf{r}}_i$$
$$d \equiv |x_1 - x_2|$$
$$b \equiv |\vec{\mathbf{b}}| = |b_x| \quad \text{one dimension}$$
$$\vec{\mathbf{r}} = x\,\hat{\imath} \quad \text{one dimension}$$
$$\vec{\mathbf{b}} = b_x\,\hat{\imath} \quad \text{one dimension}$$
$$\text{speed} = v = |\vec{\mathbf{v}}|$$
$$\vec{\mathbf{v}}_{av} \equiv \frac{\Delta\vec{\mathbf{r}}}{\Delta t}$$
$$\vec{\mathbf{v}} = \lim_{\Delta t\to 0}\frac{\Delta\vec{\mathbf{r}}}{\Delta t} \equiv \frac{d\vec{\mathbf{r}}}{dt}$$
$$a_{x,av} \equiv \frac{\Delta v_x}{dt}$$
$$a_x = \lim_{\Delta t\to 0}\frac{\Delta v_x}{\Delta t} \equiv \frac{dv_x}{dt} = \frac{d}{dt}\left(\frac{dx}{dt}\right) = \frac{d^2x}{dt^2}$$
$$v_x = \lim_{\Delta t\to 0}\frac{\Delta x}{\Delta t} \equiv \frac{dx}{dt}$$
$$x_f = x_i + v_{x,i}\Delta t + \frac{1}{2}a_x\left(\Delta t\right)^2$$
$$v_{x,f} = v_{x,i} + a_x\Delta t$$
$$x(t) = x_i + v_{x,i}t + \frac{1}{2}a_x t^2$$
$$v_x(t) = v_{x,i} + a_x t$$
$$v_{x,f}^2 = v_{x,i}^2 + 2a_x\Delta x$$

$$\Delta\vec{\mathbf{p}} = \vec{\mathbf{0}} \quad \text{isolated system}$$
$$\vec{\mathbf{p}}_f = \vec{\mathbf{p}}_i \quad \text{isolated system}$$
$$\vec{\mathbf{p}} \equiv m\vec{\mathbf{v}}$$
$$m_u = -\frac{\Delta v_{s,x}}{\Delta v_{u,x}}m_s$$
$$\vec{\mathbf{J}} = \Delta\vec{\mathbf{p}}$$
$$\Delta E = 0 \quad \text{isolated system}$$
$$K = \frac{1}{2}mv^2$$
$$\vec{v}_{12} = \vec{\mathbf{v}}_2 - \vec{\mathbf{v}}_1 \quad \text{relative velocity}$$
$$v_{12} = |\vec{\mathbf{v}}_2 - \vec{\mathbf{v}}_1| \quad \text{relative speed}$$

Power	**Prefix**	**Abbreviation**
10^{-12}	pico	p
10^{-9}	nano	n
10^{-6}	micro	µ
10^{-3}	milli	m
10^{-2}	centi	c
10^{3}	kilo	k
10^{6}	mega	M
10^{9}	giga	G
10^{12}	tera	T

Math:

$$ax^2 + bx^2 + c = 0 \Longrightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$\sin\alpha \pm \sin\beta = 2\sin\frac{1}{2}\left(\alpha \pm \beta\right)\cos\frac{1}{2}\left(\alpha \mp \beta\right)$$
$$\cos\alpha \pm \cos\beta = 2\cos\frac{1}{2}\left(\alpha + \beta\right)\cos\frac{1}{2}\left(\alpha - \beta\right)$$
$$c^2 = a^2 + b^2 - 2ab\cos\theta_{ab}$$
$$\frac{d}{dx}\,\sin^n\left(ax\right) = an\cos\left(ax\right)\sin^{n-1}\left(ax\right)$$
$$\frac{d}{dx}\,\cos^n\left(ax\right) = -an\sin\left(ax\right)\cos^{n-1}(\left(ax\right))$$
$$\frac{d}{dx}\frac{1}{\sqrt{\sin\left(ax\right)}} = -\frac{a\cos\left(ax\right)}{2\sin^{3/2}\left(ax\right)} \quad \text{oddly specific information}$$