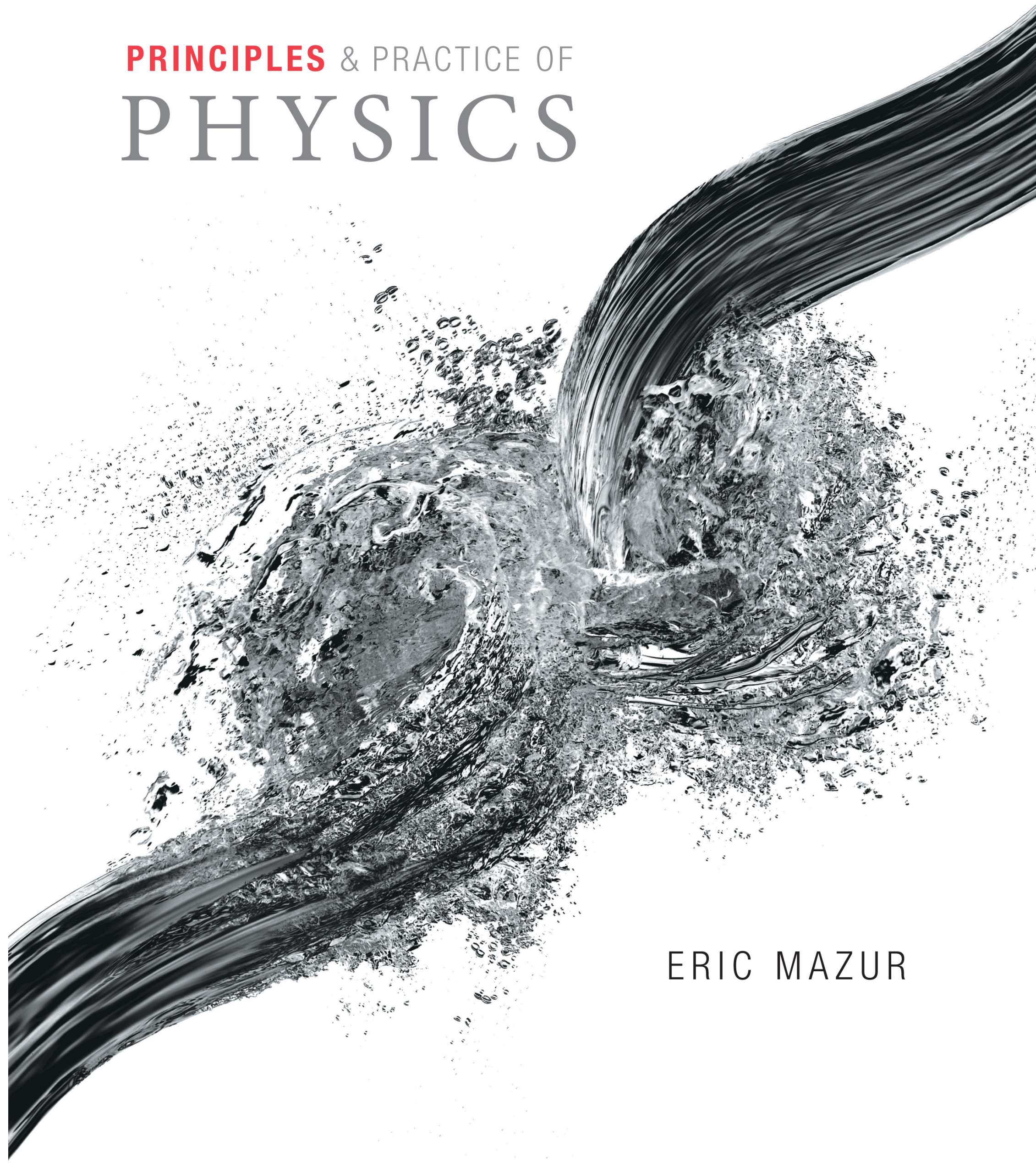
Chapter 2
Motion in One Dimension



MasteringPhysics, PackBack Answers

- You should be on both by Tuesday.
- MasteringPhysics – first reading quiz Tuesday

- PackBack – should have email & be signed up. Can use course code if you didn't get email.

PackBack Answers

- Try to ask questions you are curious about
- Don’t just use book discussion questions, ideally
  - unless that is what you are curious about …
- Can be about physics we don’t cover
- the volume of questions can be high at first
  - try to browse and see what has already been asked
  - provide answers & up/down vote as you browse

PackBack

Asked by **Parker Durkac**
at The University of Alabama

When kicking a soccer ball, how is it possible to curl a kick?

I played soccer for 12 years, and I have always noticed how we can curl a ball in the air. I was wondering how the ball is able to make a curve mid flight?

4:48 PM, 1/18/2017 Options ▾

Add your own Response

Answered by **Sam Howard**
at The University of Alabama

The ball is able to make a curve in mid flight due to the rotational spin you put on the ball when you kick it. When kicking it, you either kick it on the ball's left or right side causing it to spin the opposite way which makes it curve while in flight.

7:37 PM, 1/18/2017 Options ▾

Answered by **Hudson Nicholson**
at The University of Alabama

Just like the other response said, the ball begins to spin in the air after you kick it a certain way. However he didn't mention that it is the ball's reaction with the surrounding air then that causes it to curve.

1:49 AM, 1/19/2017 Options ▾

Labs

- first procedure – link on MasteringPhysics
- read ahead of time
- can print there (one per group!)

- I set the guidelines, but the TAs are in charge

- start of each lab – work on problems

A data-driven approach

watch an object fall

- record position vs time
- what can we find out?

t (s)	x (m)
0	0
1	5
2	20
3	45
4	80
5	125

- (clearly not real data)

A data-driven approach

what's the plan?

- see what we can figure out from data alone
- need an abstraction – a **model** for how it falls
  - come up with a hypothesis from data
  - what does it predict?
  - test it

A data-driven approach

what's the hypothesis?

- we don't have great everyday intuition
  - falling happens too fast (hang time?)
  - no accurate timing
- let's try something that seems plausible

A data-driven approach

possible hypothesis 1

for every increment of time Δt, the object falls the same distance Δx.

Mathematically: $\Delta x \propto \Delta t$, or Δx vs Δt is a straight line

A data-driven approach

nice idea, but wrong

Possible hypothesis 1: for every increment of time Δt, the object falls the same distance Δx.

If this were true, the Δx should be the same for identical time intervals, or $\Delta x/\Delta t$ should be constant

(We know now this means constant velocity.)

A data-driven approach

data says:

Hypothesis 1 is wrong. The object speeds up as it falls.

time t (s)	distance x (m)	rate distance changed $\Delta x/\Delta t$ (m/s)
0	0	–
1	5	5
2	20	15
3	45	25
4	80	35
5	125	45

A data-driven approach

look carefully

- The rate of position change also increases
- This rate of change is *velocity* – it also increases
  - but it increases by the same amount each second!
  - suggests *its* rate of change is constant.

time t (s)	distance x (m)	rate distance changed $\Delta x/\Delta t$ (m/s)
0	0	–
1	5	5
2	20	15
3	45	25
4	80	35
5	125	45

A data-driven approach

a new hypothesis

Hypothesis 2: The rate at which the velocity changes is constant.

Finding a constant of motion is a big deal.

No matter what happens, this thing doesn’t change. It must be kind of a big deal.

A data-driven approach

try it

Indeed! The position and velocity continuously increase, but the rate of velocity change is constant.

This is *acceleration:* $$\frac{dv}{dt} = \frac{d}{dt}\frac{dx}{dt} = \frac{d^2x}{dt^2}$$

time t (s)	distance x (m)	average velocity $\bar{v}$ (m/s)	rate velocity changes $\Delta\bar{v}/\Delta t$ (m/s^2)
0	0	–	–
1	5	5	–
2	20	15	10
3	45	25	10
4	80	35	10
5	125	45	10

A data-driven approach

what did we figure out?

- The rate of change of velocity is (essentially) constant for a falling object
- We call this rate of change **acceleration**
- Velocity increases linearly with time
- Since x=dv/dt, this means x(t) increases quadratically

Tuesday's lab

Uncertainty analysis

- Is there a good everyday example?
- What good is standard deviation anyway?

Sciencing

- So you have an idea.
- This idea must be *testable* ... or it is not science
- So we test it.

- How good is our test? How well did it work?
  - a measure of the result & accuracy
  - does it make any sense? predict something else ...

SCIENCE IN ACTION

"I'm *Sciencing* as fast as I can!"
- Professor Farnsworth

SCIENTIFIC METHOD

1. Make an Observation - "What is happening?"

An Observation is when you notice something in the world around you and decide you want to find out more about it.

2. Define the Question - "Why is this happening?"

Defining the Question creates an idea that can be tested using a series of Experiments.

3. Form a Hypothesis - "I think this happens because..."

A Hypothesis is a statement that uses a few observations, without any experimental evidence, to define why something happens.

4. Perform Experiments - "Let's test my Hypothesis..."

An Experiment is a series of tests to see if your Hypothesis is correct or incorrect. For each test, record the data you discover.

5. Analyze the Data - "Was my Hypothesis right?"

Analyzing data takes what you found in your experiments and compares it to your Hypothesis. If needed, perform another Experiment to gather better data.

6. Conclusion - "Experiments show my hypothesis was..."

Forming a Conclusion presents the Experimental Data and explains how it proves or disproves your Hypothesis. Often, Scientists will take this Conclusion and perform other Experiments on it to discover new things.

Example

- Does a heavier or lighter object fall faster?
- can't do just one experiment
  - by "chance" one will land first
- do it many times
  - what is the variability?
  - how much is too much?
  - at what point are they basically the same?

Measurements

- we don't do just one
- make a series of measurements, average them
- this should improve accuracy, right?
- how much? when to stop?

- need to quantify degree of uncertainty

An example: counting cards

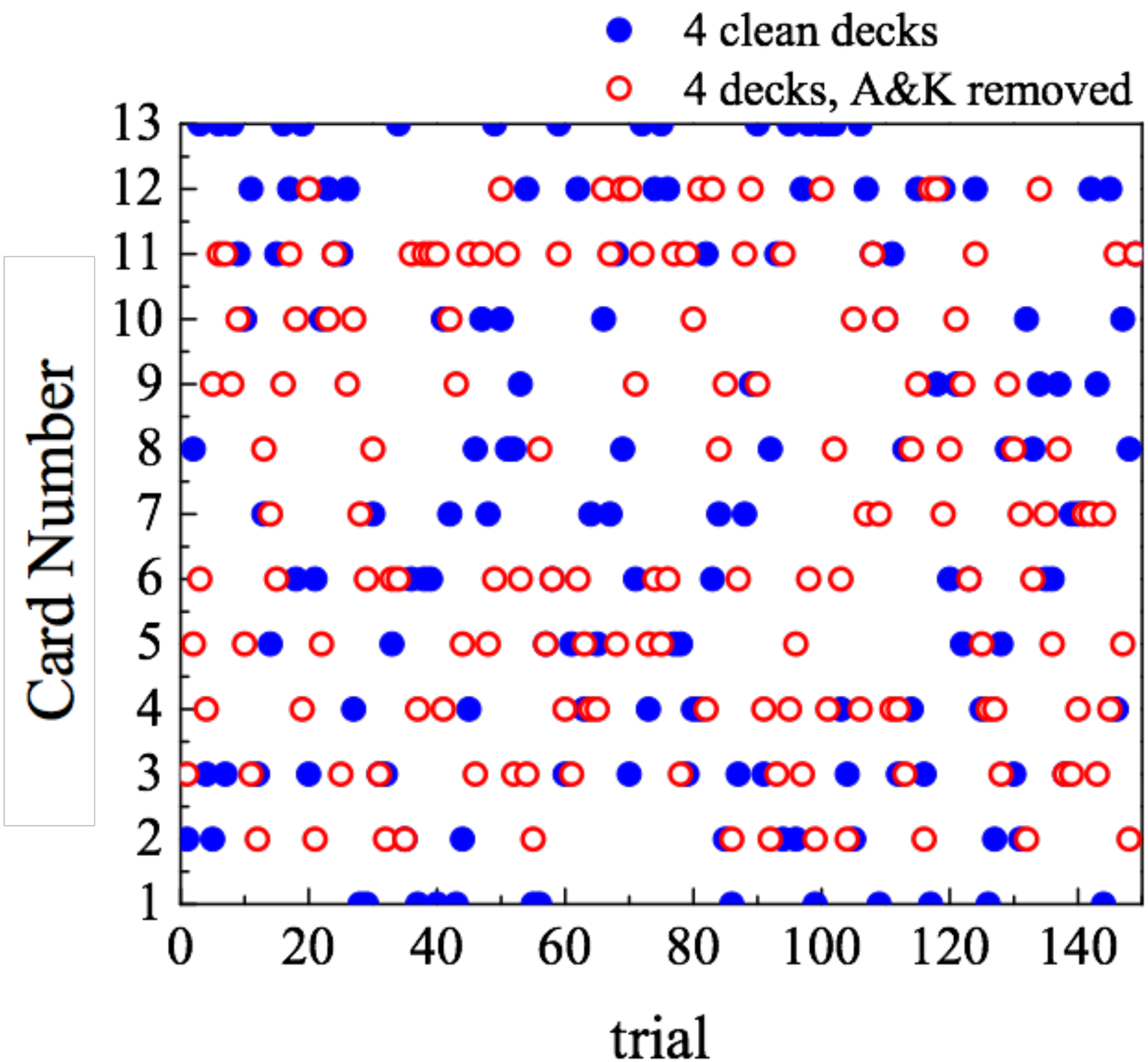
- one measurement vs. many
- how does accuracy improve?
- how to measure accuracy?
- statistical measures of uncertainty & dispersion

- if you don't see the whole deck at once, what can you still say?

The experiment: picking cards

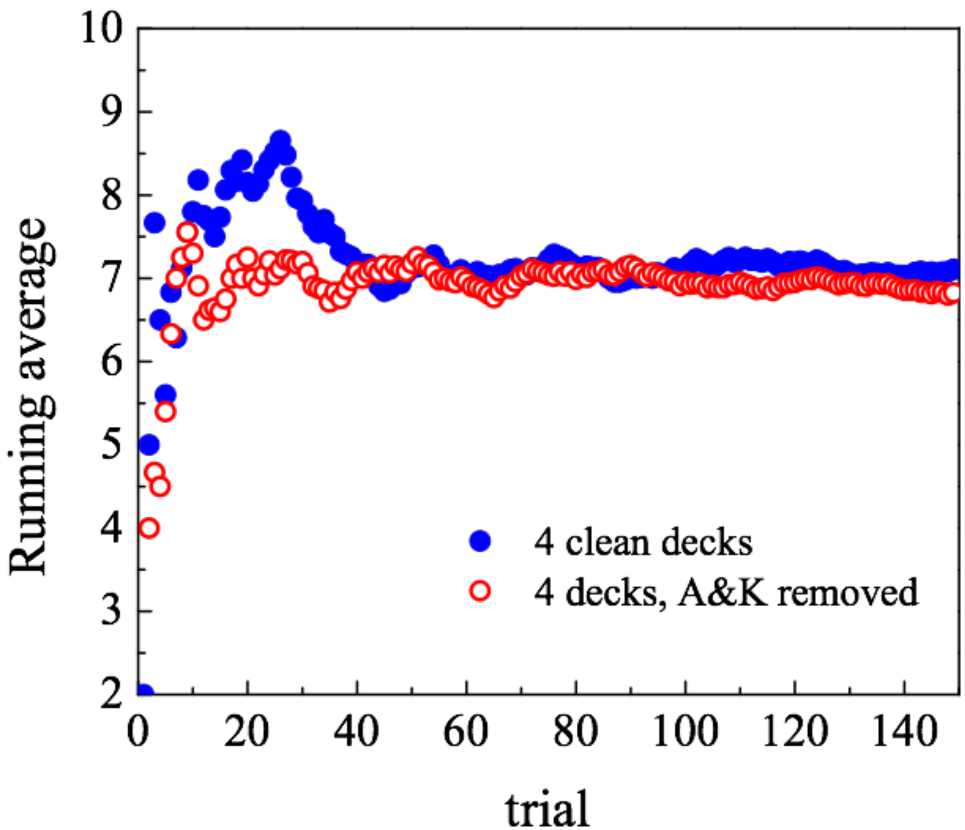
- give each one a number
- Ace = 1, 2 = 2 ... Jack = 11 ... King = 13
- what is the average card?
  - we expect it must be 7 ...
- what is the spread? how to define this?

150 trials ...



4 clean decks
4 decks, A&K removed
Card Number
trial
0
20
40
60
80
100
120
140
1
2
3
4
5
6
7
8
9
10
11
12
13

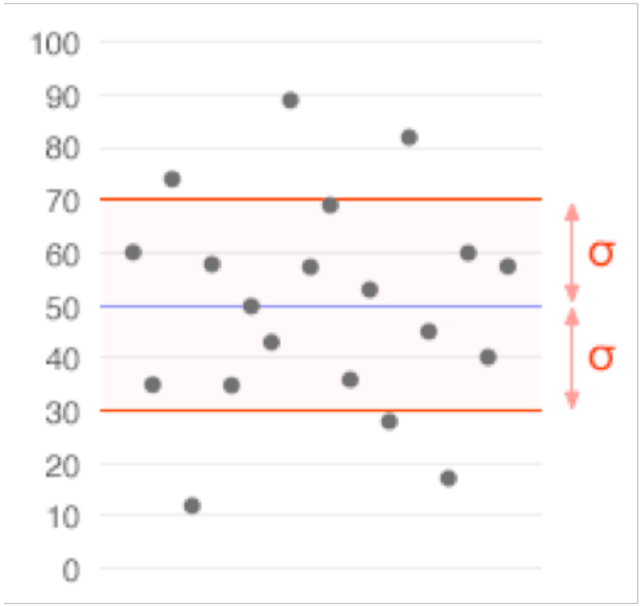
draw 1 card, record, shuffle, repeat
cards have values 1-13, equal number of each
average must be 7, if one chooses enough cards
takes ~50 before 'luck' is irrelevant!



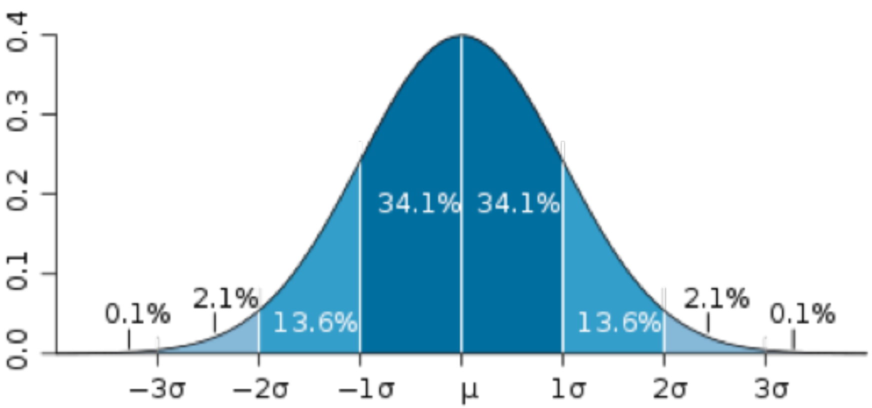
The mean isn't enough. how about the dispersion?

standard deviation is a measure of the variability dispersion in a population or data set

low standard deviation: data tends to lie close to the average (mean)
high standard deviation: data spread over a large range



data set: data clustered about average



many trials: follow a *distribution*

~68% within +/- 1 standard deviation
~95% within +/- 2 standard deviations
~99.7% within +/- 3 ...

so what?

- knowing the standard deviation tells you
  - if subsequent measurements are outliers
  - what to expect next, on average
  - *accuracy* of a set of data
  - variability in a large batch
- "six sigma" - quality control
  - means one out of 500 million!

so what?

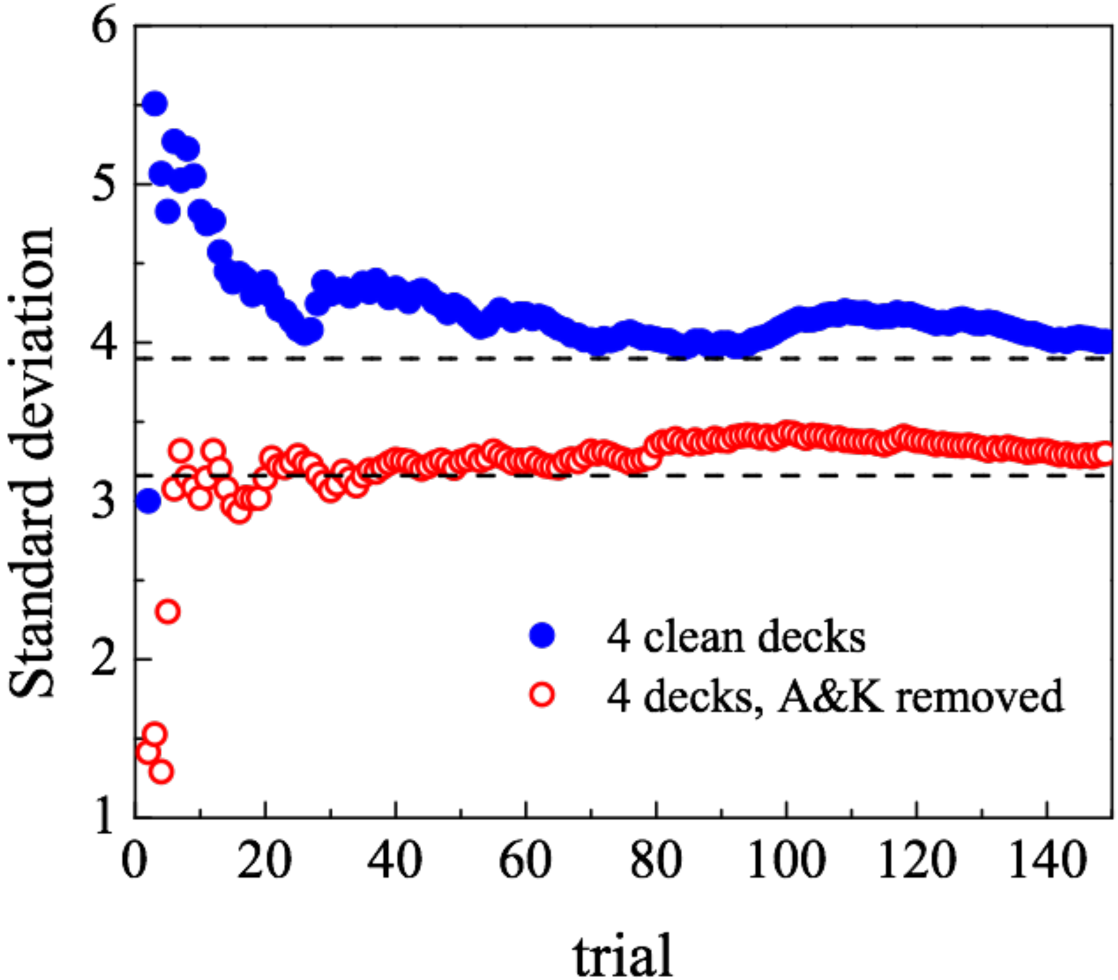
if the mean of the measurements is too far away from the prediction compared to the standard deviation, then the theory being tested probably needs to be revised!

particle physics: 5-sigma standard typical
1 out of 1.7M chance of false positive

larger signal than that ... probably a new effect!

for the cards?

take out highest and lowest cards,
data is more tightly distributed
lower standard deviation!



wait, there's more

expect 75% of cards within 2 standard deviations of average

or, 75% are within about 4 cards from the average after 100 trials

or, 75% of cards should be between 3 and Jack (inclusive)

It works!

flip side: we could estimate the distribution of cards without prior knowledge

- from a run of cards, if long enough, could say what's missing or in excess
- could take many samples of the deck though (e.g., removing all 7s)

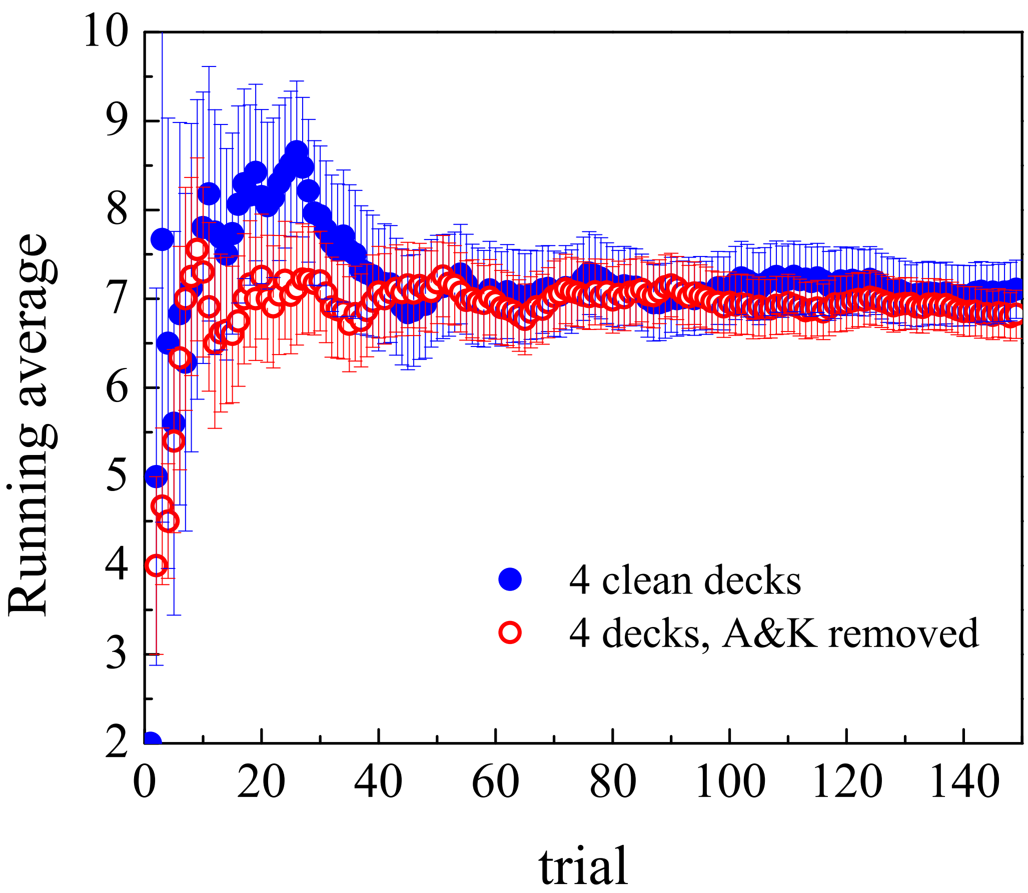
what else?

- standard deviation gives accuracy of averages
- if you do n measurements, average is more accurate for higher n. makes sense!
- uncertainty of the average is standard deviation over the root of the number of measurements

$$\sigma_{\overline{x}} = \frac{\sigma}{\sqrt{n}}$$

$$(\text{best value of } x) = \overline{x} \pm \sigma_{\overline{x}}$$

Can now add error bars to mean measurements



Accuracy goes as √N

Know when to stop
"good enough"

Know when difference is significant

Problem

1. Water is poured into a container that has a leak. The mass m of the water is as a function of time t is

$$m = 5.00t^{0.8} - 3.00t + 20.00$$

with $t \geq 0$, m in grams, and t in seconds. At what time is the water mass greatest?

Solution

Find: The time t at which the water mass m is greatest. This can be accomplished by finding the time derivative of $m(t)$ and setting it equal to zero, followed by checking the second derivative to be sure we have found a maximum.

Sketch: It is useful to plot the function $m(t)$ and graphically estimate about where the maximum should be, roughly.[i]

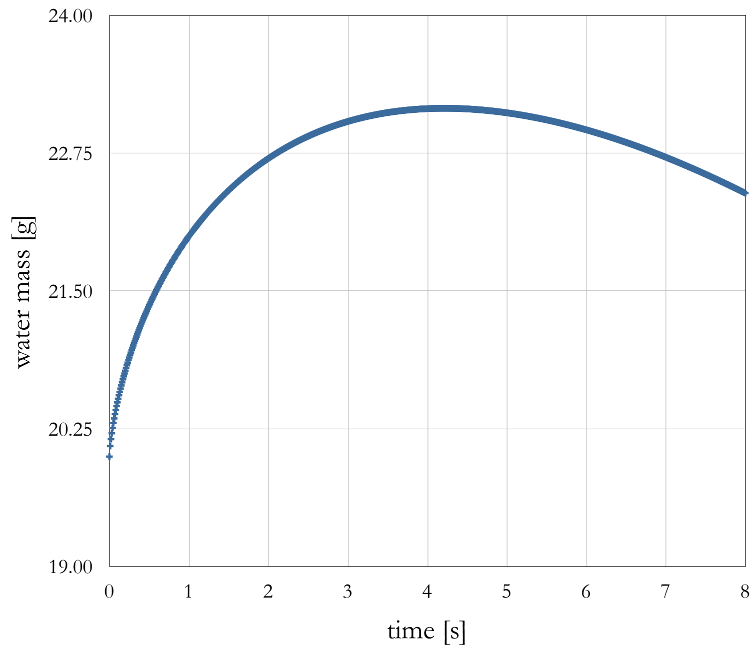


Figure 1: *Water mass versus time, problem 1. Note the rather expanded vertical axis, with offset origin.*

It is clear that there is indeed a maximum water mass, and it occurs just after $t = 4\,\mathrm{s}$.

Solution (contd.)

Relevant equations: We need to find the maximum of $m(t)$. Therefore, we need to set the first derivative equal to zero. We must also check that the second derivative is negative to ensure that we have found a maximum, not a minimum. Therefore, only two equations are needed:

$$\frac{dm}{dt} = \frac{d}{dt}\left[m(t)\right] = 0 \qquad \text{and} \qquad \frac{d^2m}{dt^2} = \frac{d^2}{dt^2}\left[m(t)\right] < 0 \quad \Longrightarrow \quad \text{maximum in } m(t)$$

Symbolic solution:

$$\begin{aligned} \frac{dm}{dt} &= \frac{d}{dt}\left[5t^{0.8} - 3t + 20\right] = 0.8\left(5t^{0.8-1}\right) - 3 = 4t^{-0.2} - 3 = 0 \\ 4t^{-0.2} - 3 &= 0 \\ t^{-0.2} &= \frac{3}{4} \\ \Longrightarrow \quad t &= \left(\frac{3}{4}\right)^{-5} = \left(\frac{4}{3}\right)^{5} \end{aligned}$$

Thus, $m(t)$ takes on an extreme value at $t = (4/3)^5$. We did not prove whether it is a maximum or a minimum however! This is important . . . so we should apply the *second derivative* test.

Recall briefly that after finding the extreme point of a function $f(x)$ via $df/dx|_{x=a} = 0$, one should calculate $d^2f/dx^2|_{x=a}$: if $d^2f/dx^2|_{x=a} < 0$, you have a maximum, if $d^2f/dx^2|_{x=a} > 0$ you have a minimum, and if $d^2f/dx^2|_{x=a} = 0$, the test basically wasted your time. Anyway:

$$\begin{aligned} \frac{d^2m}{dt^2} &= \frac{d}{dt}\left[\frac{dm}{dt}\right] = \frac{d}{dt}\left[4t^{-0.2} - 3\right] = -0.2\left(4t^{-0.2-1}\right) = -0.8t^{-1.2} \\ \frac{d^2m}{dt^2} &< 0 \quad \forall \quad t > 0 \end{aligned}$$

Since $t^{-1.2}$ is always positive for $t > 0$, $\frac{d^2m}{dt^2}$ is always less than zero[ii], which means we have indeed found a maximum.

Solution (contd.)

Numeric solution: Evaluating our answer numerically, remembering that t has units of seconds (s):

$$t = \left(\frac{4}{3}\right)^5 \approx 4.21399 \xrightarrow[\text{digits}]{\text{sign.}} 4.21\,\mathrm{s}$$

The problem as stated has only three significant digits, so we round the final answer appropriately.

Double check: From the plot above, we can already graphically estimate that the maximum is somewhere around $4\frac{1}{4}$ s, which is consistent with our numerical solution to 2 significant figures.

[ii] You can read the symbol $\forall$ above as "for all." Thus, $\forall\ t > 0$ is read as "for all t greater than zero."

Problem

6. Find for me the numerical value I of the following integral, by any means necessary. No work need be shown for this problem, but do note how you obtained the answer.

$$I = \int_{0}^{1.026955} 3\sin\left(x^2\right)\,dx$$

Solution

Solution: There is no analytic solution to this integral. You'll need a numerical technique, the simplest of which is to ask Wolfram Alpha. Try something like this (clickable link):

`http://www.wolframalpha.com/input/?i=integral+of+sin%28x%5E2%29+from+0+to+1.026955`

To 6 significant digits, the answer is 1.00000. This is one of the *Fresnel Integrals*, and they come up frequently in optics, among other things. `http://en.wikipedia.org/wiki/Fresnel_integral`

Try it!