Name & ID ______________________________

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Department of Physics and Astronomy

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Exam II Solution

1. A stationary mass M sits on level ground at the base of an inclined plane, and is struck by a mass m moving with velocity v_i. The two objects stick together, and begin to move up the (frictionless) incline. To what height above level ground does the combined mass rise?

Solution: The collision between the two masses is clearly totally inelastic, and we will have to use conservation of momentum to handle it. From the time after the collision until the combined mass reaches the top of the ramp, we can use conservation of energy. First, conservation of momentum:

$$mv_i = (M+m)v_f \tag{1}$$
$$v_f = \frac{m}{m+M}v_i \tag{2}$$

The combined mass thus starts out with a kinetic energy $\frac{1}{2}(m+M)v_f^2$, and will rise along the ramp until all of this kinetic energy has been converted to gravitational potential energy. Equating,

$$\Delta K = 0 - \frac{1}{2}(m+M)v_f^2 = -\frac{1}{2}(m+M)\left(\frac{m}{m+M}v_i\right)^2 = -\Delta U = (m+M)gh - 0 \tag{3}$$
$$(m+M)gh = \frac{1}{2}\frac{mv_i^2}{m+M} \tag{4}$$
$$h = \frac{m^2v_i^2}{2g(m+M)^2} \tag{5}$$

2. A wagon is coasting along a level sidewalk at 5.00 m/s. Its wheels have very good bearings. You are standing on a low wall, and you drop vertically into the wagon as it passes by. The wagon has an inertia of 100 kg, and your inertia is 50.0 kg. **(a)** Use conservation of momentum to determine the speed of the wagon after you are in it. **(b)** Use conservation of energy to determine that speed. **(c)** Comparing your two answers, explain which method is correct.

Solution: **(a)** This is effectively a totally inelastic collision since the two objects start with some difference in velocity and end with none. The initial horizontal momentum is that of the cart alone; after the person drops into the cart, the same horizontal momentum is split between the person and the cart. Let the cart's mass be M and its initial velocity be v_i, with the person's mass m.

$$Mv_i = (m+M)v_f \tag{6}$$
$$v_f = \frac{M}{m+M}v_i \approx 3.33\,\text{m/s} \tag{7}$$

(b) Our answer to the first part already suggests this collision is totally inelastic, which means mechanical energy is not conserved, but we can give it a try. If our answer is higher than what we get conserving momentum alone, we should be very suspicious. Equating the kinetic energy before and after,

$$\frac{1}{2}M_i^2 = \frac{1}{2}(m+M)v_f^2 \tag{8}$$

$$v_f = \sqrt{\frac{M}{m+M}}v_i \approx 4.08\,\mathrm{m/s^2} \tag{9}$$

(c) We established that this is a totally inelastic collision, which means mechanical energy is not conserved, so (a) is correct. The fact that energy conservation came up with a larger final velocity is consistent with the fact that some mechanical energy is actually lost. Since momentum is always conserved, we know that the result for (a) is reliable.

3. A 85 kg halfback on a football team runs head-on into a 120 kg opponent at an instant when neither has his feet on the ground. The halfback is initially going west at 10 m/s, and his opponent is initially going east at 4.37 m/s. The collision is totally inelastic. Suppose that the positive x axis is directed to the west. **(a)** If the collision takes 0.227 s, what are the x components of the accelerations of each player? **(b)** How much kinetic energy is converted to incoherent energy during the collision?

Solution: **(a)** We can use conservation of momentum to get the final velocity, and then use the known time interval to find the acceleration. Given $+x$ points west, let 1 represent the halfback and 2 the opponent. Assume that after the collision the final velocity is to the west.

$$m_1v_1 - m_2v_2 = (m_1+m_2)v_f \tag{10}$$

$$v_f = \frac{m_1v_1 - m_2v_2}{m_1+m_2} \approx 1.59\,\mathrm{m/s} \tag{11}$$

Now the accelerations:

$$a_1 = \frac{v_f - v_1}{\Delta t} \approx -37\,\mathrm{m/s^2} \tag{12}$$

$$a_2 = \frac{v_f - (-v_2)}{\Delta t} \approx 26\,\mathrm{m/s^2} \tag{13}$$

Note that as a double check, $m_1a_1 = -m_2a_2$ as required.

(b) The kinetic energy lost is

$$K_f - K_i = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 - \frac{1}{2}(m_1+m_2)v_f^2 \approx -5100\,\mathrm{J} \tag{14}$$

4. You are sitting on a ledge 3 m above the ground, and drop a ball to the ground. It bounces to a height of 2.5 m. How fast would you have to throw it directly downward to make it bounce to a height of 6 m? (Recall our coefficient of restitution lab. The coefficient of restitution ϵ was a **constant** relating the velocity before and after a ball bounced: $\epsilon = \frac{v_{\text{after}}}{v_{\text{before}}}$.)

Let the ball start out at height h_o, and after dropping it rebounds to a height h_1. The velocity of the ball just before it hits the ground v_b can be found from conservation of energy:

$$mgh_o = \frac{1}{2}mv_b^2 \quad \Longrightarrow \quad v_b = \sqrt{2gh_o} \tag{15}$$

The definition of the coefficient of restitution ϵ allows us to determine the velocity of the ball after it rebounds v_a:

$$v_a = \epsilon v_b = \epsilon\sqrt{2gh_o} \tag{16}$$

If the ball rebounds to a height h_1, conservation of energy dicates

$$\frac{1}{2}mv_a^2 = \frac{1}{2}m\left(\epsilon\sqrt{2gh_o}\right)^2 = mgh_1 \quad \Longrightarrow \quad \epsilon = \sqrt{h_1/h_o} \tag{17}$$

This gives us ϵ in terms of the given heights. If we throw the ball downward with velocity v_i, its velocity just before hitting the ground v_f can be found from conservation of energy:

$$\frac{1}{2}mv_f^2 = mgh_o + \frac{1}{2}mv_i^2 \tag{18}$$

$$v_f^2 = v_i^2 + 2gh_o \tag{19}$$

Just after rebounding, the velocity will be ϵv_f, implying a kinetic energy of $\frac{1}{2}m(\epsilon v_f)^2$. This kinetic energy will be converted entirely to gravitational potential energy at the maximum height h_2.

$$mgh_2 = \frac{1}{2}m\left(\epsilon v_f\right)\right)^2 = \frac{1}{2}m\left(\epsilon\sqrt{v_i^2 + 2gh_o}\right) \tag{20}$$

$$gh_2 = \frac{1}{2}\epsilon^2 v_i^2 + \epsilon^2 gh_o \tag{21}$$

$$v_i^2 = \frac{2gh_2}{\epsilon^2} - 2gh_o \tag{22}$$

$$v_i = \sqrt{\frac{2gh_2}{\epsilon^2} - 2gh_o} = \sqrt{2g\left(\frac{h_oh_2}{h_1} - h_o\right)} = \sqrt{2gh_o\left(\frac{h_2}{h_1} - 1\right)} \approx 9.1\,\mathrm{m/s} \tag{23}$$

5. Not looking where you are going, you and your bike collide at $12\,\mathrm{m/s}$ into the back of a car stopped at a red light. The car does not have its brakes applied, and so is jolted forward. The driver of the car is claiming whiplash. Before your day in court, you want to determine the acceleration of the car as a result of the collision. You note that you and your bike (combined inertia of $80\,\mathrm{kg}$) come to a complete halt, and that the rim of your front wheel was pushed all the way to the center hub. The diameter of your bike wheel before the crash was $0.75\,\mathrm{m}$, and the internet tells you the inertia of the car is $1800\,\mathrm{m}$. What was the acceleration of the car?

Solution: Initially, the bike is traveling at velocity v_i and after the collision it comes to rest. The car begins at rest and acquires velocity v_{cf} after the collision. Conservation of momentum relates the two velocities. Let m_b and m_c be the bike and car masses, respectively.

$$m_bv_i = m_cv_{cf} \quad \Longrightarrow \quad v_{cf} = \frac{m_b}{m_c}v_i \tag{24}$$

During the interaction, the bike's rim, of diameter d, is compressed through a distance $\Delta x = d/2$. The (presumed constant) acceleration can then be found from the car's initial and final velocities and the interaction distance:

$$v_{cf}^2 - v_{ci}^2 = 2a\Delta x \tag{25}$$

$$a = \frac{v_{cf}^2 - v_{ci}^2}{2\Delta x} = \frac{m_b^2 v_i^2}{2m_c^2 \Delta x} \approx 0.38\,\mathrm{m/s^2} \tag{26}$$

Equivalently, we could have said that the work done during the collision, $W = m_c a \Delta x$, must be equal to the change in the car's kinetic energy.

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Formula sheet

basics

$$g = |\vec{\mathbf{a}}_{\text{free fall}}| = 9.81\,\text{m/s}^2 \quad \text{near earth's surface}$$

$$\text{sphere} \quad V = \frac{4}{3}\pi r^3$$

$$ax^2 + bx^2 + c = 0 \Longrightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\frac{d}{dx}\sin ax = a\cos ax \qquad \frac{d}{dx}\cos ax = -a\sin ax$$

$$\int \cos ax\,\text{dx} = \frac{1}{a}\sin ax \qquad \int \sin ax\,\text{dx} = -\frac{1}{a}\cos ax$$

$$\vec{\mathbf{A}} = \vec{\mathbf{A}}_x + \vec{\mathbf{A}}_y = A_x\,\hat{\imath} + A_y\,\hat{\jmath}$$

$$\vec{\mathbf{A}} \cdot \vec{\mathbf{B}} = AB\cos\phi = A_xB_x + A_yB_y$$

$$|\vec{\mathbf{F}}| = \sqrt{F_x^2 + F_y^2} \quad \text{magnitude}$$

$$\theta = \tan^{-1}\left[\frac{F_y}{F_x}\right] \quad \text{direction}$$

1D motion

$$\Delta\vec{\mathbf{r}} = \vec{\mathbf{r}}_f - \vec{\mathbf{r}}_i$$

$$\text{speed} = v = |\vec{\mathbf{v}}| \quad \vec{\mathbf{v}}_{av} \equiv \frac{\Delta\vec{\mathbf{r}}}{\Delta t} \quad \vec{\mathbf{v}} = \lim_{\Delta t\to 0}\frac{\Delta\vec{\mathbf{r}}}{\Delta t} \equiv \frac{d\vec{\mathbf{r}}}{dt}$$

$$a_{x,av} \equiv \frac{\Delta v_x}{\Delta t} \quad a_x = \lim_{\Delta t\to 0}\frac{\Delta v_x}{\Delta t} \equiv \frac{dv_x}{dt} = \frac{d}{dt}\left(\frac{dx}{dt}\right) = \frac{d^2x}{dt^2}$$

$$x_f = x_i + v_{x,i}\Delta t + \frac{1}{2}a_x\,(\Delta t)^2$$

$$v_{x,f} = v_{x,i} + a_x\Delta t$$

$$x(t) = x_i + v_{x,i}t + \frac{1}{2}a_xt^2$$

$$v_x(t) = v_{x,i} + a_xt$$

$$v_{x,f}^2 = v_{x,i}^2 + 2a_x\Delta x$$

momentum

$$\Delta\vec{\mathbf{p}} = \vec{\mathbf{0}} \quad \vec{\mathbf{p}}_f = \vec{\mathbf{p}}_i \quad \text{isolated system}$$

$$\vec{\mathbf{p}} = m\vec{\mathbf{v}} \qquad \vec{\mathbf{J}} = \Delta\vec{\mathbf{p}}$$

$$v_{1f} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)v_{i1} + \left(\frac{2m_2}{m_1 + m_2}\right)v_{2i} \quad \text{1D elastic}$$

$$v_{2f} = \left(\frac{2m_1}{m_1 + m_2}\right)v_{1i} + \left(\frac{m_2 - m_1}{m_1 + m_2}\right)v_{2i} \quad \text{1D elastic}$$

$$\vec{v}_{12} = \vec{\mathbf{v}}_2 - \vec{\mathbf{v}}_1 \quad \text{relative velocity}$$

$$v_{12} = |\vec{\mathbf{v}}_2 - \vec{\mathbf{v}}_1| \quad \text{relative speed}$$

$$\epsilon = \frac{v_{\text{after}}}{v_{\text{before}}} \quad \text{coeff. of restitution}$$

interactions & energy

$$\Delta U^G = mg\Delta x \qquad \frac{a_{1x}}{a_{2x}} = -\frac{m_2}{m_1}$$

$$E_{\text{mech}} = K + U \quad K = \frac{1}{2}mv^2 = p^2/2m$$

$$\Delta E_{\text{mech}} = \Delta K + \Delta U = 0 \quad \text{non-dissipative, closed}$$