University of Alabama
Department of Physics and Astronomy

PH 125 / LeClair Fall 2014

Problem Set 5

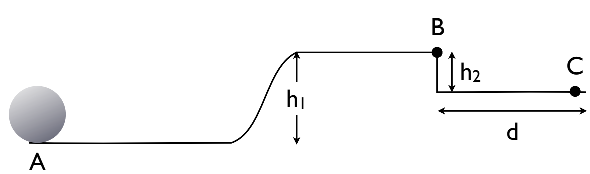
Instructions:

1. Answer all questions below. Show your work for full credit.
2. All problems are due by the end of the day on 17 Oct 2014.
3. You may collaborate, but everyone must turn in their own work.

1. A tall, cylindrical chimney falls over when its base is ruptured. Treat the chimney as a thin rod of length l. At the instant it makes an angle θ with the vertical as it falls, what is the tangential acceleration of the top? The moment of inertia of a rod about its end point is $\frac{1}{3}ml^2$.

2. A wheel is rotating freely at angular speed 800 rev/min on a shaft. The shaft has negligible rotational inertia. A second wheel, initially at rest and with twice the rotational inertia of the first wheel, is suddenly coupled to the same shaft. **(a)** What is the angular speed of the resultant combination of the shaft and the two wheels? **(b)** What fraction of the original rotational kinetic energy is lost?

3. In the figure below, a small, solid, uniform ball is to be shot from point A so that it rolls smoothly along a horizontal path, up a ramp, and onto a plateau. Then it leaves the plateau horizontally to land on a game board, a horizontal distance d from the right edge of the plateau. The vertical heights are $h_1 = 5.00\,\text{cm}$ and $h_2 = 1.60\,\text{cm}$. With what speed must the ball be shot at point A for it to land at $d = 6.00\,\text{cm}$? The moment of inertia of a solid sphere is $\frac{2}{5}mR^2$.



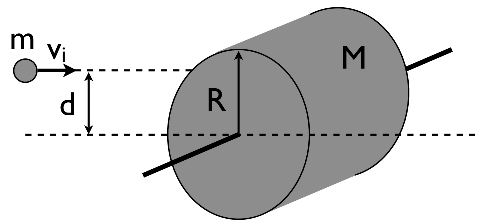
4. A (spherical) star of radius $R = 5 \times 10^5\,\text{km}$ has a rotational period of 60 days. Later in its life, its radius expands to $R = 5 \times 10^6\,\text{km}$, though its mass M remains constant. What is the new rotational period after expansion? Presume the star's moment of inertial is kMR^2 at all times.

5. A small body of mass m hangs in equilibrium at one end of a light string of length l, the upper end of which is fixed. A small body of mass m moving horizontally with velocity $2\sqrt{gl}$ strikes the former body and adheres to it. Find:

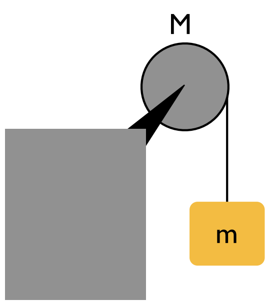
(a) the velocity with which the combined bodies begin to move,

(b) the angle through which the string turns before coming to rest for an instant,

6. A wad of sticky clay with mass m and velocity v_i is fired at a solid cylinder of mass M and radius R as shown below. The cylinder is initially at rest and mounted on a fixed horizontal axle that runs through its center of mass. The line of motion of the projectile is perpendicular to the axis and at a distance $d<R$ from the center. Find the angular speed of the system just after the clay strikes and sticks to the surface of the cylinder. The moment of inertia of a solid cylinder is $I=\frac{1}{2}MR^2$, the moment of inertia of a point particle mass m a distance R from an axis of rotation is $I=mR^2$.



7. A uniform disk with mass $M=2.5\,\mathrm{kg}$ and radius $R=20\,\mathrm{cm}$ is mounted on a fixed horizontal axle, as shown below. A block of mass $m=1.2\,\mathrm{kg}$ hangs from a massless cord that is wrapped around the rim of the disk. Find the acceleration of the falling block, the angular acceleration of the disk, and the tension in the cord. *Note: the moment of inertia of a disk about its center of mass is* $I=\frac{1}{2}MR^2$.



8. A bowler throws a bowling ball of radius R along a lane. The ball slides on the lane with initial speed v_o and initial angular speed $\omega_o=0$. The coefficient of kinetic friction between the ball and the lane is μ_k. The kinetic frictional force $\vec{\mathbf{f}}_k$ acting on the ball causes a linear acceleration of the ball while producing a torque that causes an angular acceleration of the ball. When the center of mass speed v_{cm} has decreased enough and angular speed ω has increased enough, the ball stops sliding and then rolls smoothly. **(a)** What then is the center of mass speed v_{cm} in terms of ω? During the sliding, what are the ball's **(b)** linear acceleration and **(c)** angular acceleration? **(d)** How long does the ball slide? **(e)** How far does the ball slide? **(f)** What is the linear speed of the ball when smooth rolling begins?

9. A torque τ acts on a body and rotates it about a fixed axis from angle θ_i to angle θ_f. **(a)** Prove that the work done is

$$W = \int_{\theta_i}^{\theta_f} \tau \, d\theta \tag{1}$$

(b) Show that the rate at which work is done, power, is

$$P = \frac{dW}{dt} = \tau\omega \tag{2}$$

10. **(a)** Starting from $\vec{\mathbf{F}} = \frac{d\vec{\mathbf{p}}}{dt}$, show that $\frac{d\vec{\mathbf{L}}}{dt} = \vec{\mathbf{r}} \times \vec{\mathbf{F}} = \vec{\boldsymbol{\tau}}$. **(b)** Show that if there is no external force, angular momentum is conserved.