What is the Relationship between Work and Energy?

Equipment:

PC with *DataStudio*	Pulley w/ photogate
Dynamics Track	Mass Hanger Set & String
Collision Cart	PowerLink or 2 USB Links
High Resolution Force Sensor	Digital Adapter

Purpose:

The purpose of this activity is to compare the work done on a cart to the change in kinetic energy of the cart. Determine the relationship of work done to the change in energy.

Background:

Newton's 2nd Law (F=ma) states that an object accelerates when it experiences a net force. In other words, the force causes the object to undergo a change in velocity and move through some displacement.

The work W done on an object is equal to the net force F_n multiplied by displacement d:

$$W = \mathbf{F}_n \cdot d$$

* It is important to note that because force is a vector, we only multiply the amount of force (the *component*) that acts along the direction of displacement *d*. For example, if you push a block across a table, *gravity* does zero work on the block, because gravity acts straight down and the block moves straight across. In that case, only the force of friction and the force of your push do work.

Since the object's velocity changes, we can also calculate its change in kinetic energy.

$$\Delta KE = ½ \, m \, v_f^2 - ½ \, m \, v_i^2$$

Where m is the object mass, v_f is the final speed of the object, and v_i is initial speed. If the object starts from rest, as in this experiment:

$$\Delta KE = ½ \, m \, v_f^2$$

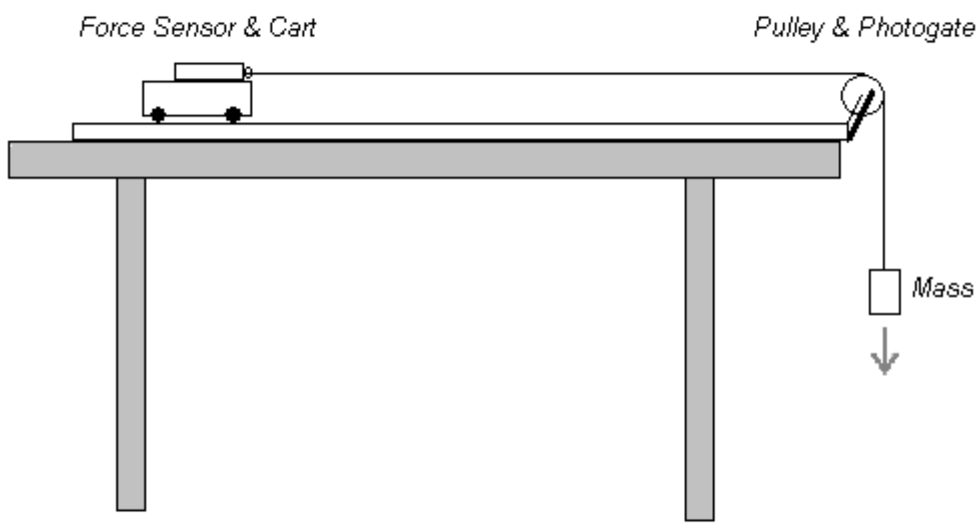
The Work-Energy Theorem states that the work done on an object is equal to its change in kinetic energy:

$$W = \Delta KE$$

Equipment Setup:

In this experiment, you will use a *Half-Atwood* machine to investigate the relationship between work and kinetic energy. A Half-Atwood machine is shown below: a string connects two objects (one object is on a surface while the other object hangs from the string) over a pulley.

1. Place the track on a horizontal surface. If the cart rolls freely toward either end, then adjust the track so it is level (the cart should not roll).
2. One end of the track should hang about two inches off the edge of the table. Attach the pulley to this end. The pulley clamps to the track, and the photogate mounts to the slot below the pulley wheel.
3. Mount the force sensor to the top of the cart.
4. Attach a string to the hook on the force sensor and put the string over the pulley. Tie the other end of the string to the mass hanger. The string should not be longer than the distance from the pulley to the floor (the height of the table).
5. Adjust the pulley so that the stretched string is parallel to the track. Adjust the photogate so that its beam travels through the spokes of the pulley. Your setup should look like this:



6. Connect the force sensor and photogate to the inteface.
7. Open DataStudio and connect the force sensor and photogate in software.
8. Create a new activity in DataStudio, and create three graphs: Force vs. Position, Acceleration vs. Position, and Velocity vs. Position.

Recording Data:

NOTE: The procedure is easier if one person handles the cart and a second person handles data collection in DataStudio.

1. Before taking data, hold the mass up with your hand so that no tension is pulling on the force sensor. Zero the force sensor. Repeat this step before each new data run.
2. Pull the cart back until the hanging mass is just below the pulley.

   NOTE: Hold the force sensor cable out to the side of the cart, so that the cable does not drag and the cart can roll freely. You will need to keep holding the cable after you let the cart go.
3. Press Start to record data. Release the cart so it moves toward the pulley. DataStudio will take data before stopping automatically.
4. Catch the cart just before it reaches the pulley. Do not let the cart hit the pulley.

NOTE: Repeat data runs until you obtain Force and Acceleration graphs that are flat. If you stop the cart too soon (before data recording stops) your data will be thrown off at the end, and there will be spikes in your data curves. Your Force graph will show a negative value because the force sensor reads negative when something *pulls* on it. For your calculations in the analysis section, take the absolute value of your Force reading.

Analysis

1. Measure the mass of the cart/force sensor together; record this value in the Data table. If the cart's mass is given, add 95 grams (0.095 kg) due to the mass of the force sensor.
2. Print the graphs from your best data run. If a printer is not available, sketch your graphs in the Student Data section.
3. DataStudio should display maximum/minimum/mean values for each graph. If not, select the Force graph. Under the **Σ** menu, select 'Mean' and 'Area'. DataStudio will highlight and display the average force value and the total area under the curve. Record these values in the Data table.

   NOTE: The area under the force curve equals the total work done on the cart. Taking the area is equivalent to multiplying the force value from the y-axis times the total displacement from the x-axis -- $W = F \cdot d$. In calculus, taking this area under the curve is called 'taking an integral'.
4. Mean and Maximum values should also be displayed in the Velocity and Acceleration graphs. If not, select the Velocity graph. Under the **Σ** menu, select 'Maximum'. DataStudio will display the maximum velocity value from your graph. Next select the Acceleration graph; under the **Σ** menu, click on 'Mean'. DataStudio will display the average acceleration value from your data. Record these values in the Data table.
5. Find the cart's displacement due to the tension force. You can find this value using your graphs in DataStudio. The x-axis of your graphs shows how far the cart traveled. To find this distance exactly, select the 'Smart Tool' cursor () and click on the last data point in your Velocity graph; the x-coordinate is the distance.

Data Sheet

Name(s):____

Period: ___________

Date: ___________

Predictions

1. As work is done to accelerate a cart, what will happen to its kinetic energy?

2. If the cart starts from rest, how would the work done on the cart compare to its final kinetic energy? How about if the cart is already moving when we do work on it?

Sketch your graphs below. Each graph displayed in DataStudio shares a position axis. Include units and labels for your axes.

Data Table

Total mass of cart (kg)	
Maximum velocity	
Average acceleration	
Average force	
Area under Force curve	
Displacement	

Calculations

Use the total mass of the cart and the final (maximum) velocity to calculate the final kinetic energy of the cart. Compare this value to the work done (the area under the position curve). We can also use our acceleration graph to calculate work done. From the average acceleration value, calculate the tension force on the cart (F=ma). Check this value by comparing it with the average force value you entered above. Then, to calculate work done, multiply the tension force times the displacement.

Calculate the percent difference between the final kinetic energy and the work done on the cart, for both work values you calculated.

$$\%Difference = \left| \frac{W - KE}{\left(\frac{W + KE}{2} \right)} \right| \times 100 .$$

Questions

3. What happens to the kinetic energy as work is done on the cart?

4. How does the final kinetic energy compare to the work done?

5. The kinetic energy is measured in joules and the work done is measured in newton•meters (N m). What is the relationship between a joule and a newton•meter?

6. Do your results support your predictions?