**Name & ID** ______________________________

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# Exam I Solution

**1.** A particle sliding down a frictionless ramp of angle $\theta$ is to attain a given displacement $\Delta x$ along the ramp in a minimum amount of time. **(a)** What is the best angle for the ramp? *Hint: find the time in terms of the angle and minimize it. See the formula sheet.* **(b)** What is the minimum time?

***Solution:*** We know the acceleration down the ramp and the displacement along the ramp $\Delta x$, from that we can find the time down the ramp. With $+x$ pointing down the ramp,

$$a = g\sin\theta \tag{1}$$

$$\Delta x = \frac{1}{2}at^2 = \frac{1}{2}(g\sin\theta)t^2 \tag{2}$$

$$t = \sqrt{\frac{2\Delta x}{g\sin\theta}} \tag{3}$$

Clearly, this is a minimum when $\sin\theta$ is a maximum, which is at $\theta = 90°$ when $\sin\theta = 1$. You can also find $dt/d\theta$ and set it equal to zero, which gives the same result. This makes sense: the quickest way to move along the ramp is if it were no ramp at all, but just falling straight downward.

The minimum time is then easily found, $t_{\text{min}} = \sqrt{2\Delta x/g}$.

**2.** The position of a particle in meters can be described by $x = 10t - 2.5t^2$, where $t$ is in seconds. **(a)** What is the position of the particle when it changes direction? **(b)** For the particle in the question above, what is its velocity when it returns to its original $t = 0$ position?

***Solution:*** **(a)** The particle changes direction when the velocity $v$ changes sign, which will happen when $v = dx/dt = 0$:

$$v = \frac{dx}{dt} = 10 - 5.0t = 0 \quad \Longrightarrow \quad t = 2.0\,\text{s} \tag{4}$$

$$x(t=2) = 10(2.0) - 2.5(2.0)^2 = 10\,\text{m/s} \tag{5}$$

**(b)** Clearly, at $t = 0$, $x = 0$. We need to find when the particle returns to $x = 0$ and find its velocity there.

$$x = 10t - 2.5t^2 = t(10 - 2.5t) \quad \Longrightarrow \quad \text{zero at } t = \{0, 4\}\,\text{s} \tag{6}$$

$$v(t=4) = 10 - 5.0(4.0) = -10\,\text{m/s} \tag{7}$$

**3.** A hockey puck on a frozen pond is given an initial speed of $20.0\,\text{m/s}$. The puck always remains on the ice and slides $115\,\text{m}$ before coming to rest. Determine the acceleration due to friction between the puck and the ice.

***Solution:*** We know the displacement, the initial velocity, and the final velocity (zero!), and want the acceleration. Since the one thing we don't know or want to know is time, if we assume the acceleration due to friction is constant we can use $v_f = v_i^2 + 2a\Delta x$.

$$v_f^2 = 0 = v_i^2 - 2a\Delta x \tag{8}$$
$$a = \frac{v_i^2}{2\Delta x} \approx 1.74\,\mathrm{m/s^2} \tag{9}$$

**4.** A bullet of mass $m$ is fired at velocity $v_i$. It strikes a wooden block of mass $M$, resulting in a completely inelastic collision in which the bullet ends up embedded in the block. After the collision, block plus bullet move smoothly along a surface with coefficient of kinetic friction $\mu$, giving rise to an acceleration of $\mu g$. **(a)** What is the velocity of the block plus bullet immediately after the collision? **(b)** How far does the block slide before coming to a stop?

***Solution:*** **(a)** The initial collision is clearly totally inelastic since the objects stick together, which means the only thing we can really rely on is conservation of momentum. That's enough to get us the final velocity.

$$mv_i = (m+M)v_f \tag{10}$$
$$v_f = \frac{m}{m+M}v_i \tag{11}$$

**(b)** Now we proceed just as in the previous problem, using the known acceleration due to friction along with the known initial and final velocities. The "final" velocity after the block stops sliding is zero, and the "initial" velocity is the velocity just after the collision we found above.

$$0 = v_f^2 - 2a\Delta x = \left(\frac{m}{m+M}\right)^2 v_i^2 - 2a\Delta x \tag{12}$$
$$d = \frac{v_i^2}{2a}\left(\frac{m}{m+M}\right)^2 = \frac{v_i^2}{2\mu g}\left(\frac{m}{m+M}\right)^2 \tag{13}$$

**5.** A 70 kg person standing on a frictionless surface fires a 0.025 kg arrow horizontally at 100 m/s. With what velocity does the archer move backwards across the ice after firing the arrow?

***Solution:*** Conservation of momentum is all we need. The initial momentum is zero, the final momentum is that of the arrow plus the person.

$$0 = m_p v_{pf} + m_a v_{af} \tag{14}$$
$$v_{pf} = -\frac{m_a}{m_p}v_{af} \approx -0.036\,\mathrm{m/s} \tag{15}$$

# Formula sheet

$$g = |\vec{\mathbf{a}}_{\text{free fall}}| = 9.81\,\mathrm{m/s^2} \quad \text{near earth's surface}$$

$$\Delta\vec{\mathbf{r}} = \vec{\mathbf{r}}_f - \vec{\mathbf{r}}_i$$

$$d \equiv |x_1 - x_2|$$

$$b \equiv |\vec{\mathbf{b}}| = |b_x| \quad \text{one dimension}$$

$$\vec{\mathbf{r}} = x\,\hat{\boldsymbol{\imath}} \quad \text{one dimension}$$

$$\vec{\mathbf{b}} = b_x\,\hat{\boldsymbol{\imath}} \quad \text{one dimension}$$

$$\text{speed} = v = |\vec{\mathbf{v}}|$$

$$\vec{\mathbf{v}}_{av} \equiv \frac{\Delta\vec{\mathbf{r}}}{\Delta t}$$

$$\vec{\mathbf{v}} = \lim_{\Delta t \to 0} \frac{\Delta\vec{\mathbf{r}}}{\Delta t} \equiv \frac{d\vec{\mathbf{r}}}{dt}$$

$$a_{x,av} \equiv \frac{\Delta v_x}{dt}$$

$$a_x = \lim_{\Delta t \to 0} \frac{\Delta v_x}{\Delta t} \equiv \frac{dv_x}{dt} = \frac{d}{dt}\left(\frac{dx}{dt}\right) = \frac{d^2x}{dt^2}$$

$$v_x = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} \equiv \frac{dx}{dt}$$

$$x_f = x_i + v_{x,i}\Delta t + \frac{1}{2}a_x\left(\Delta t\right)^2$$

$$v_{x,f} = v_{x,i} + a_x\Delta t$$

$$x(t) = x_i + v_{x,i}t + \frac{1}{2}a_x t^2$$

$$v_x(t) = v_{x,i} + a_x t$$

$$v_{x,f}^2 = v_{x,i}^2 + 2a_x\Delta x$$

$$\Delta\vec{\mathbf{p}} = \vec{\mathbf{0}} \quad \text{isolated system}$$

$$\vec{\mathbf{p}}_f = \vec{\mathbf{p}}_i \quad \text{isolated system}$$

$$\vec{\mathbf{p}} \equiv m\vec{\mathbf{v}}$$

$$m_u = -\frac{\Delta v_{s,x}}{\Delta v_{u,x}} m_s$$

$$\vec{\mathbf{J}} = \Delta\vec{\mathbf{p}}$$

$$\Delta E = 0 \quad \text{isolated system}$$

$$K = \frac{1}{2}mv^2$$

$$\vec{v}_{12} = \vec{\mathbf{v}}_2 - \vec{\mathbf{v}}_1 \quad \text{relative velocity}$$

$$v_{12} = |\vec{\mathbf{v}}_2 - \vec{\mathbf{v}}_1| \quad \text{relative speed}$$

| **Power** | **Prefix** | **Abbreviation** |
|---|---|---|
| $10^{-12}$ | pico | p |
| $10^{-9}$ | nano | n |
| $10^{-6}$ | micro | µ |
| $10^{-3}$ | milli | m |
| $10^{-2}$ | centi | c |
| $10^{3}$ | kilo | k |
| $10^{6}$ | mega | M |
| $10^{9}$ | giga | G |
| $10^{12}$ | tera | T |

**Math:**

$$ax^2 + bx^2 + c = 0 \Longrightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\sin\alpha \pm \sin\beta = 2\sin\frac{1}{2}\left(\alpha \pm \beta\right)\cos\frac{1}{2}\left(\alpha \mp \beta\right)$$

$$\cos\alpha \pm \cos\beta = 2\cos\frac{1}{2}\left(\alpha + \beta\right)\cos\frac{1}{2}\left(\alpha - \beta\right)$$

$$c^2 = a^2 + b^2 - 2ab\cos\theta_{ab}$$

$$\frac{d}{dx}\sin^n\left(ax\right) = an\cos\left(ax\right)\sin^{n-1}\left(ax\right)$$

$$\frac{d}{dx}\cos^n\left(ax\right) = -an\sin\left(ax\right)\cos^{n-1}(\left(ax\right))$$

$$\frac{d}{dx}\frac{1}{\sqrt{\sin\left(ax\right)}} = -\frac{a\cos\left(ax\right)}{2\sin^{3/2}\left(ax\right)} \quad \text{oddly specific information}$$