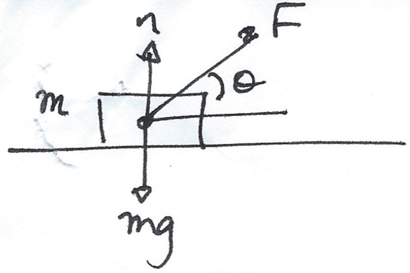
PHD5 HW4
F2014

1)



$F\sin\theta + n - mg = 0$

$n = mg - F\sin\theta$

$F\cos\theta = ma = 0$ for const speed

$F = 122\,N$
$\theta = 37°$
$v = 5\,m/s$

rate of work: $P = \frac{dW}{dt} = \underbrace{\frac{d}{dt}(\vec{F}\cdot\Delta x)}_{\text{const } F} = \vec{F}\cdot\frac{d\Delta\vec{x}}{dt} = \vec{F}\cdot\vec{v}$

$P = \vec{F}\cdot\vec{v} = |F||v|\cos\theta = F_x v_x = \underline{487\,W}$

---

2. $k = 2.5\,N/cm = 250\,N/m$ $\quad \left(2.50\,\frac{N}{cm}\cdot\frac{100\,cm}{1\,m} = 250\,N/m\right)$

$\Delta x = 0.12\,m$

$m = 0.25\,kg$

a) $W_g = +mg\Delta x = \underline{0.29\,J}$ (energy added to block)

b) $W_s = -\frac{1}{2}k(\Delta x)^2 = \underline{-1.8\,J}$ (energy removed from block)

c) must be that initial KE striking equals network

$\Delta K = \overset{0}{\cancel{\frac{1}{2}mv_f^2}} - \frac{1}{2}mv_i^2 = W_s + W_g$

$v_i = \sqrt{\frac{-2(W_s + W_g)}{m}} \simeq \underline{3.5\,m/s}$

$\frac{1}{2}mv_i^2 + \overset{\phi}{U_i} = \frac{1}{2}mv_f^2 + U_{sf} + U_{gf}$

$(W = -U)$ $\quad U_{sf} = \frac{1}{2}kx^2$

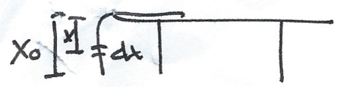
$U_{gf} = -mgx$

d) rearranging, $W_s = -\frac{1}{2}mv_i^2 - W_g = -\frac{1}{2}mv_i^2 - mg\Delta x = -\frac{1}{2}kx^2$

$\frac{1}{2}kx^2 = \frac{1}{2}mv_i^2 + mg\Delta x \qquad \frac{1}{2}kx^2 - mgx - \frac{1}{2}mv_i^2 = 0$

$x = \frac{mg \pm \sqrt{m^2g^2 + kmv_i^2}}{k} \approx \{0.23\,m, \underbrace{-0.21\,m}_{\text{unphys.}}\} = 0.23\,m$

$\frac{x_{2v}}{x_v} \simeq 1.93$ times

3.  rope has mass/length $\lambda = \frac{M}{L}$

bit dx has $dm = \lambda dx = \frac{M}{L} dx$

to lift dx through height x: $dW = dm\, g x = \frac{M}{L} g x\, dx$

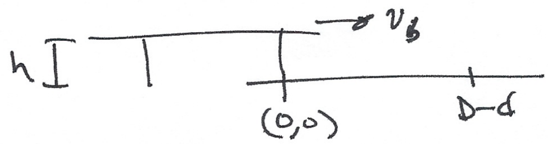
$$W_{tot} = \int_0^{x_0} \frac{M}{L} g x \, dx = \frac{1}{2} \frac{Mg}{L} x_0^2 = \frac{1}{32} MgL \simeq \underline{1.0 \times 10^{-3}\ J}$$

$$x_0 = \frac{L}{4}$$

---

4. Bobby reaches $D - d = 2.20 - 0.27\ m$ $\qquad R \propto v^2 \propto K \propto U \propto x^2$

$\Rightarrow$



<u>Bobby's launch</u>

$$y(x) = h - \frac{gx^2}{2v_b^2} \qquad y(D-d) = 0 = h - \frac{g(D-d)}{2v_b^2}$$

$$v_b^2 = \frac{g(D-d)^2}{2h} \qquad \underline{\text{conv } E \text{ for launch}}: \ \frac{1}{2} m v_b^2 = \frac{1}{2} k x_b^2$$

combine $\Rightarrow$ $\frac{1}{2} m \frac{g(D-d)^2}{2h} = \frac{1}{2} k x_b^2 \Rightarrow k = \frac{mg(D-d)^2}{2h x_b^2}$

<u>for J's launch</u>, $y(D) = 0 = h - \frac{gD^2}{2v_J^2} \Rightarrow \underline{v_J^2 = \frac{gD^2}{2h}}$

<u>CoE</u>: $\frac{1}{2} m v_J^2 = \frac{1}{2} k x_J^2 = \frac{1}{2} \frac{mg(D-d)^2}{2h x_b^2} x_J^2$ (use k from Bobby)

combine $\Rightarrow$ $\frac{1}{2} m \frac{gD^2}{2h} = \frac{1}{2} m \frac{g(D-d)^2}{2h x_b^2} x_J^2$ (use v_J from $y(x)$)

$$D^2 = (D-d)^2 \left(\frac{x_J}{x_b}\right)^2$$

$$\Rightarrow \boxed{x_J = \left(\frac{D}{D-d}\right) x_b \simeq 1.25\ cm}$$

⑤ $\delta K = \frac{1}{2}(\delta m)v^2$

$$P = \frac{\delta K}{\delta t} = \frac{1}{2}\frac{\delta m}{\delta t}v^2$$

$$\delta m = \rho\,\delta V = \rho A \delta x = \rho A v \delta t$$

$$P = \frac{1}{2}\frac{\delta m}{\delta t}v^2 = \frac{1}{2}\rho A v \cdot v^2 = \frac{1}{2}\rho A v^3 = \vec{F}\cdot\vec{v} = Fv$$

$$\Rightarrow \underline{F = \frac{1}{2}\rho A v^2}$$

⑥ at equ, $-F = \frac{\partial U}{\partial r} = 0$

$$4\epsilon\left[\left(\frac{-12\sigma^{12}}{r^{13}}\right) - \left(\frac{-6\sigma^6}{r^7}\right)\right] = 0$$

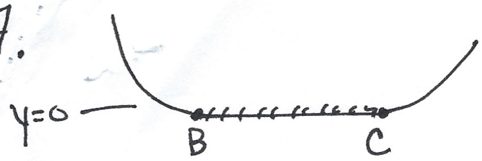
$$\frac{-12\sigma^{12}}{r^{13}} + \frac{6\sigma^6}{r^7} = 0$$

$$\frac{-2\sigma^6}{r^6} + 1 = 0$$

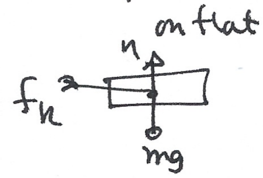
$$\boxed{r_{eq} = \sigma\sqrt[6]{2}}$$

at r_{eq}, $U(r_{eq}) = 4\epsilon\left[\frac{\sigma^{12}}{\sigma^{12}\cdot 4} - \frac{\sigma^6}{\sigma^6\cdot 2}\right] = 4\epsilon\left(\frac{1}{4} - \frac{1}{2}\right) = \underline{-\epsilon}$

7.



going up & down curves doesn't change E_{tot}



$f_k = -\mu_k mg$

$$E_A = K_A + U_A = 0 + mgh$$

$$E_B = K_B + U_B = \frac{1}{2}mv^2 + 0 = mgh$$

$$E_C = E_B + W = E_B + \int_B^C -\mu_k mg \cdot dx = E_B - \mu_k mg L$$

on each pass, we lose $\mu_k mg L$

we start at A w/ $E_A = mgh = mg\frac{L}{2}$

$\Rightarrow$ n passes, $E_A = mg\frac{L}{2} = n \cdot \mu_k mg L$

$$\frac{1}{2} = \mu_k n \qquad n = \frac{1}{2\mu_k} = \frac{1}{0.4} = 2.5$$

goes $\longrightarrow$ $\longleftarrow$ then stops halfway while going right

8. $W = \frac{1}{2}k(2d)^2 - \frac{1}{2}kd^2 = 3 \cdot \frac{1}{2}kd^2 = \underline{\underline{3W_0}}$

a) $W_0 = \frac{1}{2}kd^2 - 0 = \frac{1}{2}kd^2$

b) $W = \frac{1}{2}k(N+1)^2 d^2 - \frac{1}{2}kN^2 d^2$

$$W = \frac{1}{2}kd^2\left[N^2 + 2N + 1 - N^2\right] = \underline{\underline{(2N+1)W_0}}$$