

Exam II

Instructions

1. **Solve 3 of 5 problems below.** All problems have equal weight.
2. Do your work on separate sheets rather than on the exam.
3. Do not hesitate to ask if you are unsure what a problem is asking for.
4. Show your work for full credit. Significant partial credit will be given.
5. **Symbolic** solutions give more partial credit than purely numerical ones.
6. You are allowed 2 sides of a standard 8.5 x 11 in paper and a calculator.

1. A stationary mass M sits on level ground at the base of an inclined plane, and is struck by a mass m moving with velocity v_i . The two objects stick together, and begin to move up the (frictionless) incline. To what height above level ground does the combined mass rise?
2. A wagon is coasting along a level sidewalk at 5.00 m/s. Its wheels have very good bearings. You are standing on a low wall, and you drop vertically into the wagon as it passes by. The wagon has an inertia of 100 kg, and your inertia is 50.0 kg. **(a)** Use conservation of momentum to determine the speed of the wagon after you are in it. **(b)** Use conservation of energy to determine that speed. **(c)** Comparing your two answers, explain which method is correct.
3. A 85 kg halfback on a football team runs head-on into a 120 kg opponent at an instant when neither has his feet on the ground. The halfback is initially going west at 10 m/s, and his opponent is initially going east at 4.37 m/s. The collision is totally inelastic. Suppose that the positive x axis is directed to the west. **(a)** If the collision takes 0.227 s, what are the x components of the accelerations of each player? **(b)** How much kinetic energy is converted to incoherent energy during the collision?
4. You are sitting on a ledge 3 m above the ground, and drop a ball to the ground. It bounces to a height of 2.5 m. How fast would you have to throw it directly downward to make it bounce to a height of 6 m? (Recall our coefficient of restitution lab. The coefficient of restitution ϵ was a **constant** relating the velocity before and after a ball bounced: $\epsilon = \frac{v_{\text{after}}}{v_{\text{before}}}$.)
5. Not looking where you are going, you and your bike collide at 12 m/s into the back of a car stopped at a red light. The car does not have its brakes applied, and so is jolted forward. The driver of the car is claiming whiplash. Before your day in court, you want to determine the acceleration of the car as a result of the collision. You note that you and your bike (combined inertia of 80 kg) come to a complete halt, and that the rim of your front wheel was pushed all the way to the center hub. The diameter of your bike wheel before the crash was 0.75 m, and the internet tells you the inertia of the car is 1800 m. What was the acceleration of the car?

Formula sheet

basics

$$g = |\vec{a}_{\text{free fall}}| = 9.81 \text{ m/s}^2 \quad \text{near earth's surface}$$

$$\text{sphere } V = \frac{4}{3}\pi r^3$$

$$ax^2 + bx^2 + c = 0 \implies x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\frac{d}{dx} \sin ax = a \cos ax \quad \frac{d}{dx} \cos ax = -a \sin ax$$

$$\int \cos ax \, dx = \frac{1}{a} \sin ax \quad \int \sin ax \, dx = -\frac{1}{a} \cos ax$$

$$\vec{A} = \vec{A}_x + \vec{A}_y = A_x \hat{i} + A_y \hat{j}$$

$$\vec{A} \cdot \vec{B} = AB \cos \phi = A_x B_x + A_y B_y$$

$$|\vec{F}| = \sqrt{F_x^2 + F_y^2} \quad \text{magnitude}$$

$$\theta = \tan^{-1} \left[\frac{F_y}{F_x} \right] \quad \text{direction}$$

1D motion

$$\Delta \vec{r} = \vec{r}_f - \vec{r}_i$$

$$\text{speed } v = |\vec{v}| \quad \vec{v}_{av} \equiv \frac{\Delta \vec{r}}{\Delta t} \quad \vec{v} = \lim_{\Delta t \rightarrow 0} \frac{\Delta \vec{r}}{\Delta t} \equiv \frac{d\vec{r}}{dt}$$

$$a_{x,av} \equiv \frac{\Delta v_x}{\Delta t} \quad a_x = \lim_{\Delta t \rightarrow 0} \frac{\Delta v_x}{\Delta t} \equiv \frac{dv_x}{dt} = \frac{d}{dt} \left(\frac{dx}{dt} \right) = \frac{d^2 x}{dt^2}$$

$$x_f = x_i + v_{x,i} \Delta t + \frac{1}{2} a_x (\Delta t)^2$$

$$v_{x,f} = v_{x,i} + a_x \Delta t$$

$$x(t) = x_i + v_{x,i} t + \frac{1}{2} a_x t^2$$

$$v_x(t) = v_{x,i} + a_x t$$

$$v_{x,f}^2 = v_{x,i}^2 + 2a_x \Delta x$$

momentum

$$\Delta \vec{p} = \vec{0} \quad \vec{p}_f = \vec{p}_i \quad \text{isolated system}$$

$$\vec{p} = m\vec{v} \quad \vec{J} = \Delta \vec{p}$$

$$v_{1f} = \left(\frac{m_1 - m_2}{m_1 + m_2} \right) v_{1i} + \left(\frac{2m_2}{m_1 + m_2} \right) v_{2i} \quad \text{1D elastic}$$

$$v_{2f} = \left(\frac{2m_1}{m_1 + m_2} \right) v_{1i} + \left(\frac{m_2 - m_1}{m_1 + m_2} \right) v_{2i} \quad \text{1D elastic}$$

$$\vec{v}_{12} = \vec{v}_2 - \vec{v}_1 \quad \text{relative velocity}$$

$$v_{12} = |\vec{v}_2 - \vec{v}_1| \quad \text{relative speed}$$

$$\epsilon = \frac{v_{\text{after}}}{v_{\text{before}}} \quad \text{coeff. of restitution}$$

interactions & energy

$$\Delta U^G = mg\Delta x \quad \frac{a_{1x}}{a_{2x}} = -\frac{m_2}{m_1}$$

$$E_{\text{mech}} = K + U \quad K = \frac{1}{2}mv^2 = p^2/2m$$

$$\Delta E_{\text{mech}} = \Delta K + \Delta U = 0 \quad \text{non-dissipative, closed}$$