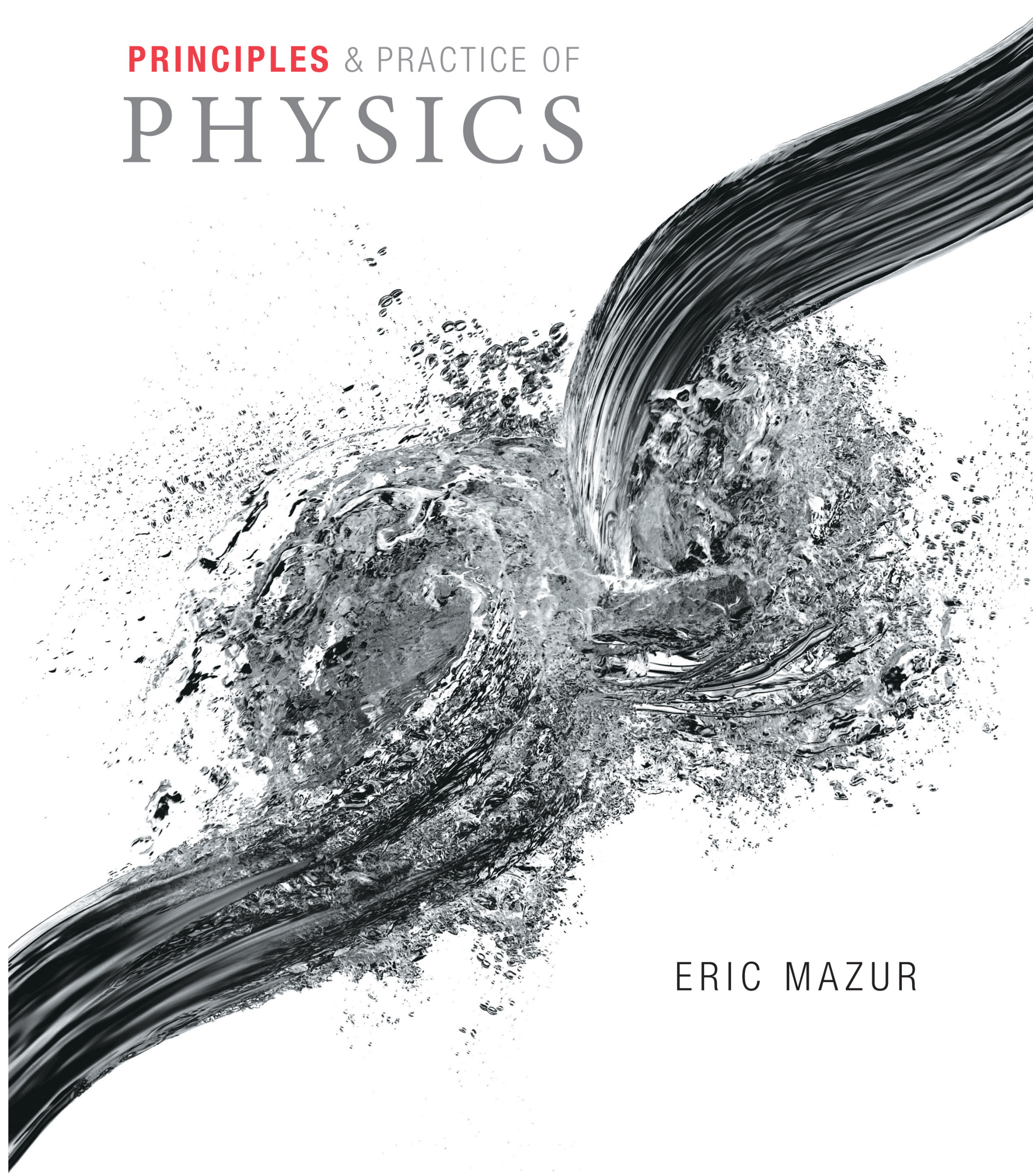
Chapter 16 Waves in One Dimension



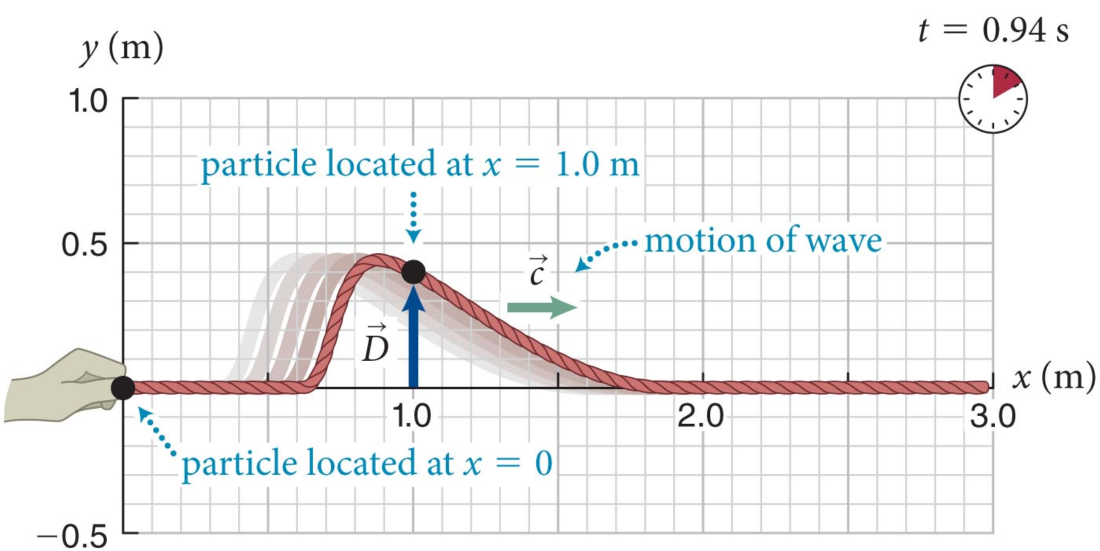
Concepts

Section Goals

You will learn to

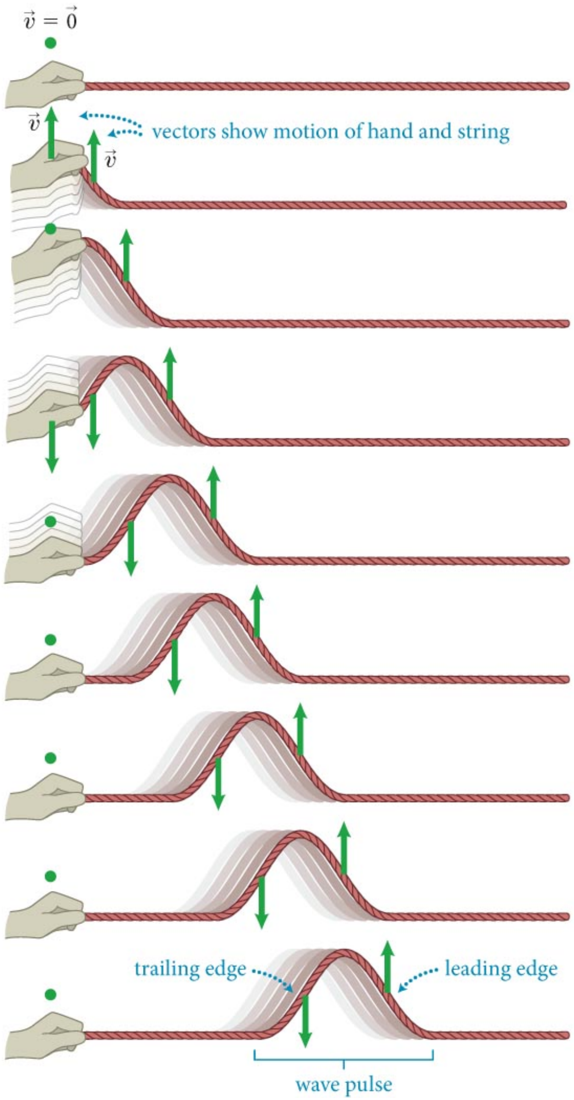
- Visualize waves in one dimension using **frame-sequence diagrams** and displacement curves.
- Differentiate the **wave speed**, *c*, and the speed of the **particles** of the medium, *v*

(*a*) Snapshot of wave at $t = 0.94$ s



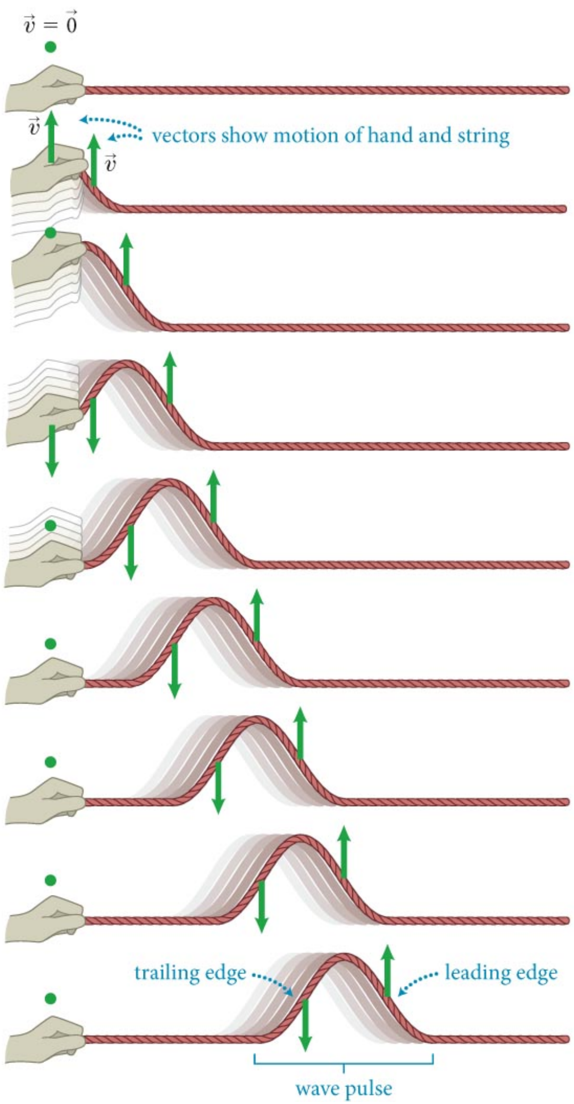
Section 16.1: Representing waves graphically

- A **transverse wave** is a wave in which the medium movement is *perpendicular* to the **wave pulse** movement.
- For example, a wave pulse travels along a string in a horizontal direction while the particles that make up the string move up and down.



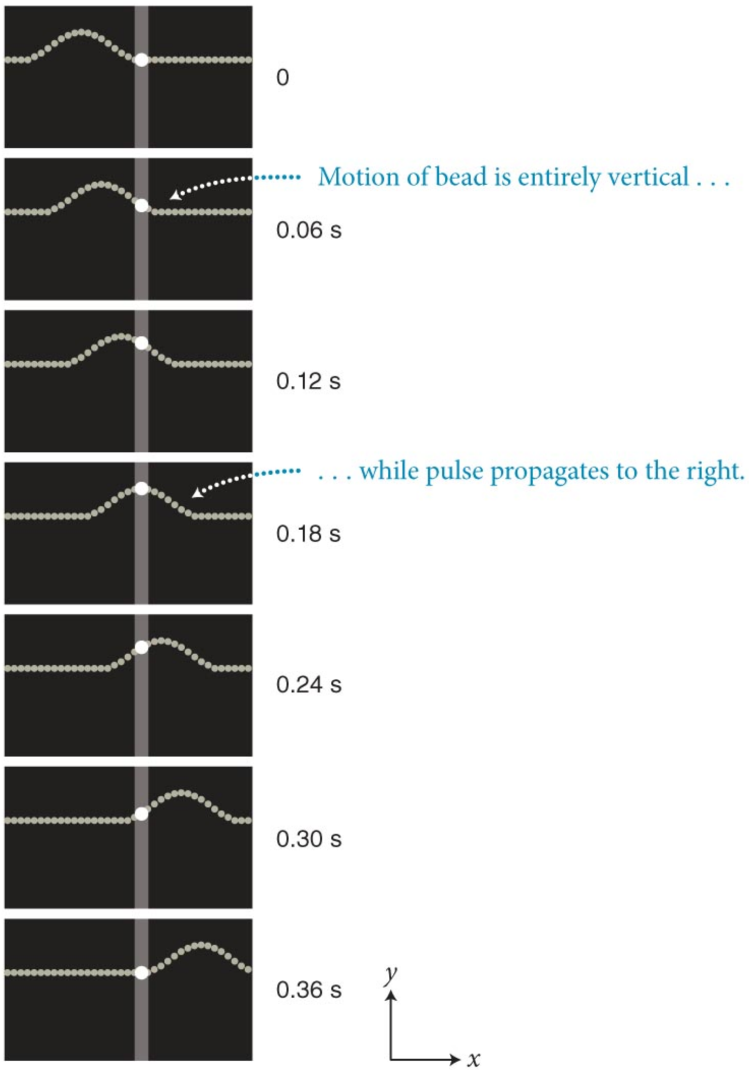
Section 16.1: Representing waves graphically

- As we can see from the figure, it is important to realize that
  - **The motion of a wave (or of a single wave pulse) is distinct from the motion of the particles in the medium that transmits the wave (or pulse).**
  - **Wave moves right, particles move up & down**



Section 16.1: Representing waves graphically

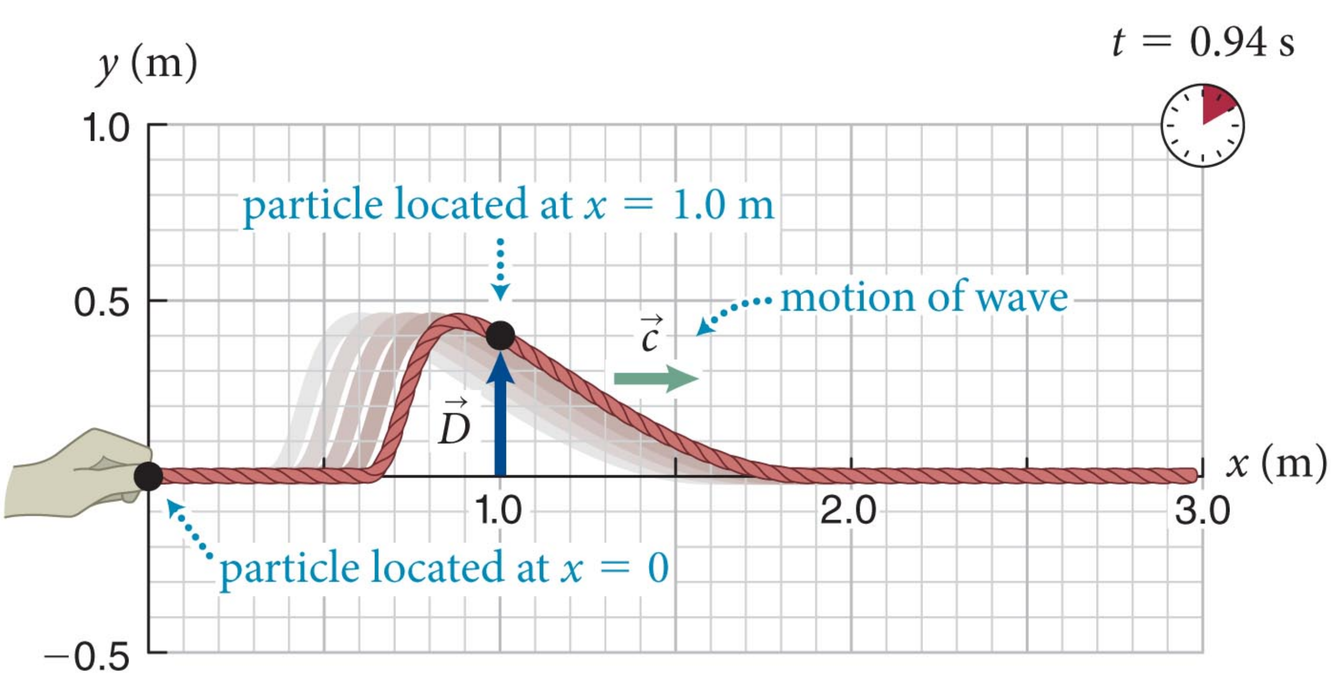
- **The wave speed *c* of a wave pulse along a string is constant.**



Section 16.1: Representing waves graphically

- Part (*a*) shows a “snapshot” of a triangular pulse traveling along a string.
- The vector $\vec{D}$ represents the displacement of one particular particle located at a position x along the string at a given instant t.

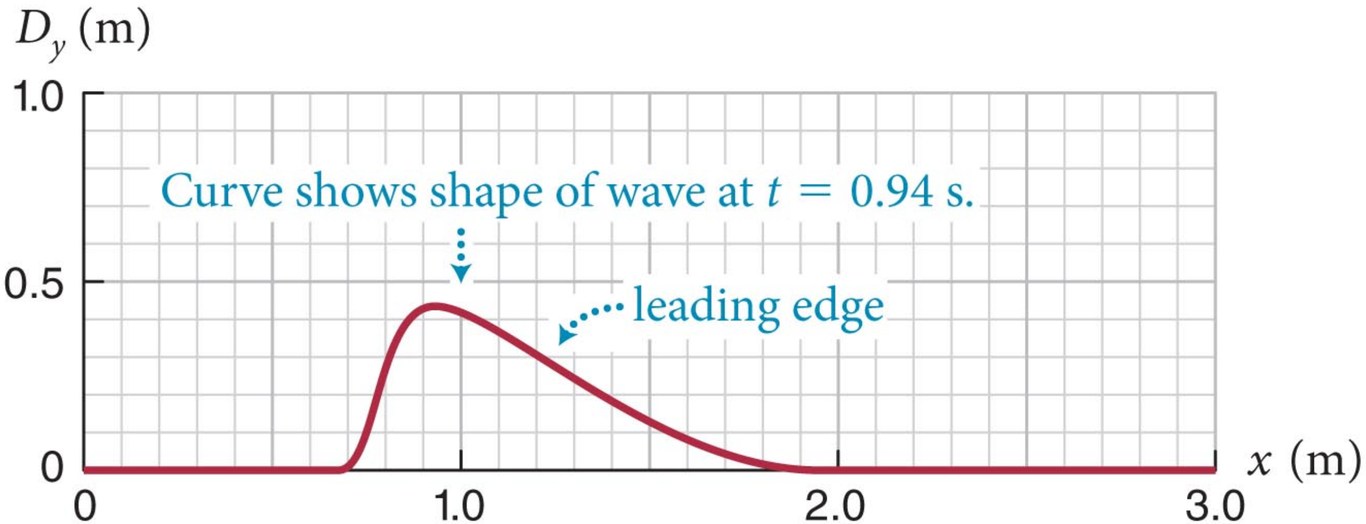
(*a*) Snapshot of wave at t = 0.94 s



Section 16.1: Representing waves graphically

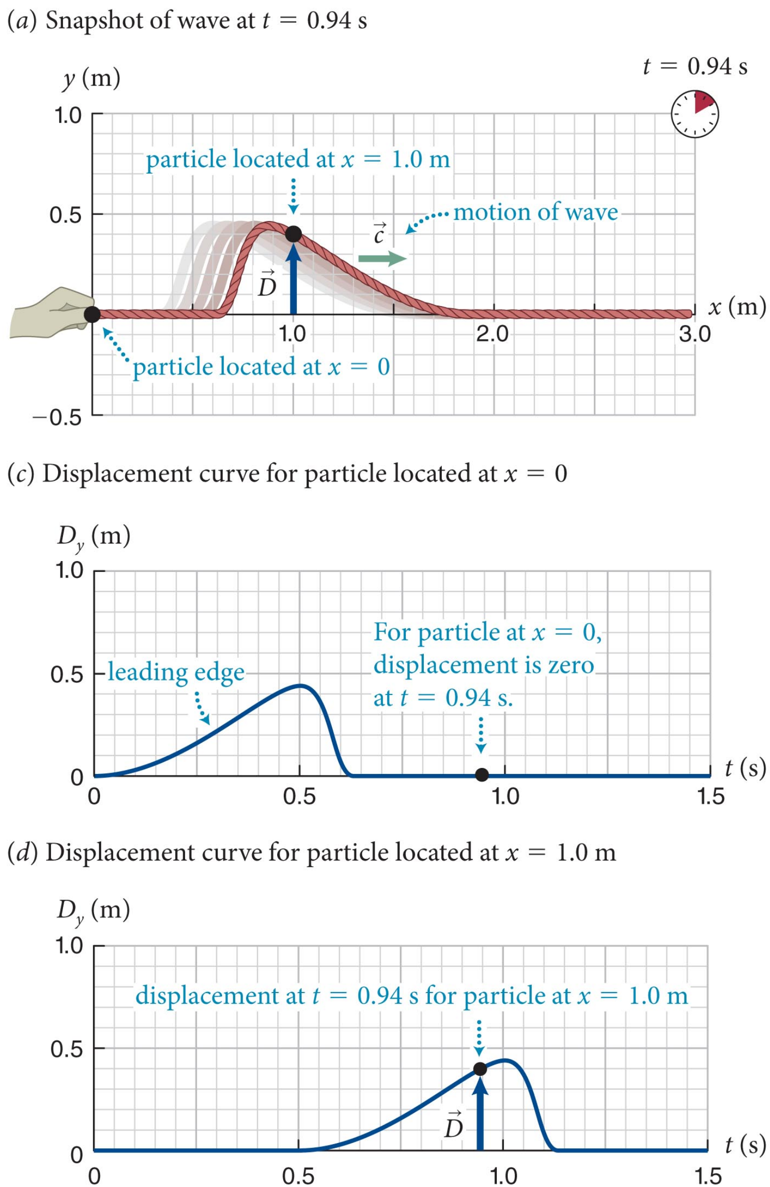
- The graphical representation of all the particle displacements at a given instant is shown in part (*b*).
- The curve gives the *y* components of the displacements of the particles of the string as a function of the position *x* along the string.
- This is called the **wave function.**

(*b*) Wave function at $t = 0.94$ s



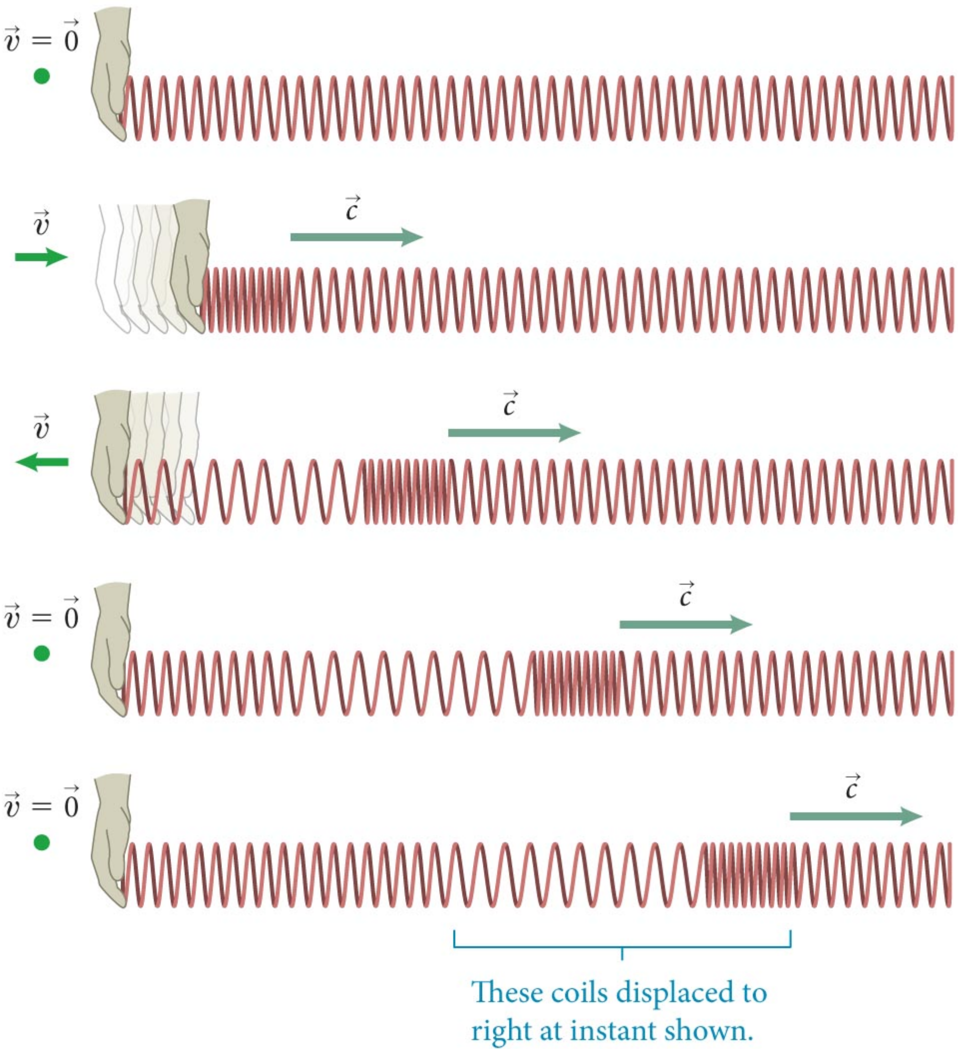
Section 16.1: Representing waves graphically

- The wave pulse shown in part (*a*) can also be represented by plotting the displacement of one particle on the string as a function of time.
- Such a plot gives us the **displacement curve** of the wave pulse.
- Parts (*c*) and (*d*) show the displacement curve for the particle at $x = 0$ m and $x = 1.0$ m. Note they are mirror images of each other



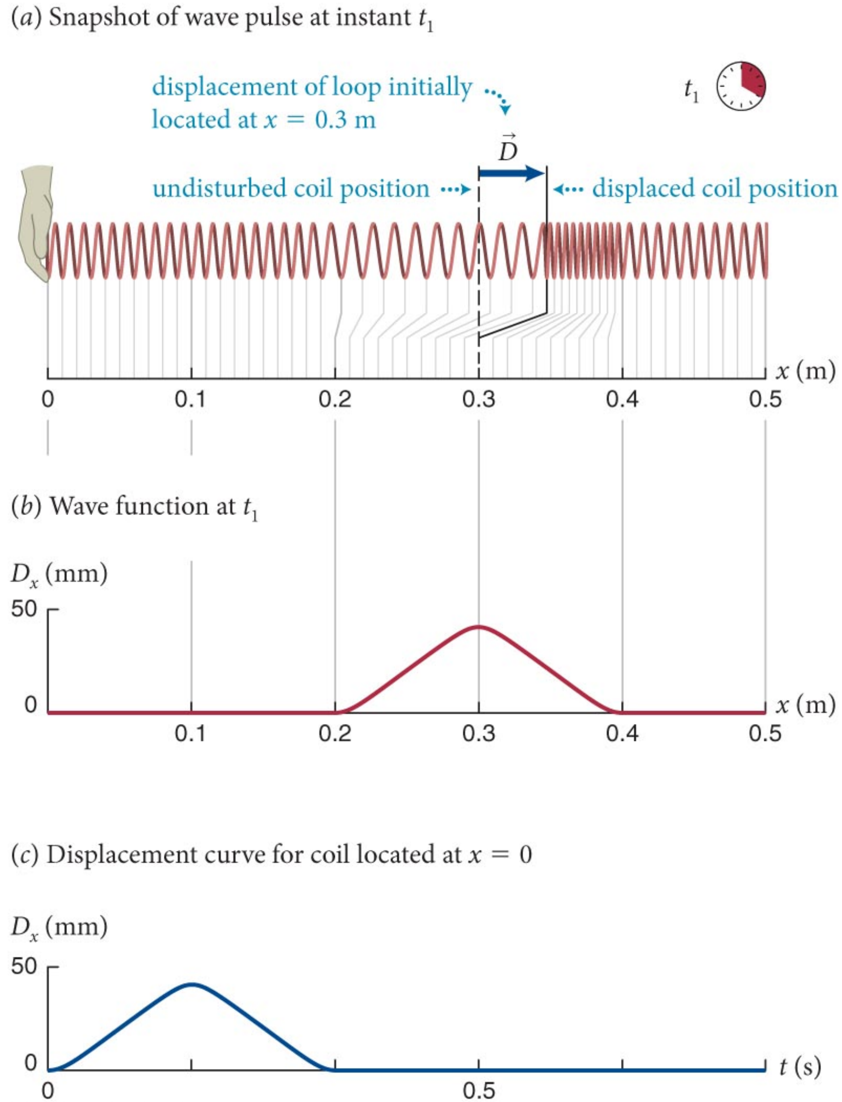
Section 16.1: Representing waves graphically

- In a **longitudinal wave**, the medium movement is *parallel* to the pulse movement.
- Here we see a longitudinal wave propagating along a spring.
- If you rapidly displace the left end of the spring back and forth, a disturbance travels down the spring.
- Both waves and medium elements move to the right



Section 16.1: Representing waves graphically

- As shown in the figure, a longitudinal wave can also be represented by wave functions and displacement curves.



Section 16.2: Wave propagation

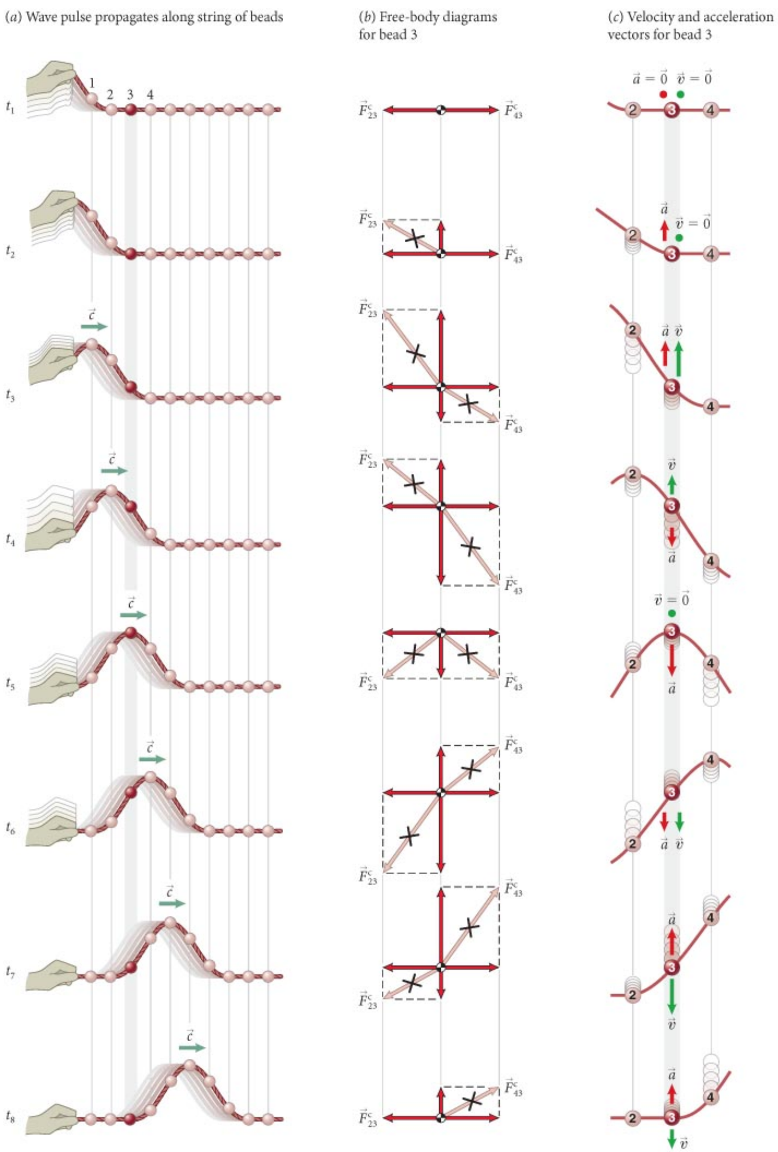
Section Goals

You will learn to

- Analyze the **kinematics** of wave motion and the **time-evolution** of the initial disturbance.
- Differentiate carefully between the **motion of the wave** itself and the **motion of the particles** of the medium the wave propagates through.
- Define the **wavelength** of a periodic wave.

Section 16.2: Wave propagation

- Let us consider a collection of beads connected by short strings.
- You pull on the first bead, it pulls on the second …
- Initial disturbance is the only cause of motion, so motion of any bead related to it alone

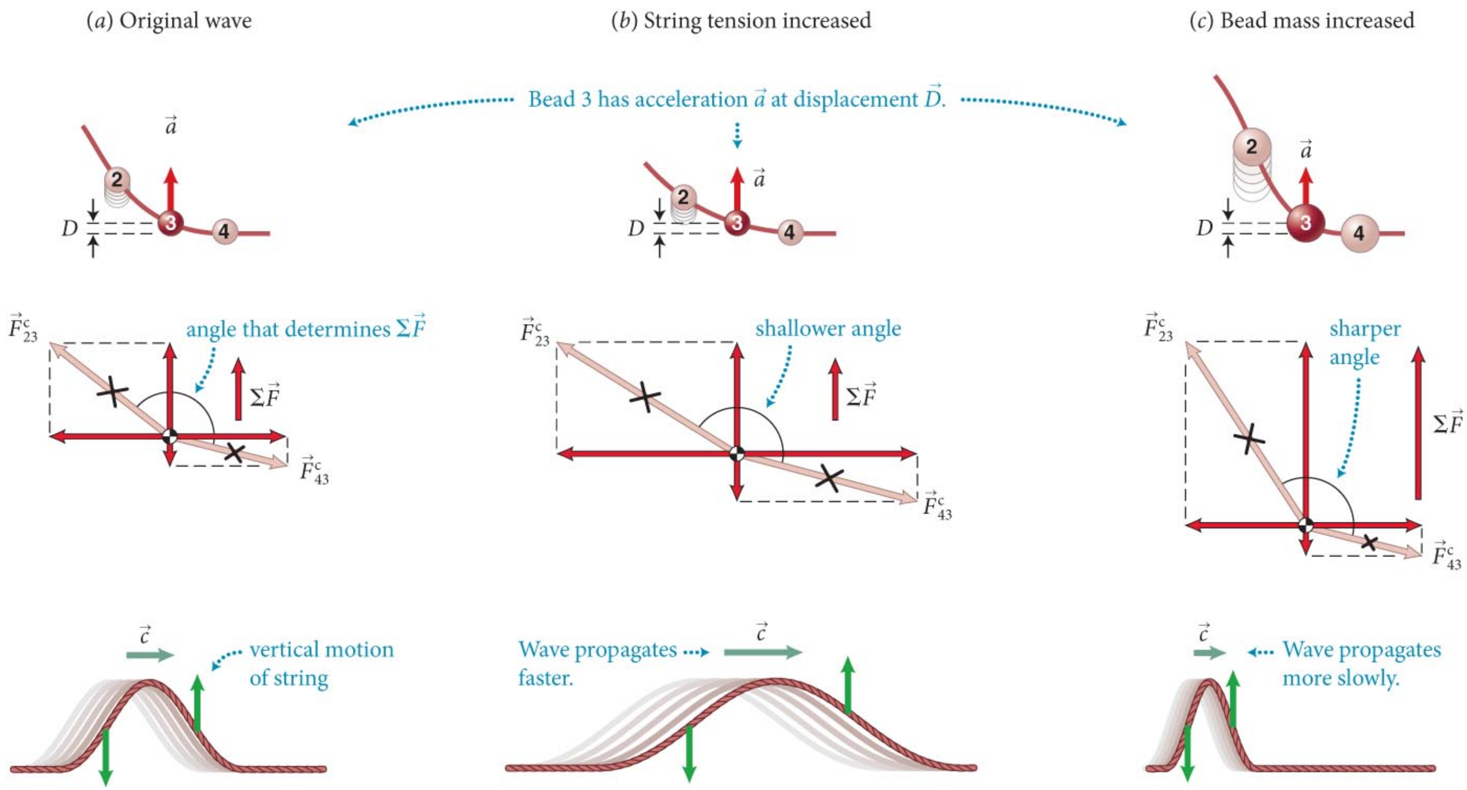


Section 16.2: Wave propagation

- Two somewhat counterintuitive points:
  1. **When a particle of the string is displaced from its equilibrium position, its velocity *v* and acceleration *a* are determined only by the initial disturbance and are independent of the wave speed *c*.**
  2. **For a given disturbance, high wave speeds yield wave pulses that are stretched out and low wave speeds result in pulses that are more compressed.**

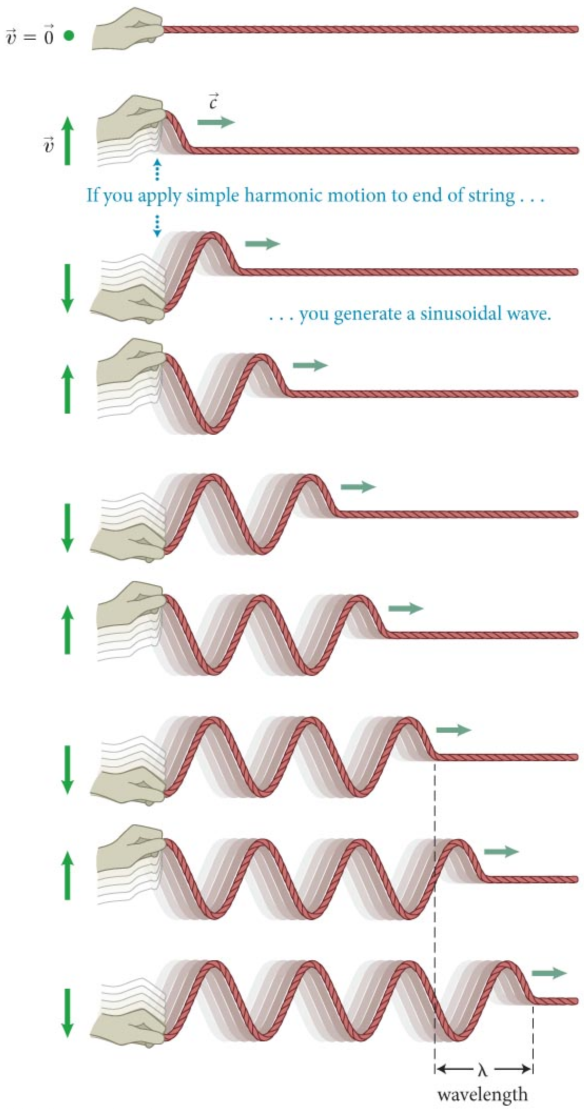
Section 16.2: Wave propagation

- As illustrated in the figure, we can determine that
  - **The speed *c* of a wave propagating along a string increases with increasing tension in the string and decreases with increasing mass per unit length along the string.**
  - **Think about guitar strings**



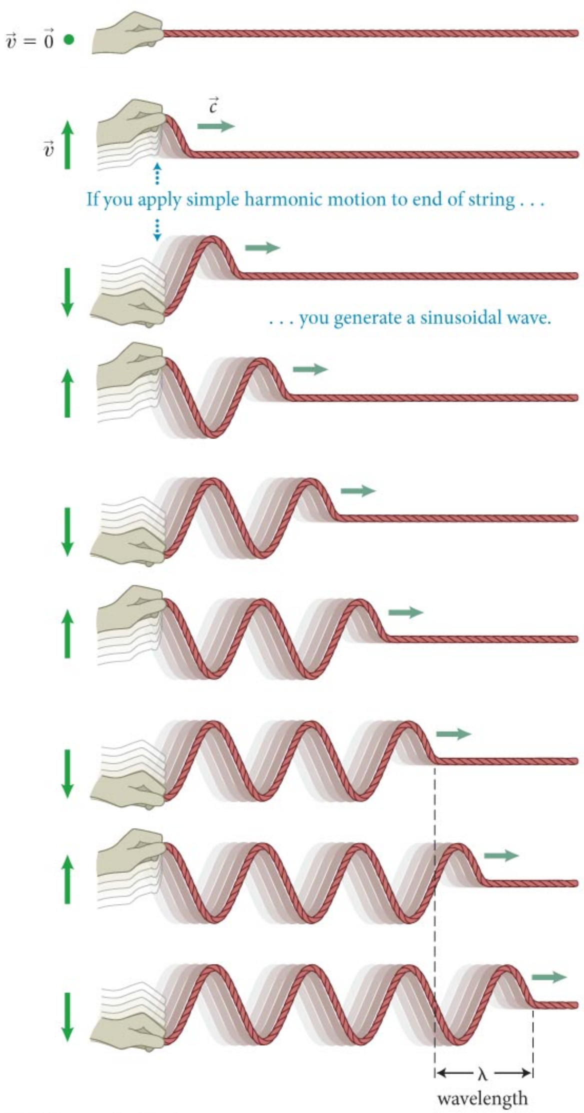
Section 16.2: Wave propagation

- If one end of a string is made to execute a periodic motion, the resulting wave is called a **periodic wave.**
- A **harmonic wave,** shown in the figure, is a type of periodic wave obtained by moving the end of the string so that it oscillates harmonically. (Sinusoidal)
- Not all periodic waves are harmonic



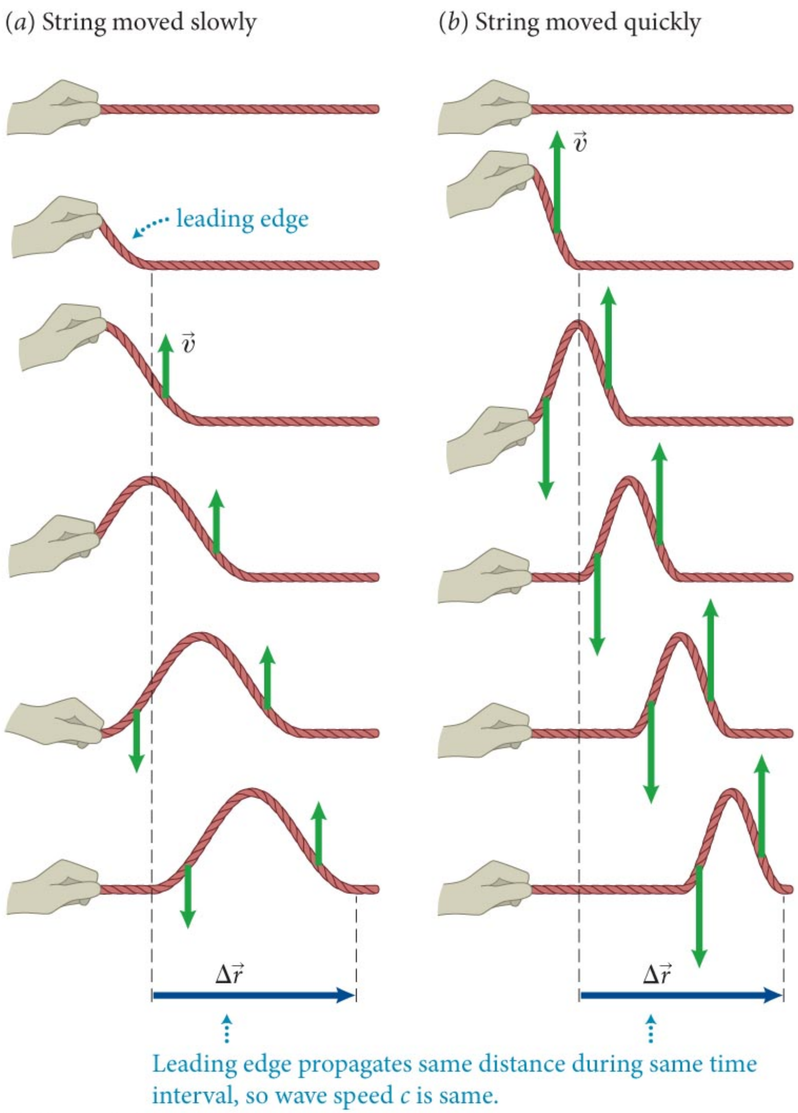
Section 16.2: Wave propagation

- A periodic wave repeats itself over a distance called the **wavelength**, denoted by λ.
- Each time one point on the string executes a complete oscillation, the wave advances by one wavelength.
- Therefore
  - **The wavelength** of a periodic wave is equal to the **product of the wave speed and the period** of the wave motion.



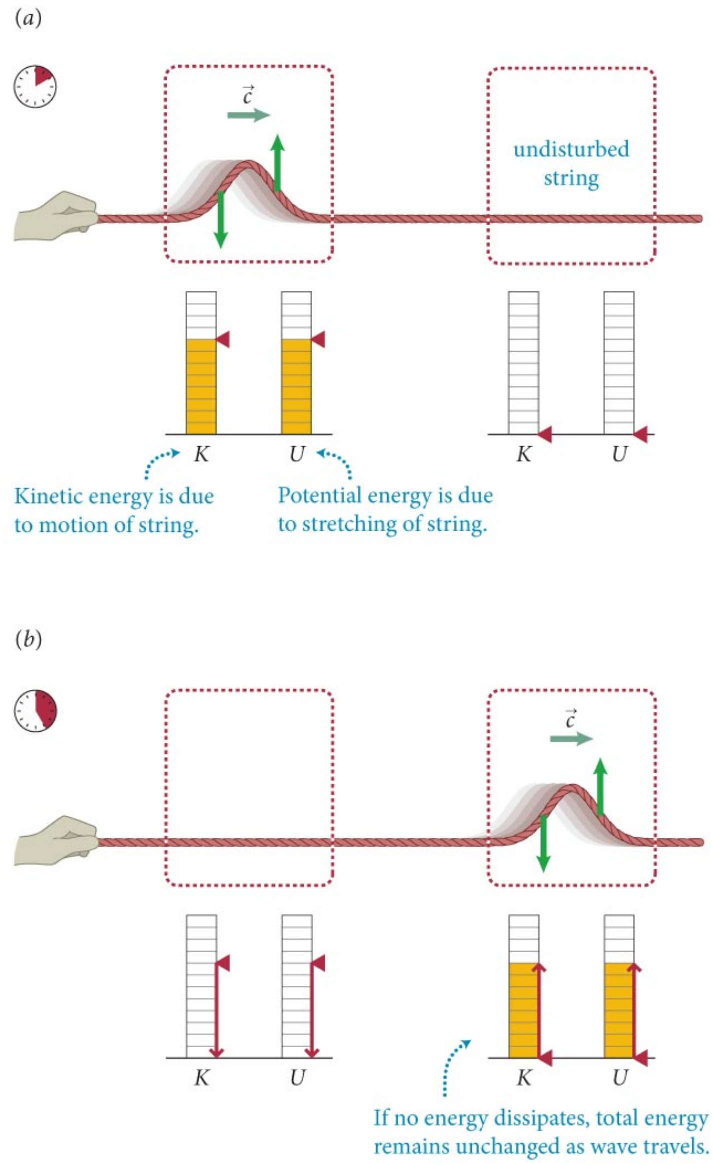
Section 16.2: Wave propagation

- As shown in the figure, moving your hand up and down more quickly does not generate a faster-traveling pulse.
- To a good approximation, we determine experimentally that
  - **The speed c of a wave propagating along a string is independent of the velocities $\vec{v}$ of the individual pieces of string. The value of c is determined entirely by the properties of the medium.**



Section 16.2: Wave propagation

- The figure illustrates how a propagating wave pulse caries two forms of energy along with it:
  - Kinetic energy associated with individual particles
  - Elastic potential energy associated with the stretching of the string

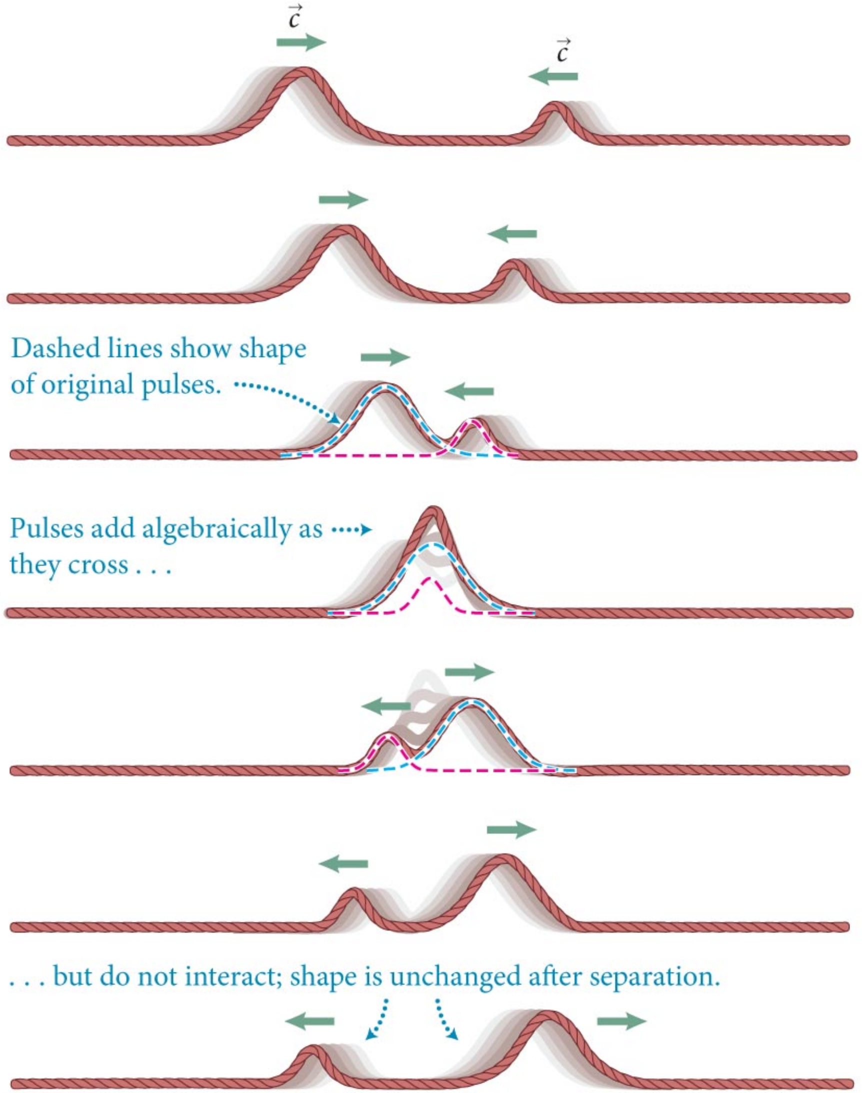


Section 16.3: Superposition of waves

Section Goals

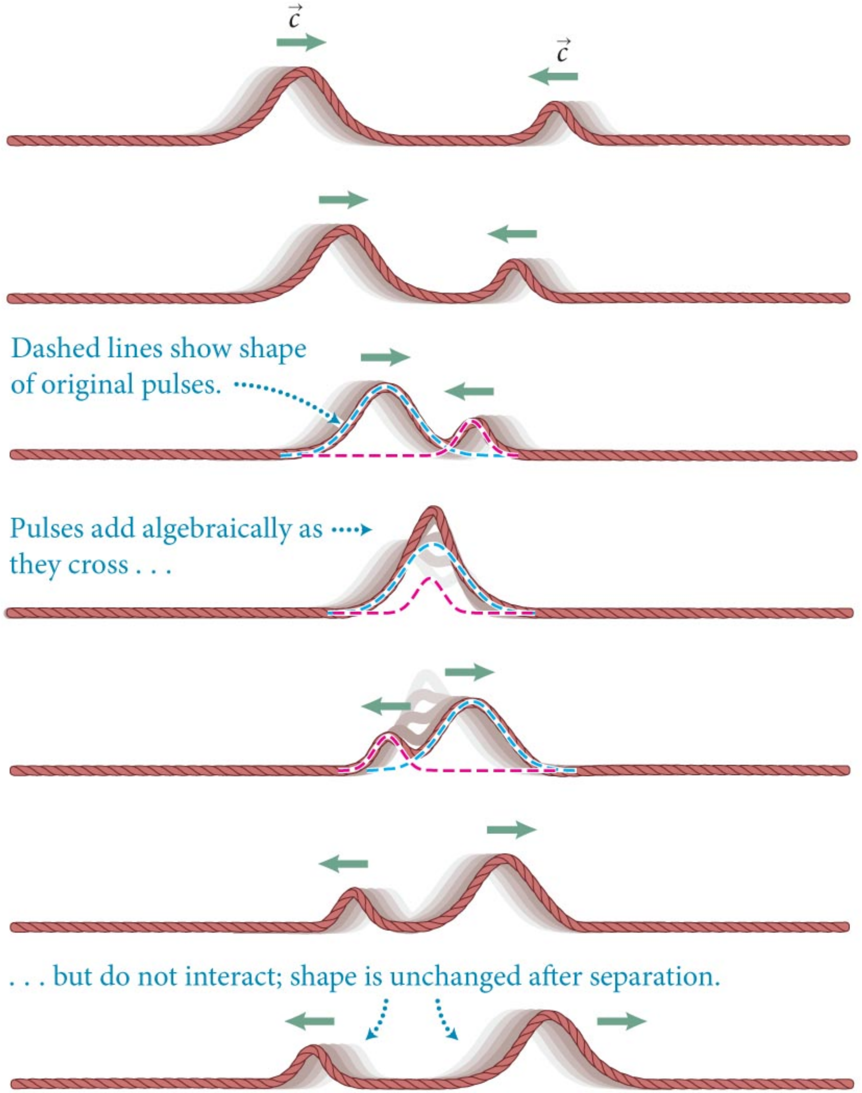
You will learn to

- Visualize the superposition of two or more waves traveling through the same region of a medium at the same time.
- Define the **nodes** for a stationary wave disturbance.



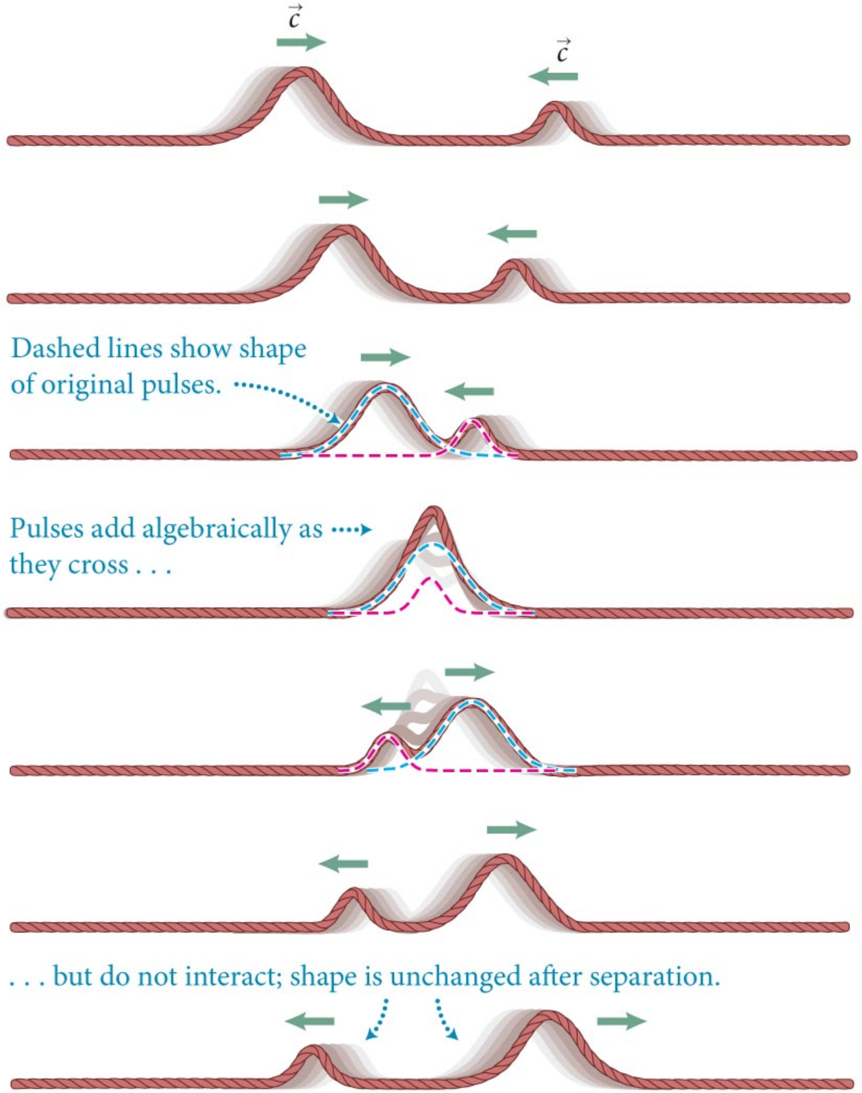
Section 16.3: Superposition of waves

- Waves exhibit a property known as the **superposition of waves:**
  - **If two or more waves overlap in a medium that obeys Hooke's law, then the resulting wave function at any instant is the algebraic sum of the individual waves.**
  - **Hooke = linear F(x)**

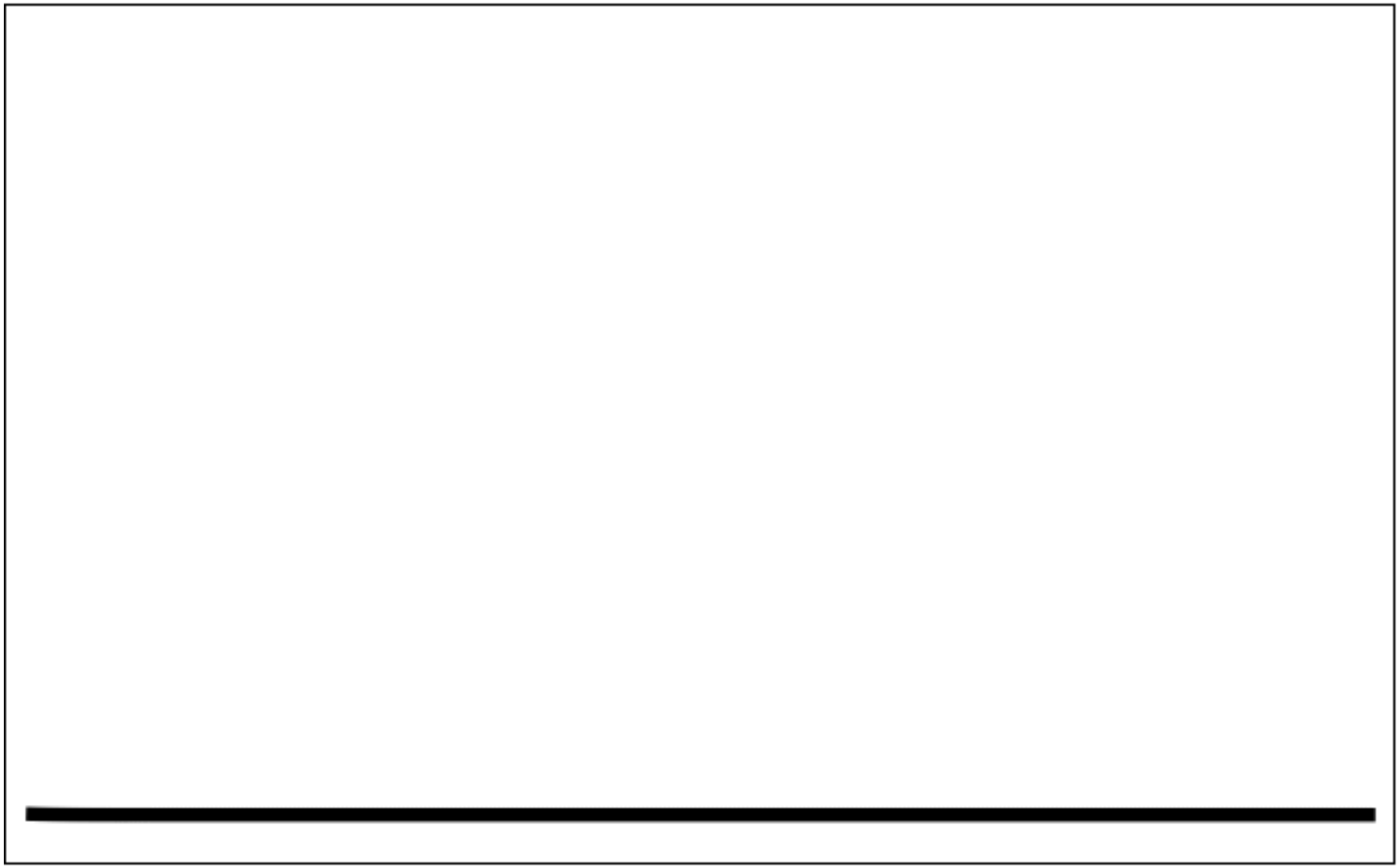


Section 16.3: Superposition of waves

- The phenomenon of two waves overlapping is called **interference.**
- If two waves with the same sign overlap, the resultant displacement is greater than that of either wave.
- This is called *constructive interference*.



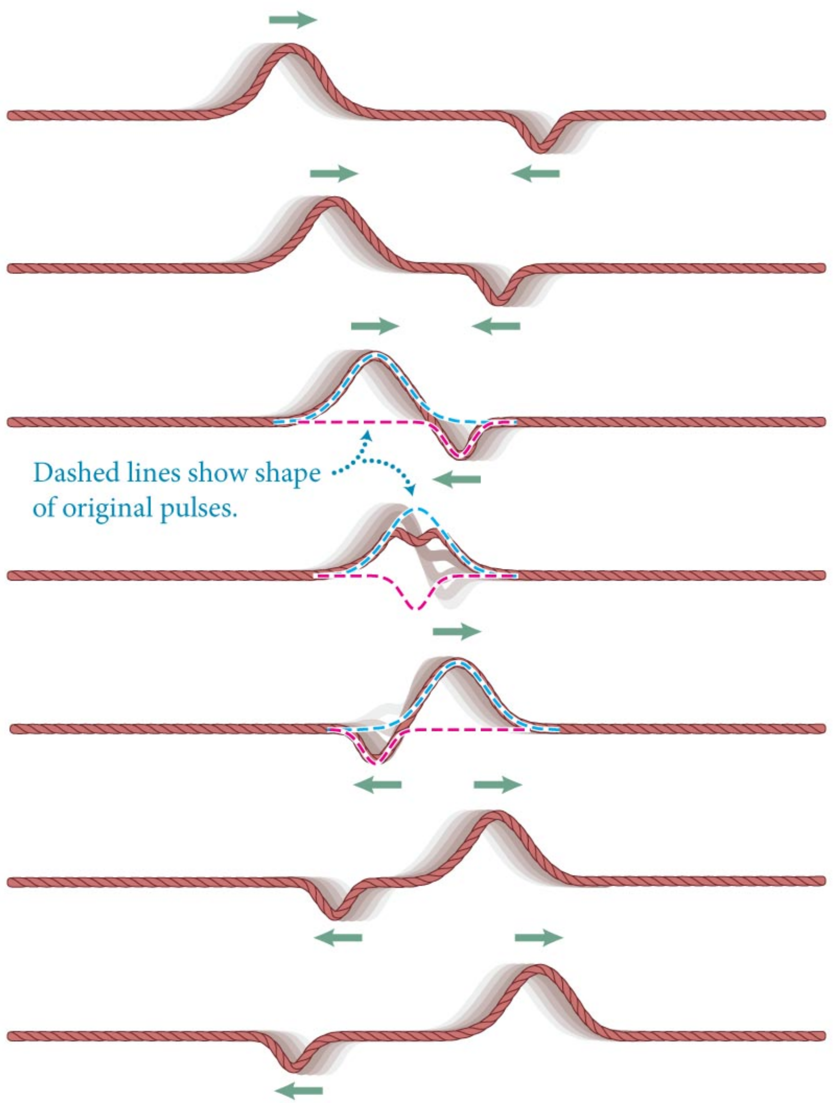
Constructive



http://www.acs.psu.edu/drussell/Demos/superposition/superposition.html

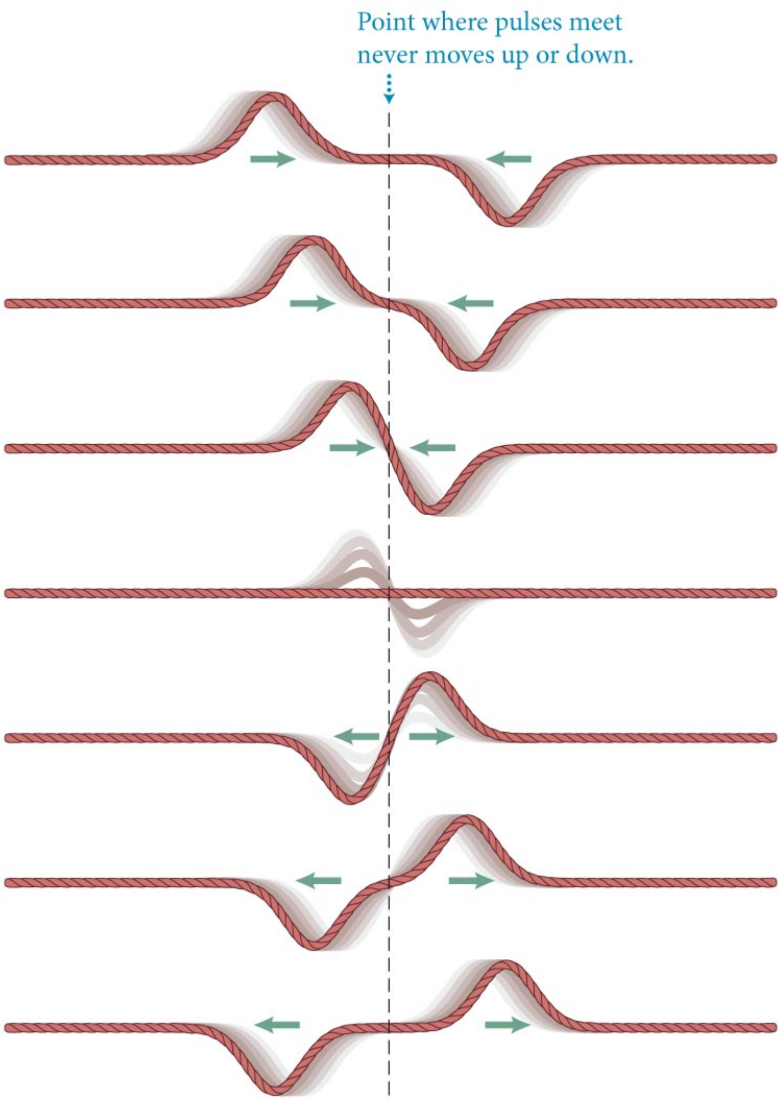
Section 16.3: Superposition of waves

- If two waves with opposite signs overlap, the resultant displacement is smaller than that of either wave.
- This is called *destructive interference*.



Section 16.3: Superposition of waves

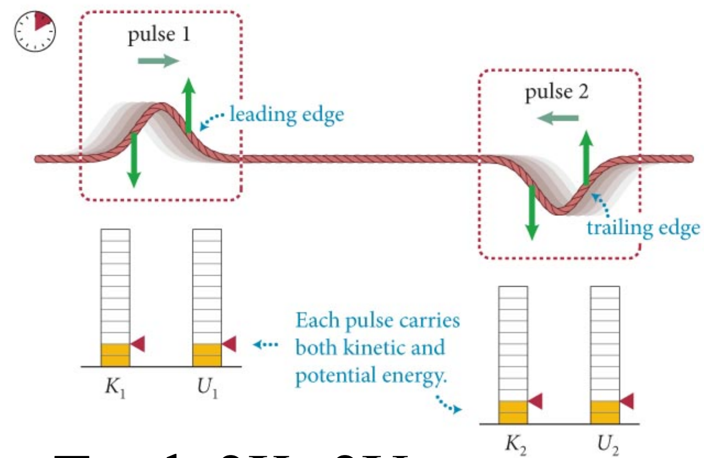
- If two waves of same size and shape but having opposite signs cross each other, the displacement of each wave cancels out.
- A point that remains stationary in a medium through which waves move is called a **node**.



Section 16.3: Superposition of waves

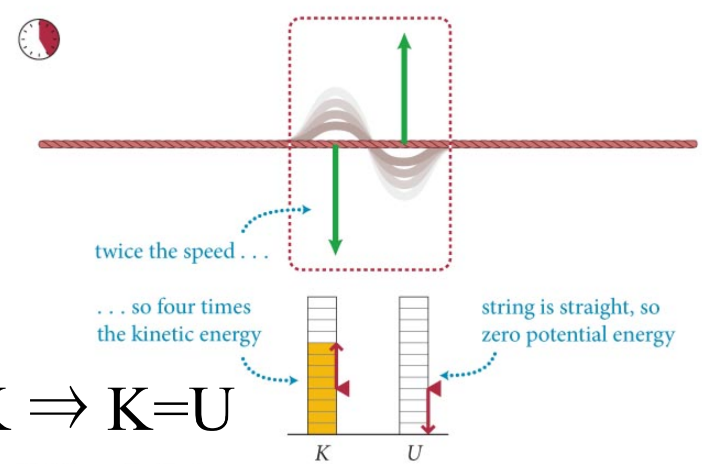
- The figure shows the energy makeup of two pulses traveling in opposite directions.
- The instant the pulses overlap, the displacement is zero and all the energy is kinetic.
- From this we can determine that
  - **A wave contains equal amounts of kinetic energy and potential energy.**

(*a*) Two pulses of identical shape traveling in opposite directions and having displacements in opposite directions



Total: 2K+2U

(*b*) At the instant the pulses overlap, the string displacement is zero everywhere, and so the potential energy is zero; all the energy is kinetic



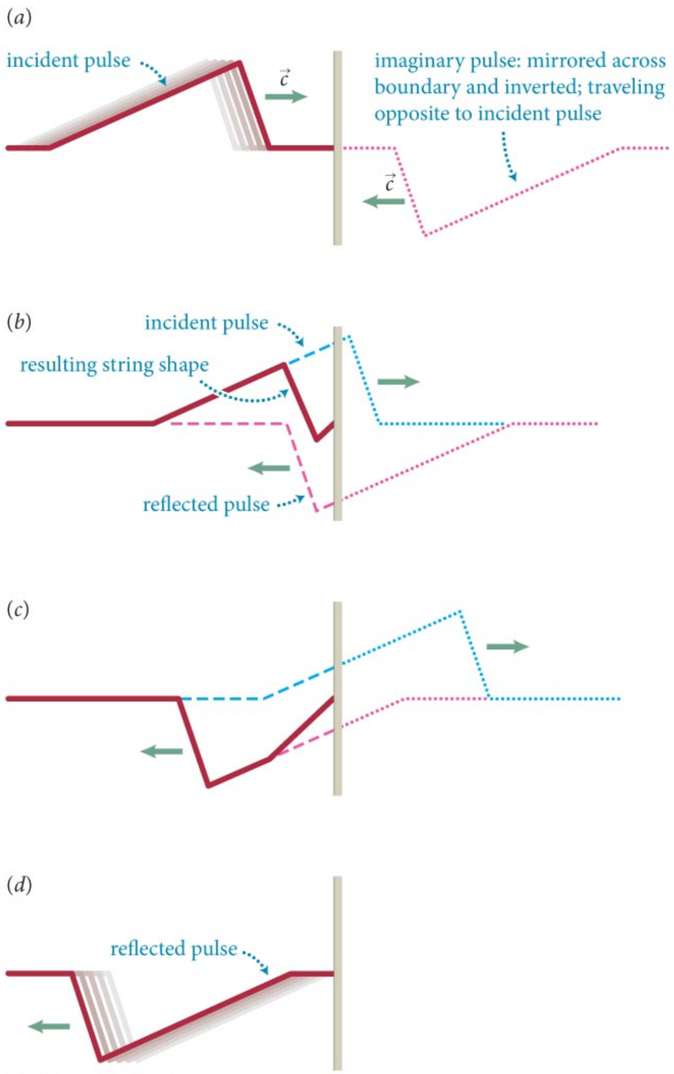
Total: 4K ⇒ K=U

Section 16.4: Boundary effects

Section Goals

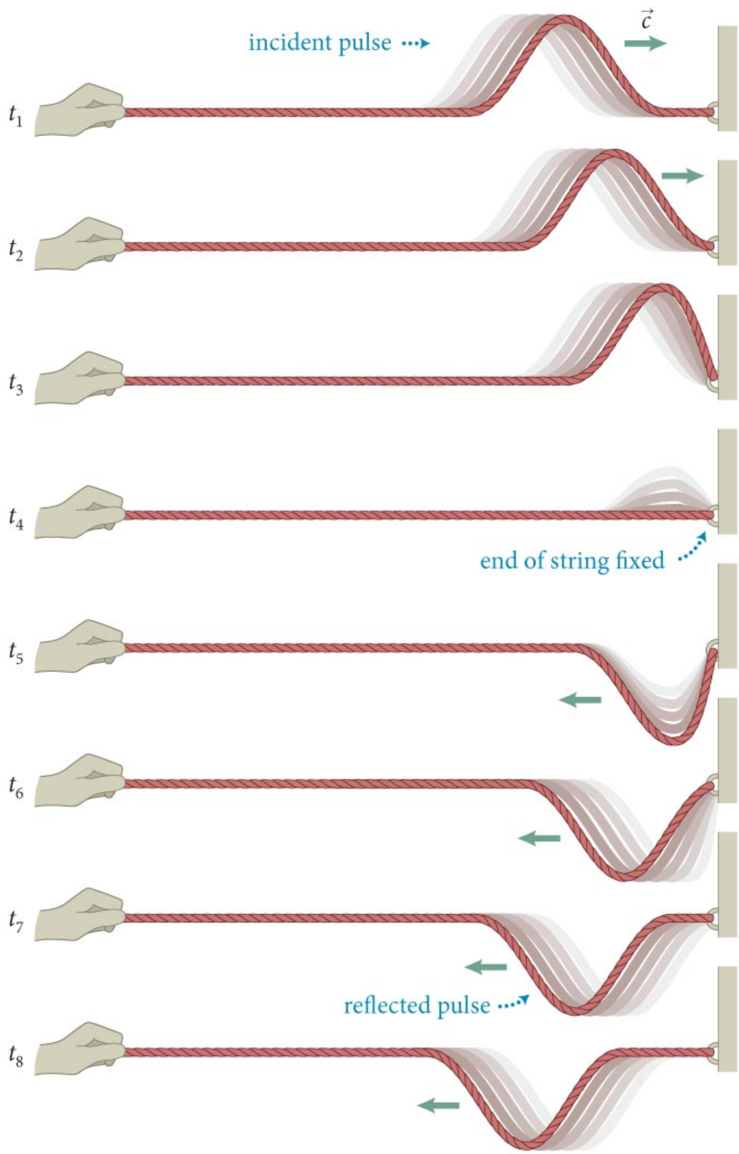
You will learn to

- Model the reflection of waves from fixed and free ends.
- Represent the boundary effects for overlapping waves graphically.



Section 16.4: Boundary effects

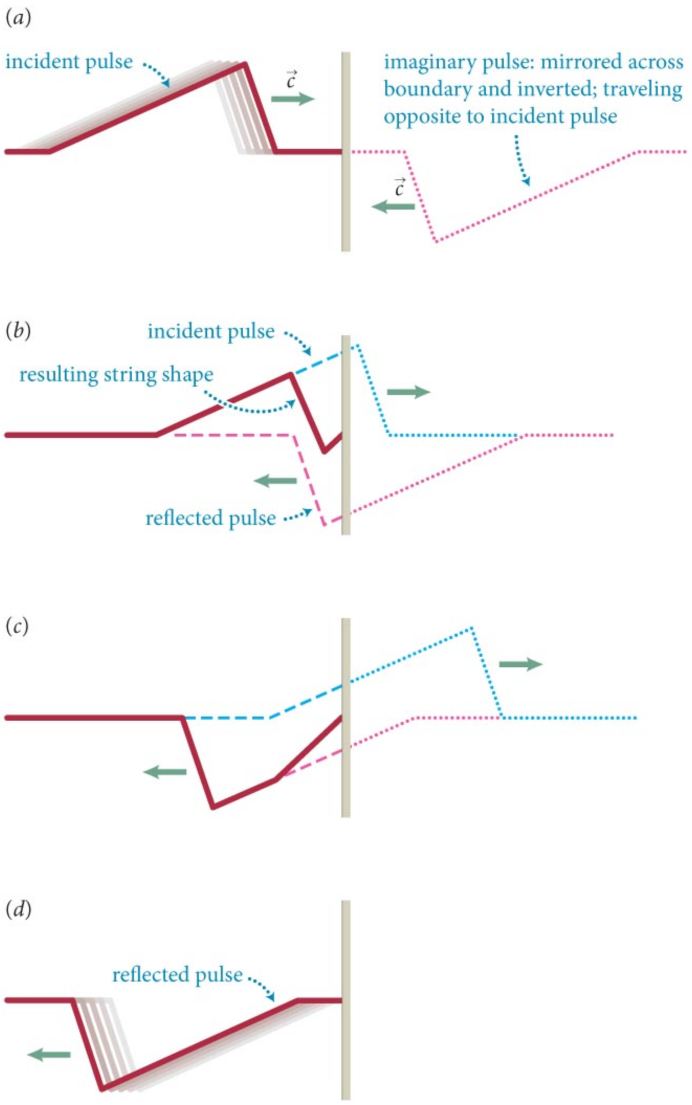
- When a wave pulse reaches a boundary where the transmitting medium ends, the pulse is reflected back.
- Consider a pulse propagating along a string that is anchored to an immovable wall.
- The leading edge of the pulse pulls up on the wall, and the wall pulls down on the string.
- Therefore, the reflected pulse is inverted.



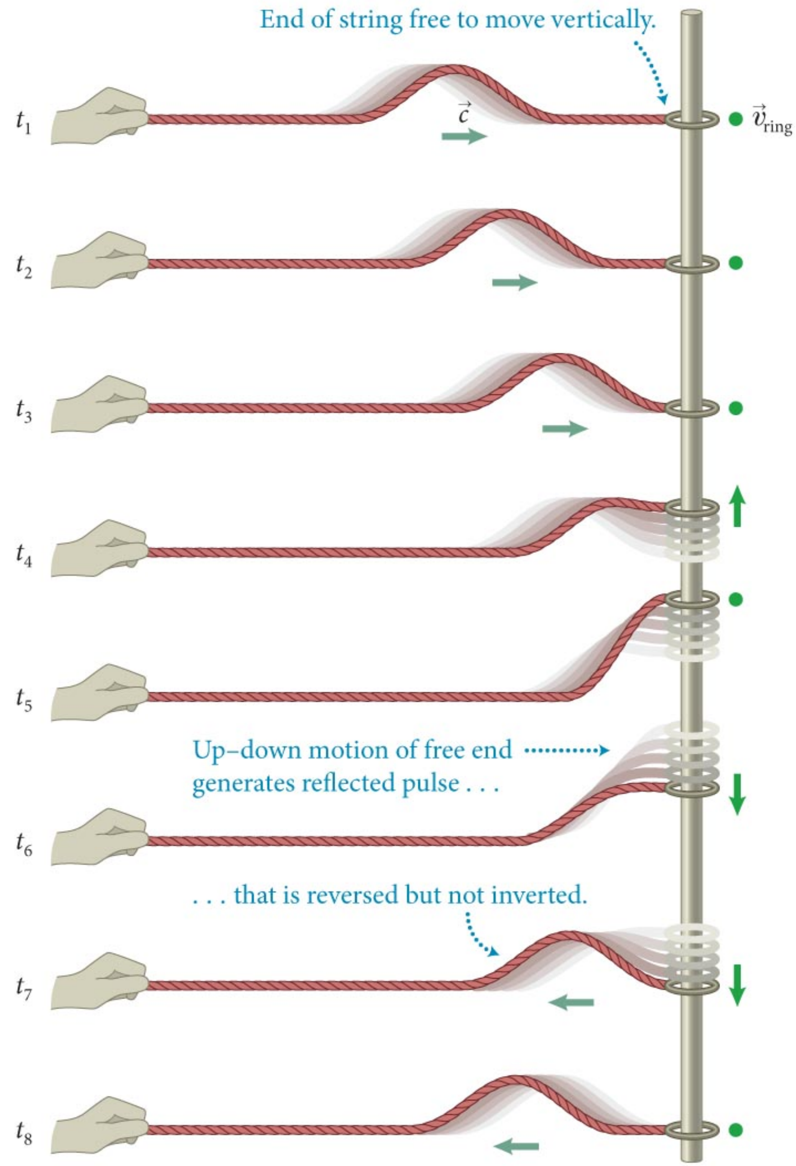
Section 16.4: Boundary effects

- The procedure for determining the shape of a reflected wave pulse can be illustrated using the example shown.

- Sum of real pulse and its reflection

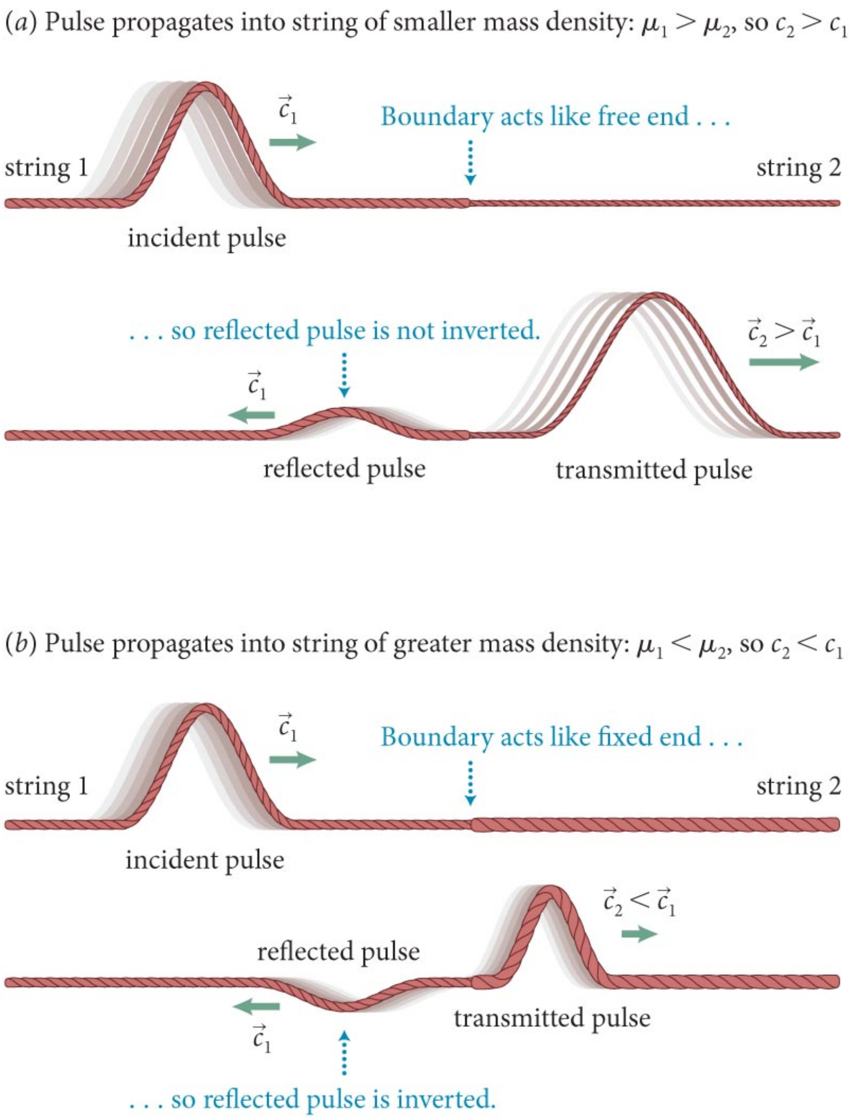


- Now let us consider reflection from a free end.
- As the figure shows, the reflected wave in this case is not inverted.



Section 16.4: Boundary effects

- When a pulse reaches a boundary between two media, the pulse is partially transmitted and partially reflected.
- Mass per unit length is called the **linear mass density** μ.
- The nature of the reflected and transmitted waves depends on whether μ_1 is greater or smaller than μ_2.
- A given pulse in a heavy string has a greater affect on a light string (and vice versa)



Quantitative Tools

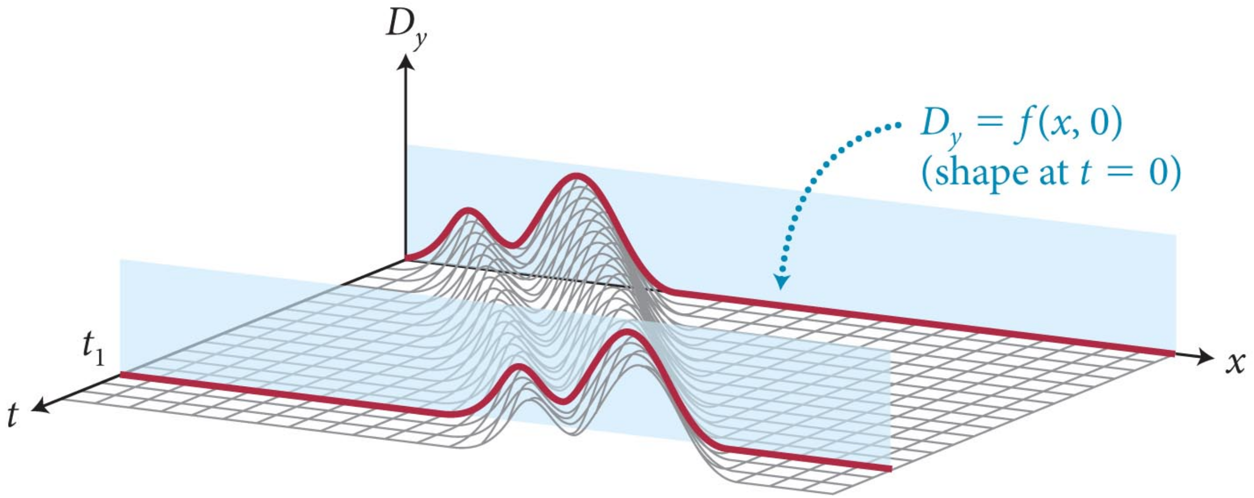
Section 16.5: Wave functions

Section Goals

You will learn to

- Represent **traveling waves** in one dimension mathematically using wave functions.
- Model **harmonic** traveling waves using trigonometric functions.
- Define the **wave number**, k, for a traveling harmonic wave.

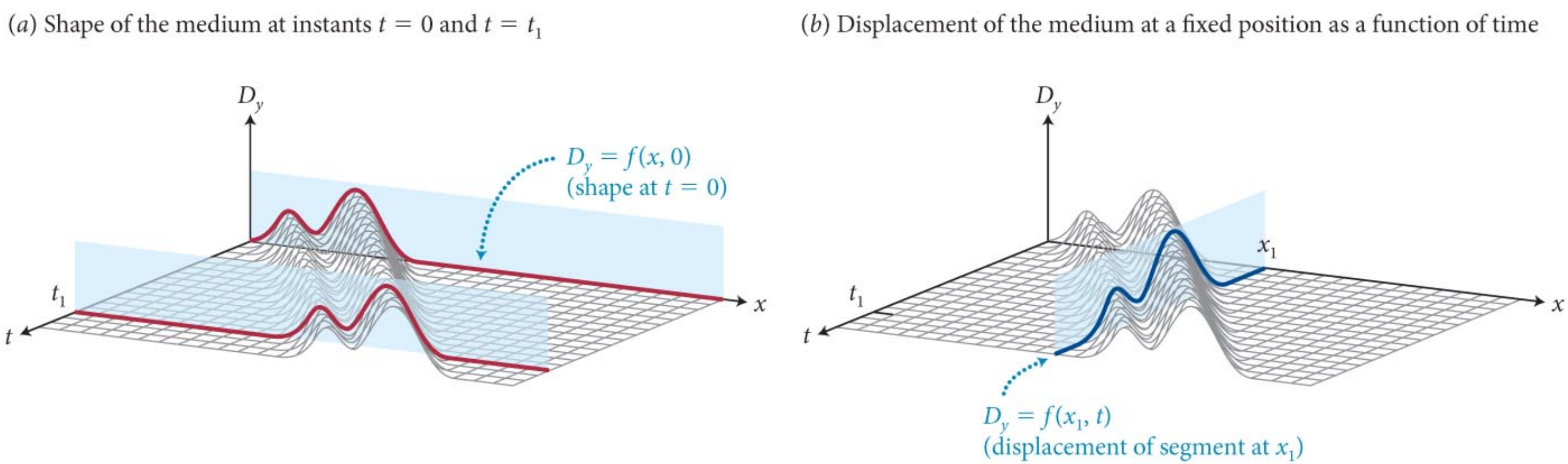
(a) Shape of the medium at instants $t = 0$ and $t = t_1$



Section 16.5: Wave functions

- D_y is the displacement of any particle on the string measured in Earth's reference frame.
- We can write $D_y = f(x, t)$, where $f(x, t)$ is called the *time-dependent wave function*.
- $f(x, t)$ completely specifies the changing shape of the wave as seen from Earth's reference frame.
- We can show that for a wave propagating at speed c:

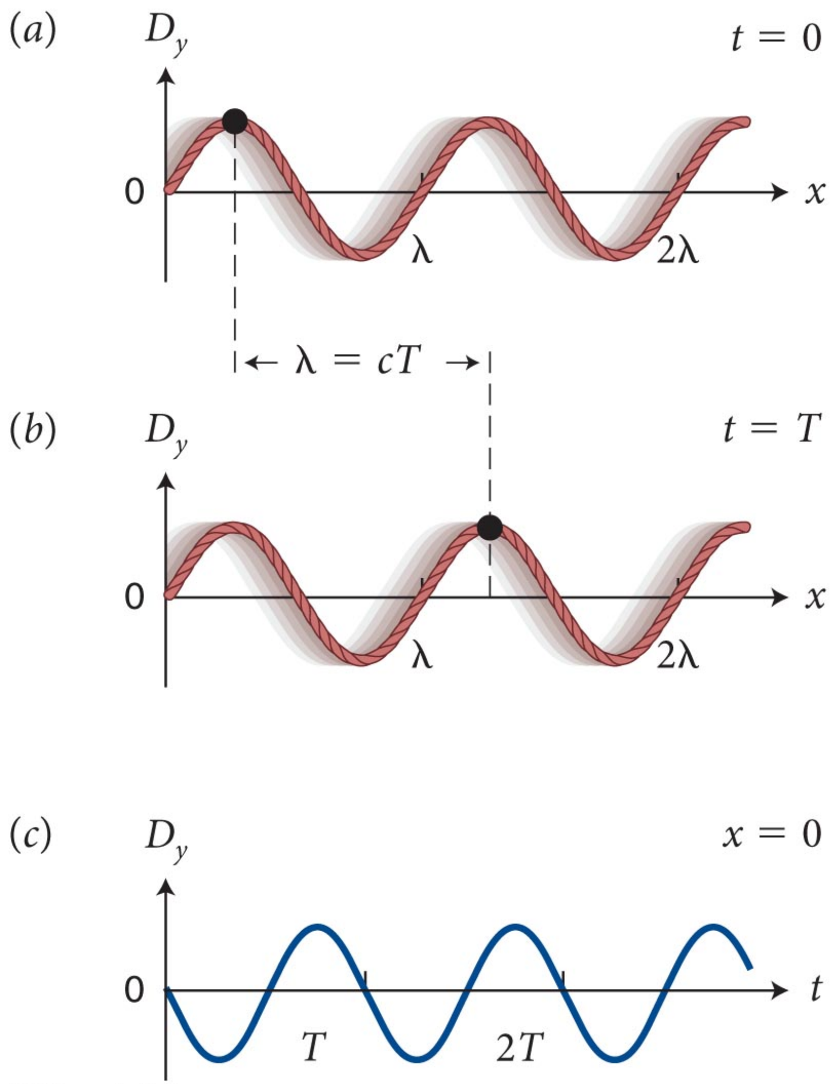
$$D_y = f(x - ct)$$



Section 16.5: Wave functions

- Let us consider a transverse harmonic wave traveling along a string.
- We can see from part (*b*) that during a time interval of one period, the wave advances by a wavelength.
- Because the wave moves at a speed c, and $f = 1/T$, we get

$$\lambda f = c$$



Section 16.5: Wave functions

- We have previously defined the angular frequency:

$$\omega = 2\pi f = 2\pi/T \qquad \text{"how many periods per sec"}$$

- We define the **wave number** k in a similar way

$$k = \frac{2\pi}{\lambda}$$

k has SI units of m^{-1} "how many waves fit in 1 m"

- For a transverse harmonic wave traveling in the positive x direction, the y component of the displacement is

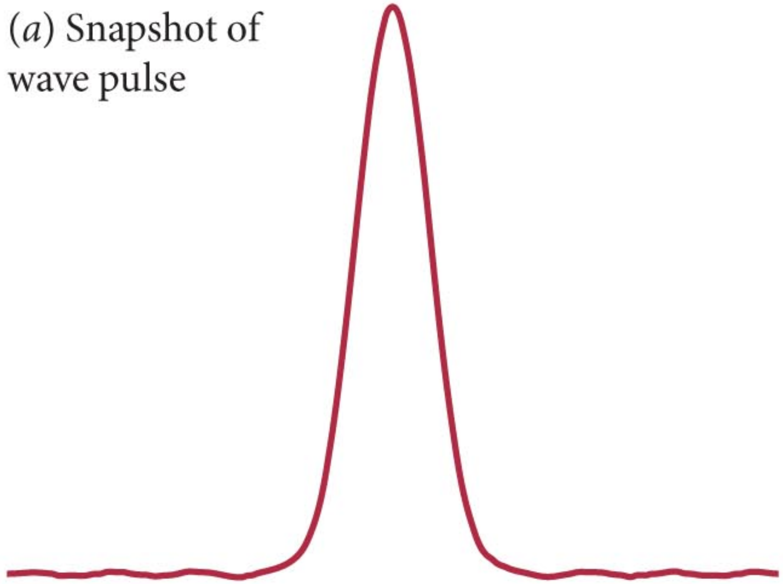
$$D_y = f(x, t) = A \sin(kx - \omega t + \varphi_i)$$

  - φ_i is the initial phase or the phase at $x = 0$, $t = 0$.
  - A is the amplitude.

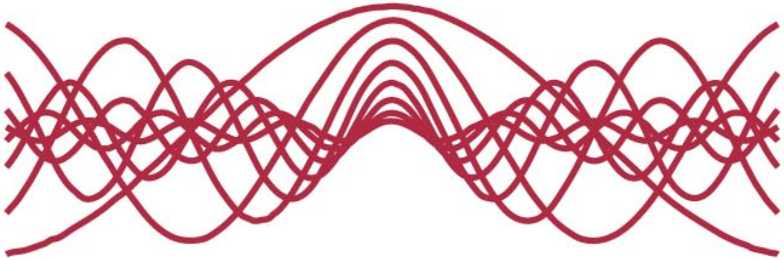
Section 16.5: Wave functions

- Fourier's theorem from Section 15.3 can be applied to waves.
- Any wave can be expressed in terms of a sum of sinusoidally varying waves. (sine waves form a complete orthonormal set)
- The figure shows a wave pulse obtained by adding together a set of harmonic waves.

(*a*) Snapshot of wave pulse

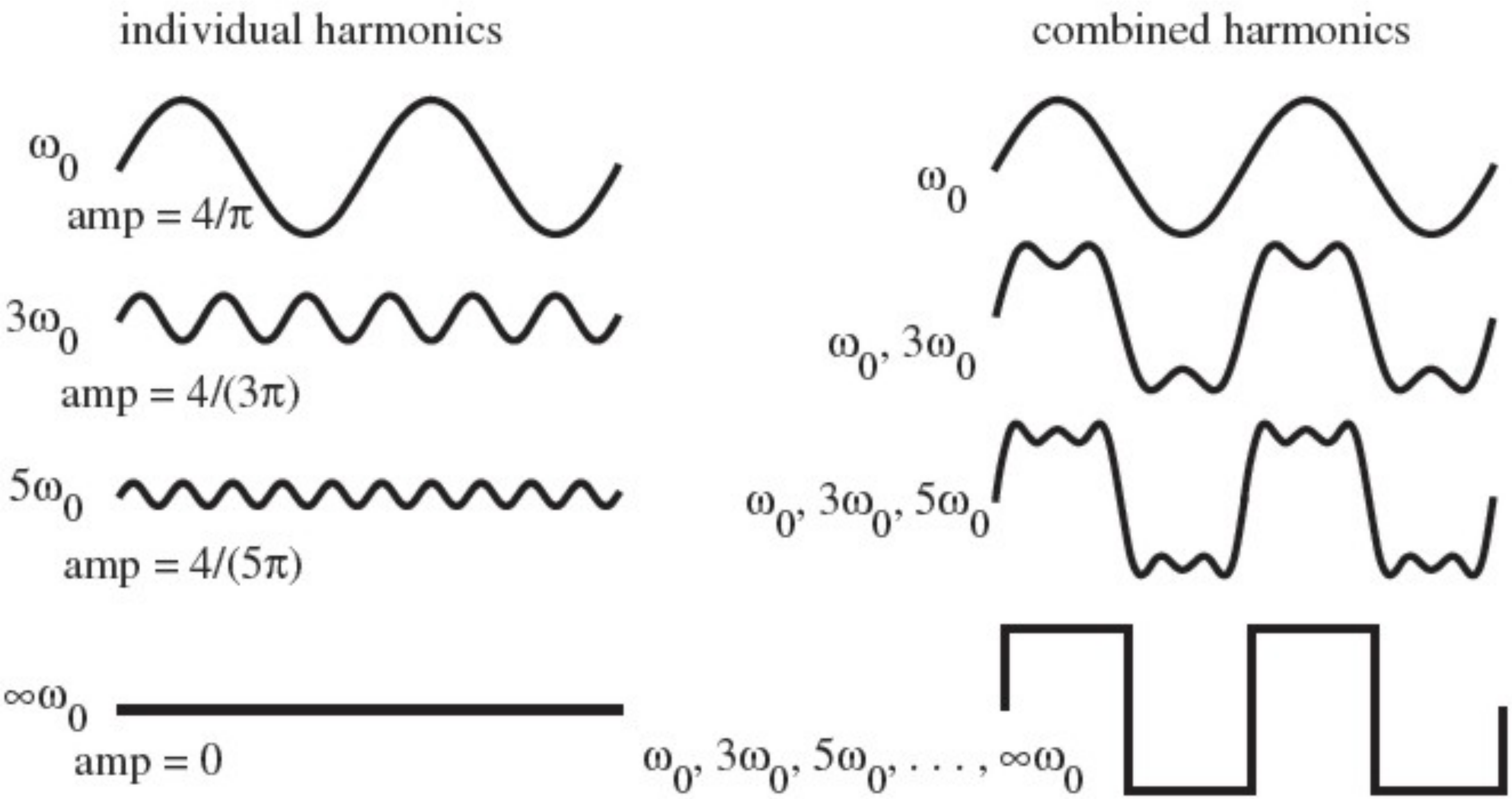


(*b*) Set of harmonic waves that add up to produce wave pulse



Building a square wave

- Infinite series of sine waves
- Odd frequencies



http://mechatronics.colostate.edu/figures/4-4.jpg

Checkpoint 16.17

16.17 (a) Which of the following functions could represent a traveling wave?

(i) $A\cos(kx + \omega t)$ (ii) $e^{-k|x-ct|^2}$

(iii) $b(x - ct)^2 e^{-x}$ (iv) $-(b^2 t - x)^2$

(b) Which of the following functions can be made into a traveling wave?

(i) $x/(1 + bx^2)$ (ii) xe^{-kx} (iii) x^2

Checkpoint 16.17

16.17 (a) Which of the following functions could represent a traveling wave?

(i) $A\cos(kx + \omega t)$ (ii) $e^{-k|x-ct|^2}$

~~(iii) $b(x - ct)^2 e^{-x}$~~ (iv) $-(b^2 t - x)^2$

All but (iii). The rest have the form *f(x-ct)*.

(b) Which of the following functions can be made into a traveling wave?

(i) $x/(1 + bx^2)$ (ii) xe^{-kx} (iii) x^2

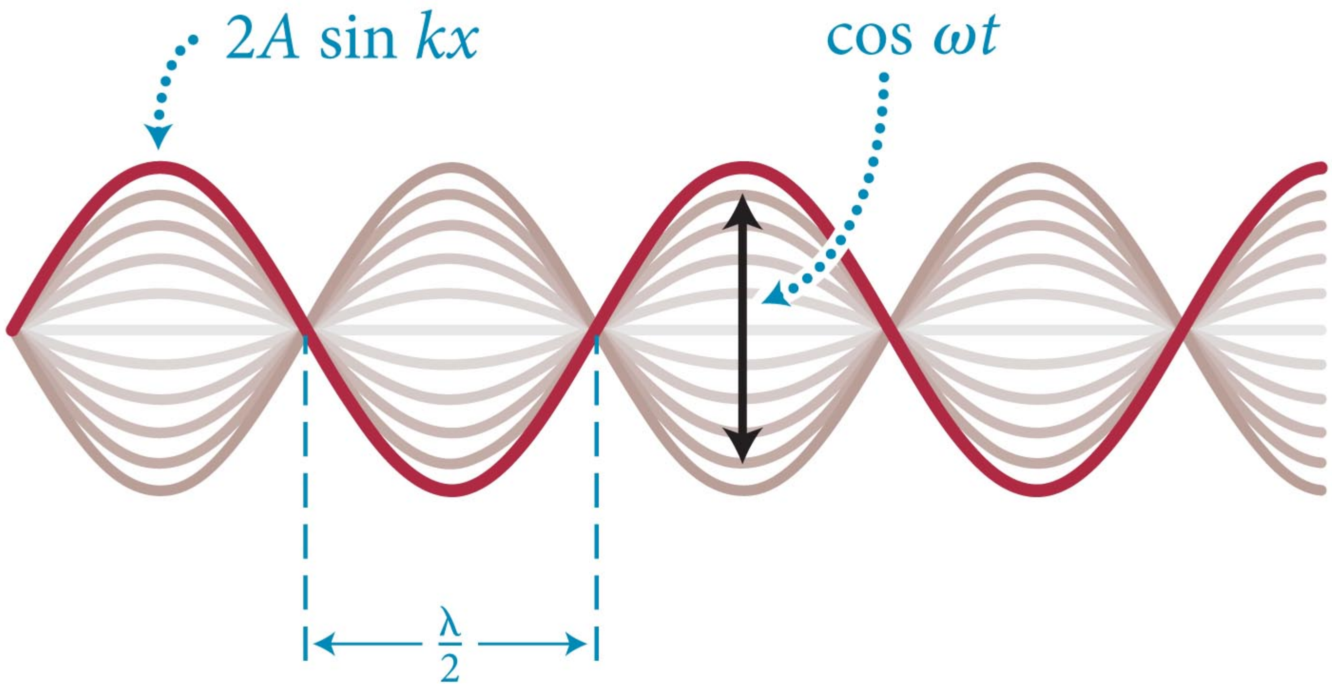
All of them, with the substitution $x \rightarrow x\text{-}ct$ or $x\text{+}ct$

Section 16.6: Standing waves

Section Goals

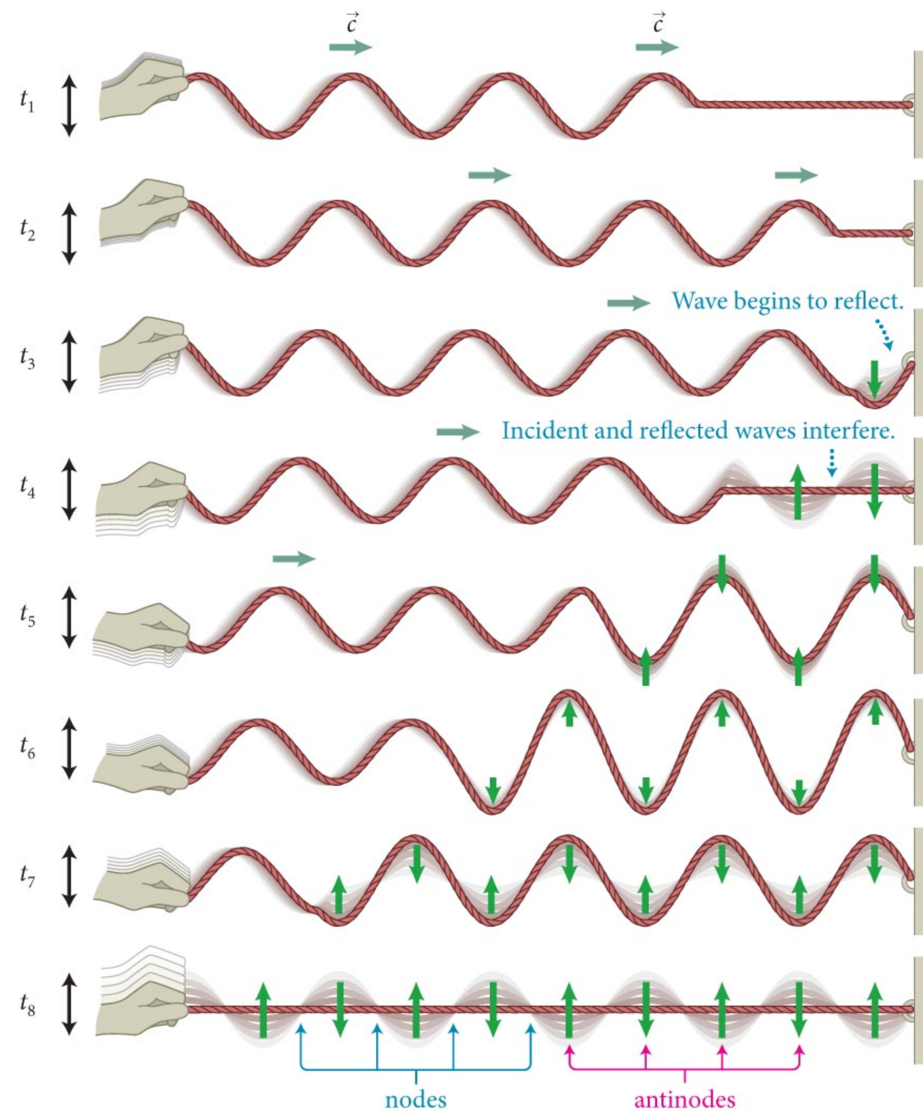
You will learn to

- Establish the geometric and mathematical properties of standing waves.
- Derive the mathematical formulas that give the **positions of the nodes and antinodes** for standing waves on a string fixed at each end.



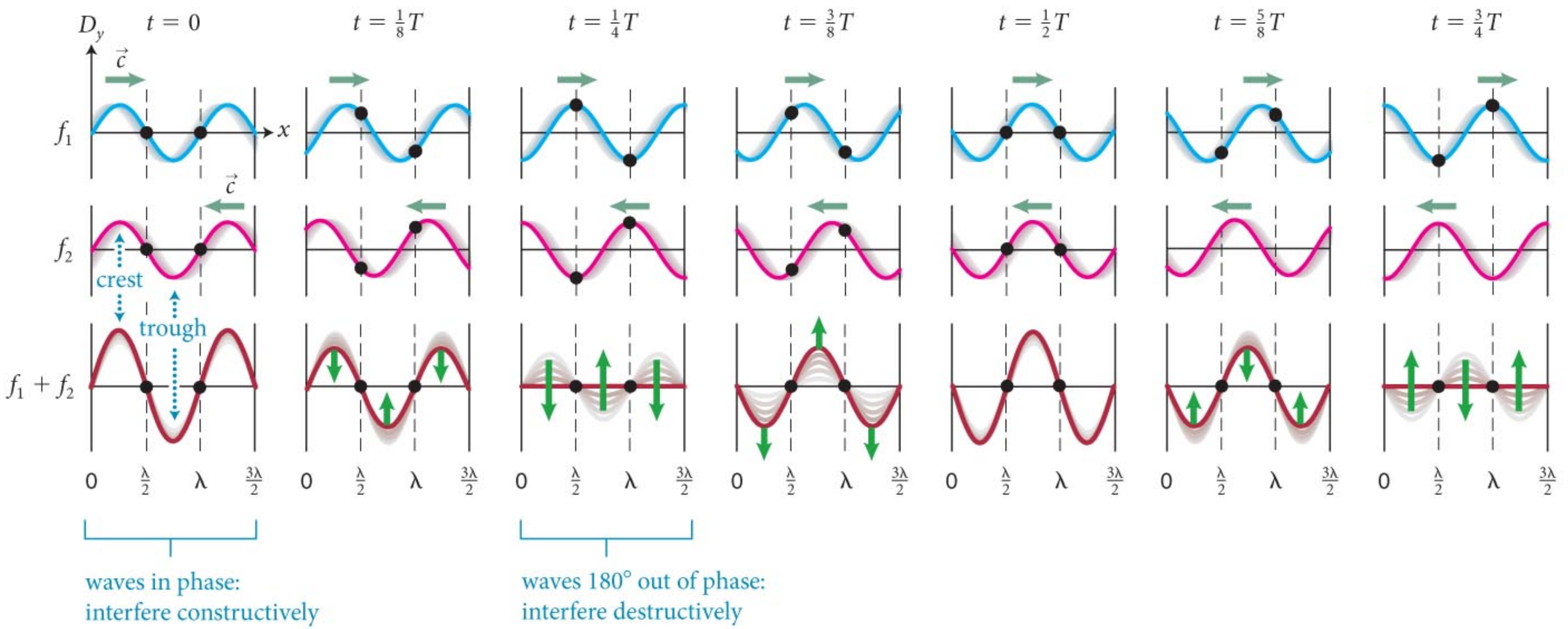
Section 16.6: Standing waves

- The figure shows the interference between a incident harmonic wave on a string and the reflected wave from the fixed end of the string.
- The points on the string that have zero displacement are called **nodes**.
- Halfway between the nodes are the **antinodes** where the particles in the medium oscillate with maximum displacement.



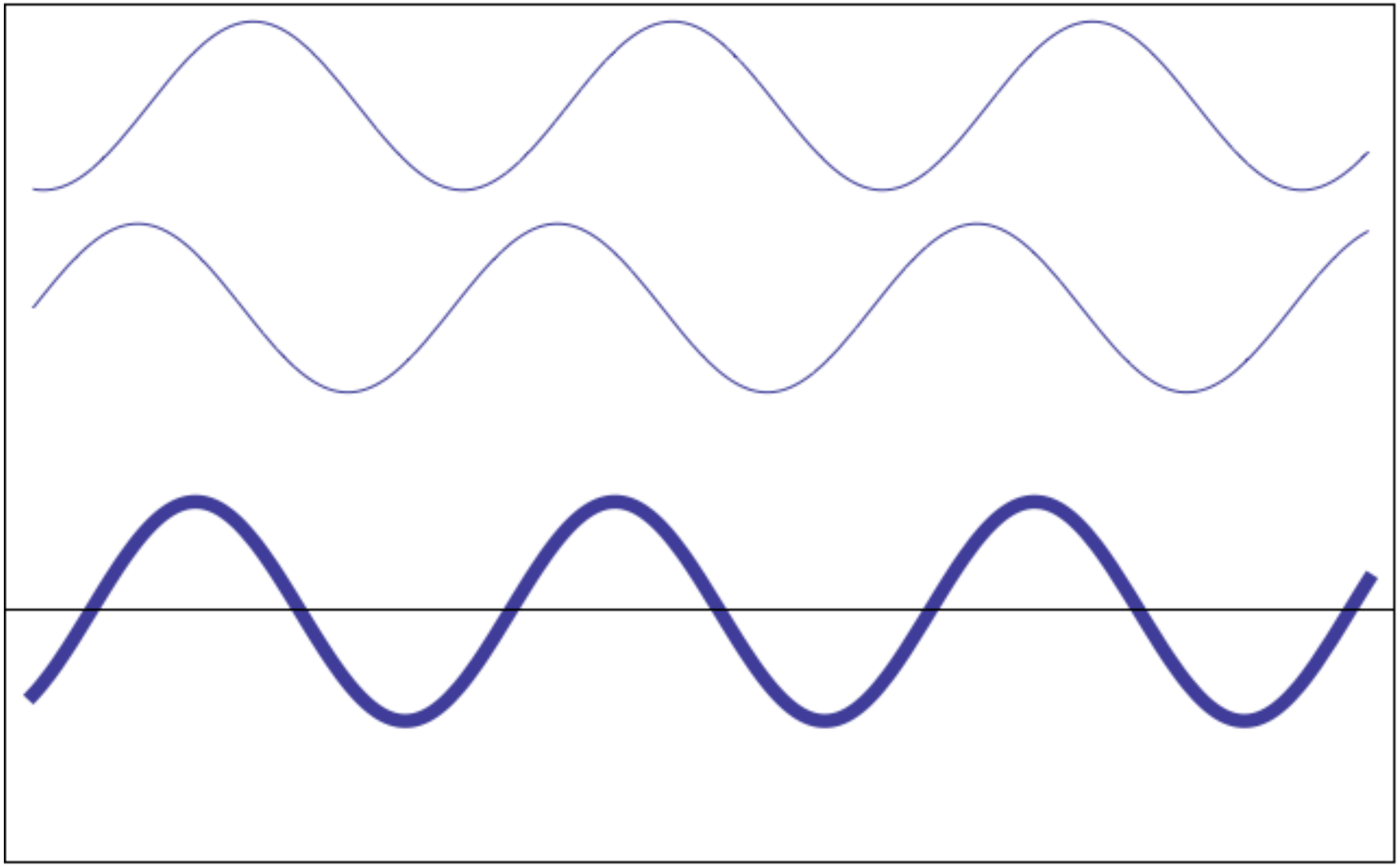
Section 16.6: Standing waves

- The pulsating stationary pattern caused by harmonic waves of the same amplitude traveling in opposite direction is called a **standing wave**.
- The figure illustrates how standing waves come about.



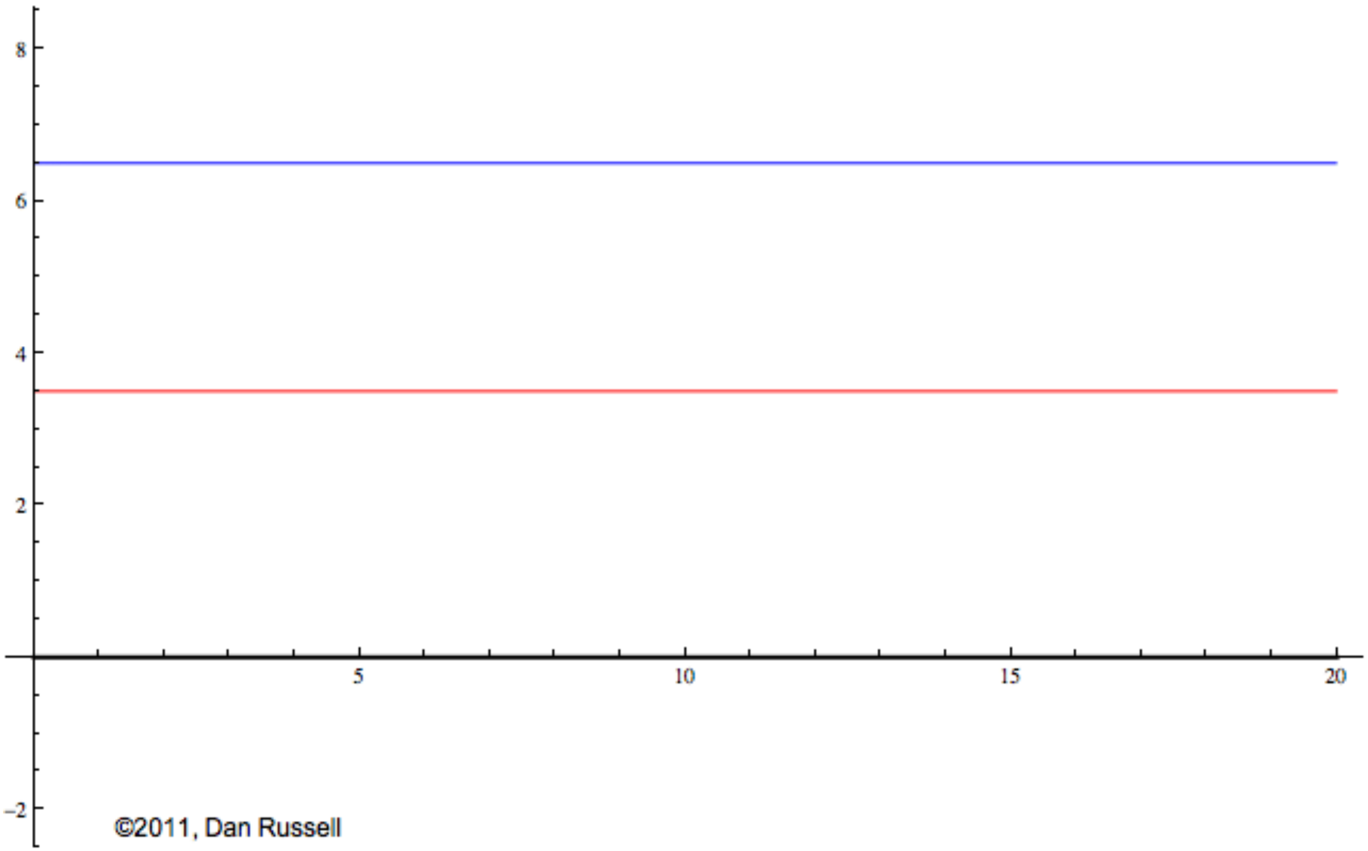
Section 16.6: Standing waves

- One wave moves: constructive & destructive interference



Section 16.6: Standing waves

- Waves move at same speed in opposite directions: standing wave



Section 16.6: Standing waves

- A sinusoidal wave traveling to the right along the *x*-axis with angular frequency ω, wave number k, and amplitude A is

$$D_{1y} = f_1(x, t) = A \sin(kx - \omega t)$$

- The wave traveling to the left is

$$D_{2y} = f_2(x, t) = A \sin(kx + \omega t)$$

- The combined wave is

$$D_y = f_1(x, t) + f_2(x, t) = A[\sin(kx - \omega t) + \sin(kx + \omega t)]$$

- By simplifying this using trigonometric identities, we get

$$D_y = f_1(x, t) + f_2(x, t) = 2A \sin kx \cos \omega t = \boxed{[2A \sin kx]} \boxed{\cos \omega t}$$

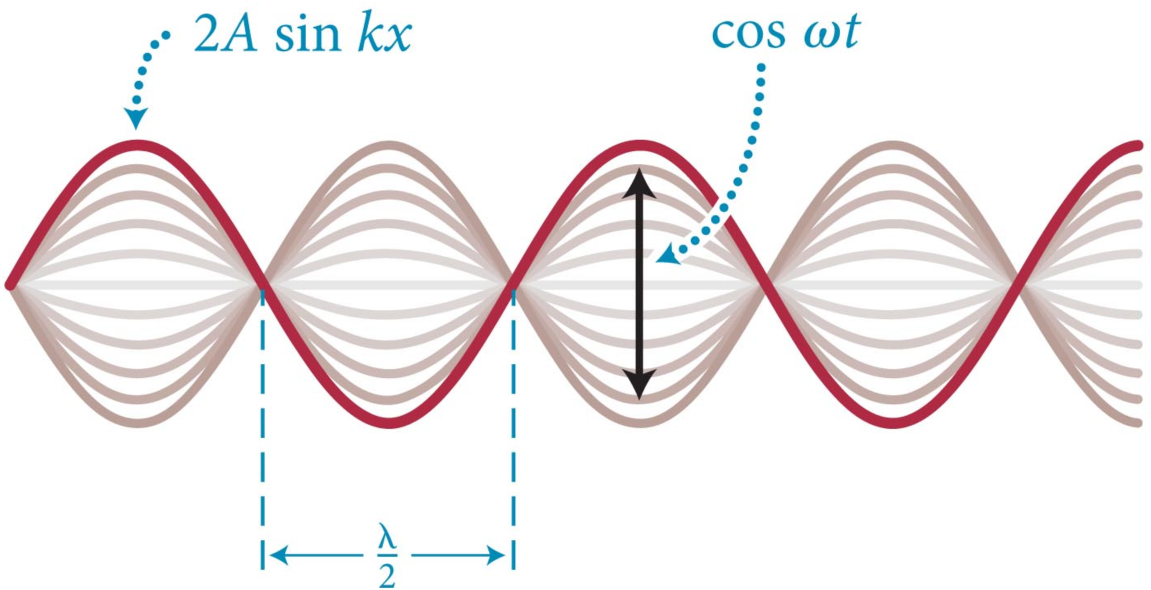
Section 16.6: Standing waves

- At points that are nodes in a standing wave sin *kx* must be zero.
- Therefore, at these points $kx = n\pi$, where *n* is a whole number.
- Using the definition of $k = 2\pi/\lambda$, we obtain

$$\frac{2\pi}{\lambda}x = n\pi \quad n = 0, \pm 1, \pm 2, \ldots$$

- So, the nodes occur at

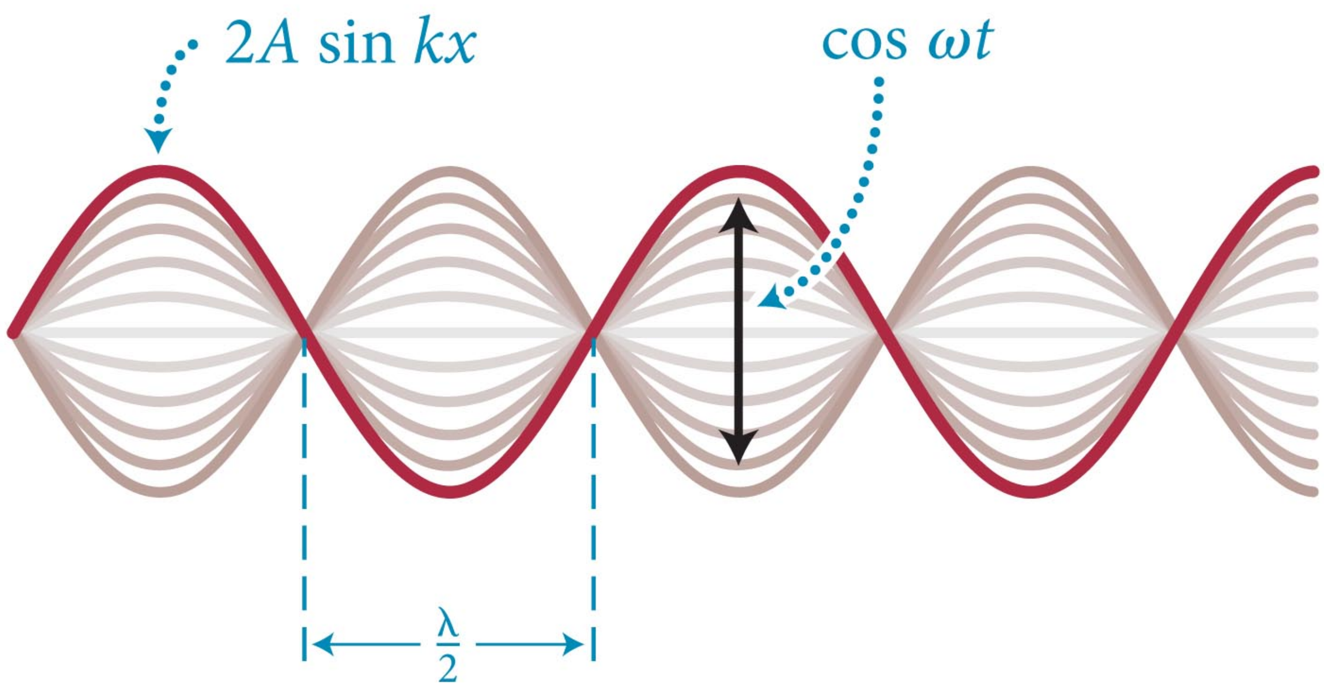
$$x = 0, \pm\frac{\lambda}{2}, \pm\lambda, \pm\frac{3\lambda}{2}, \ldots$$



Section 16.6: Standing waves

- Antinodes occur when $\sin kx = \pm 1$.
- This requires that $kx = n(\pi/2)$, where n is an odd whole number.
- So, the antinodes occur at

$$x = \pm\frac{\lambda}{4}, \pm\frac{3\lambda}{4}, \frac{5\lambda}{4}, \ldots$$



Checkpoint 16.19

16.19 (*a*) Do two counter-propagating waves that have the same wavelength but different amplitudes cause standing waves?

(*b*) Do two counter-propagating waves that have the same amplitude but different wavelengths cause standing waves?

Checkpoint 16.19

16.19 (*a*) Do two counter-propagating waves that have the same wavelength but different amplitudes cause standing waves?

No. If one amplitude were twice the other, half of it forms a standing wave with the whole of the other one. That leaves half a traveling wave yet.

(*b*) Do two counter-propagating waves that have the same amplitude but different wavelengths cause standing waves?

No, in this case interference for a given position is constructive at some instants and destructive at others. For a standing wave, it has to be the same at all instants for a given position.

Section 16.7: Wave speed

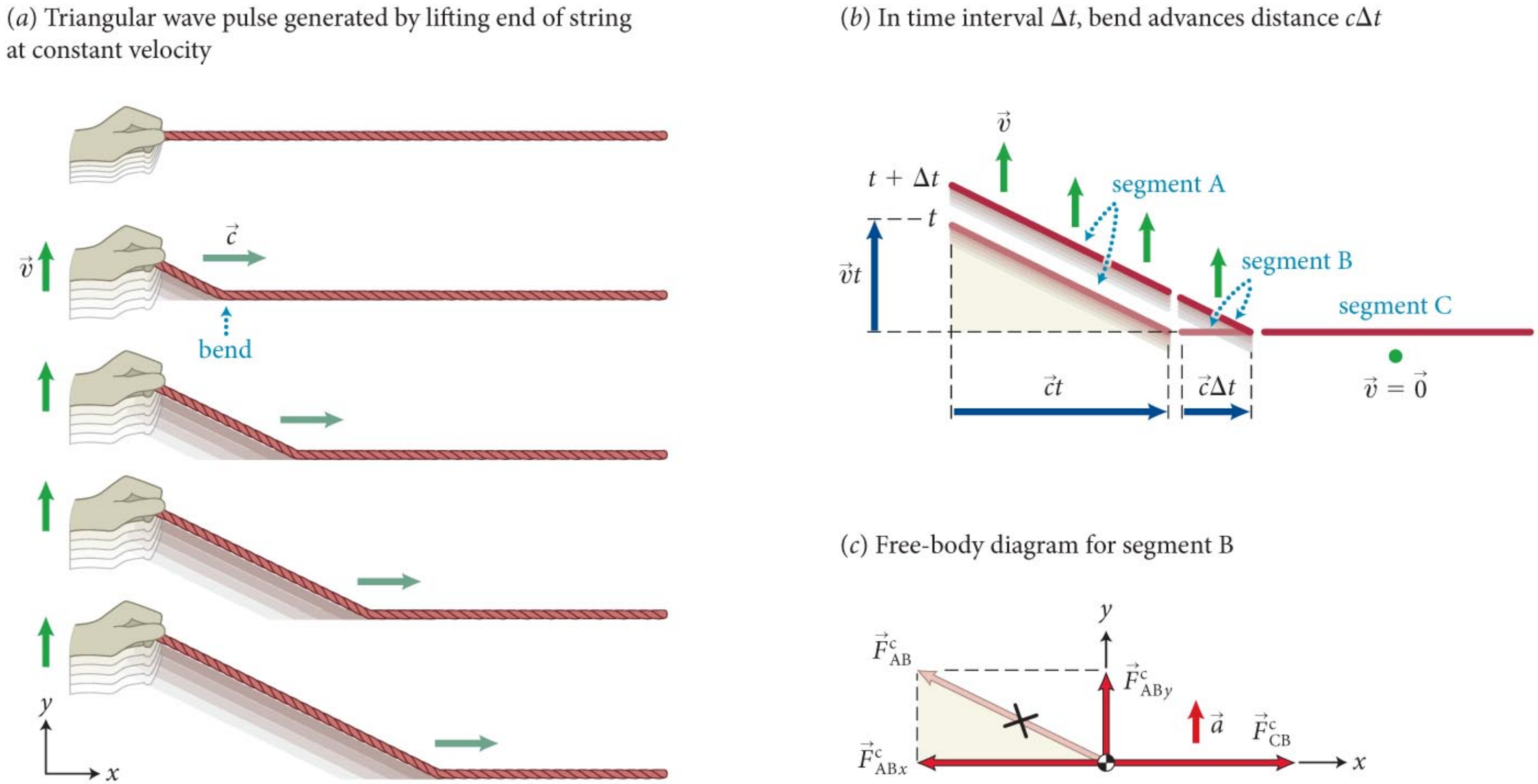
Section Goal

You will learn to

- Derive equations that give the **speed of wave propagation** for traveling waves on a uniform mass density string.

Section 16.7: Wave speed

- The procedure for deriving an expression for the wave speed *c* is illustrated in the figure. The derivation is in the text.



Section 16.7: Wave speed

- The mass per unit length of a uniform string is called the **linear mass density** (μ), and is defined as

$$\mu \equiv \frac{m}{\ell} \quad \text{(uniform linear object)}$$

- We can derive the wave speed of a transverse wave on a string to be

$$c = \sqrt{\frac{\mathcal{T}}{\mu}}$$

where $\mathcal{T}$ is the string tension.

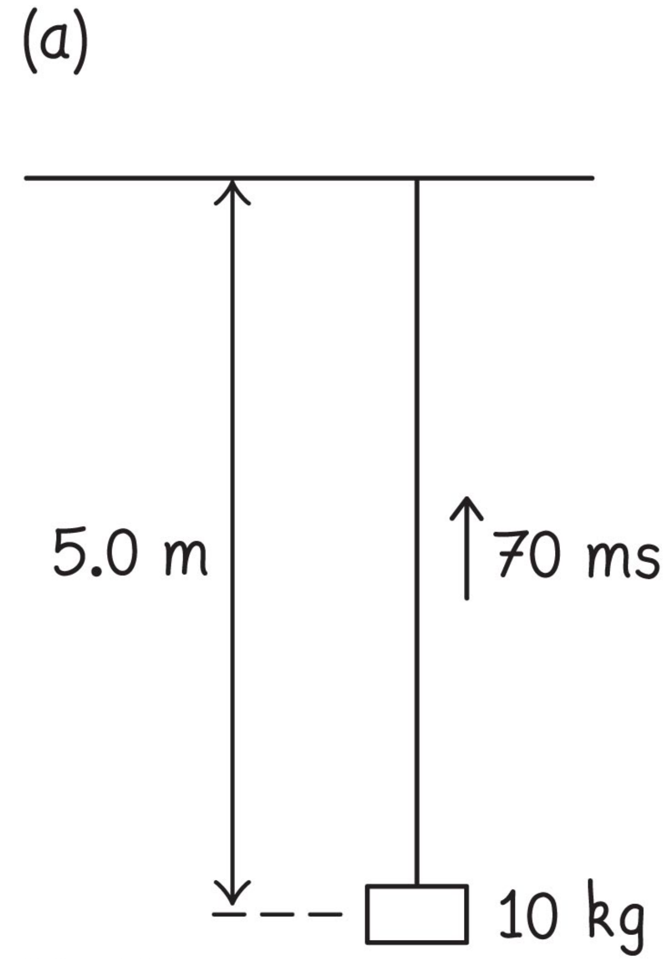
- Guitar/piano: tighten to tune, vary string weight to change fundamental tone.

Example 16.4 Measuring mass

You use a hammer to give a sharp horizontal blow to a 10-kg lead brick suspended from the ceiling by a wire that is 5.0 m long. It takes 70 ms for the pulse generated by the sudden displacement of the brick to reach the ceiling. What is the mass of the wire?

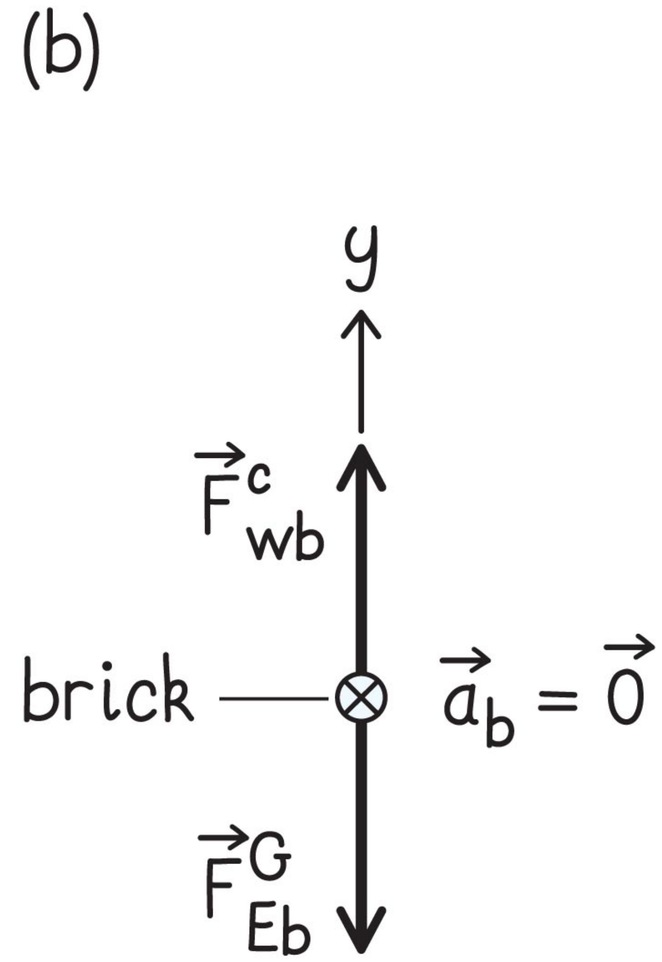
Example 16.4 Measuring mass (cont.)

❶ GETTING STARTED I begin by making a sketch of the situation (Figure 16.40*a*). Because I know the pulse travels 5.0 m in 70 ms, I can determine the wave speed along the wire: $c = (5.0\ \text{m})/(0.070\ \text{s}) = 71\ \text{m/s}$. This wave speed depends on the linear mass density of the wire and the tension in the wire. The latter is determined by the force exerted by the brick on the wire.



Example 16.4 Measuring mass (cont.)

❶ GETTING STARTED Because the force exerted by the brick on the wire and the force exerted by the wire on the brick are equal in magnitude, I draw a free-body diagram for the brick (Figure 16.40*b*) to help me determine that magnitude.



Example 16.4 Measuring mass (cont.)

❷ DEVISE PLAN To determine the mass of the wire, I can use Eq. 16.25. To use this equation, I must know the length of the wire, which is given, and the linear mass density μ, which I can calculate from Eq. 16.30.

To obtain μ from Eq. 16.30, I need to know c (which I already calculated) and the tension $\mathcal{T}$ in the wire. The tension is just the weight of the brick.

Example 16.4 Measuring mass (cont.)

❸ EXECUTE PLAN The tension is equal to the downward force of gravity exerted on the brick:

$$\mathcal{T} = F^G_{\text{Eb}} = (10\ \text{kg})(9.8\ \text{m/s}^2) = 98\ \text{N}.$$

Substituting the values for $\mathcal{T}$ and c into Eq. 16.30, I calculate the linear mass density of the wire $\mu = \mathcal{T}/c^2 = (98\ \text{N})/(71\ \text{m/s})^2 = 0.019\ \text{kg/m}$. The mass of the wire is thus $m = (0.019\ \text{kg/m})(5.0\ \text{m}) = 9.5 \times 10^{-2}\ \text{kg}$. ✔

Example 16.4 Measuring mass (cont.)

❹ EVALUATE RESULT The value I obtain—about 0.1 kg—is not unreasonable for a 5.0-m-long wire that can support a lead brick.

Section 16.7
Question

A vibrating string is clamped at both ends, with one of the clamps being a tension-adjustment screw. By what factor must you change the tension in the string to double its frequency of vibration without changing the wavelength?

1. 2
2. 1/2
3. 2^2
4. $(1/2)^2$
5. $2^{1/2}$
6. $(1/2)^{1/2}$
7. None of the above

Question

A vibrating string is clamped at both ends, with one of the clamps being a tension-adjustment screw. By what factor must you change the tension in the string to double its frequency of vibration without changing the wavelength?

1. 2
2. 1/2
3. ✔ 2^2 – $f = c/\lambda \sim \sqrt{T}$. Doubling f means T increases fourfold
4. $(1/2)^2$
5. $2^{1/2}$
6. $(1/2)^{1/2}$
7. None of the above

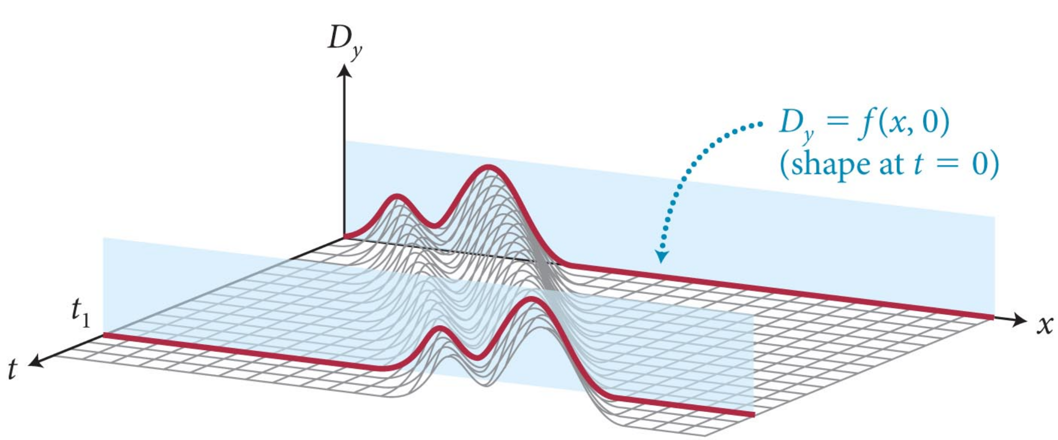
Section 16.8: Energy transport in waves

Section Goals

You will learn to

- Calculate the **kinetic and potential energy** for a wave disturbance in one dimension.
- Express the **average power** carried by a harmonic wave.

(*a*) Shape of the medium at instants $t = 0$ and $t = t_1$



Section 16.8: Energy transport in waves

- Let E_λ denote the energy that must be supplied over a period T to generate a wave on a string. (one wavelength)
- Then the average power that must be supplied to generate the wave is given by

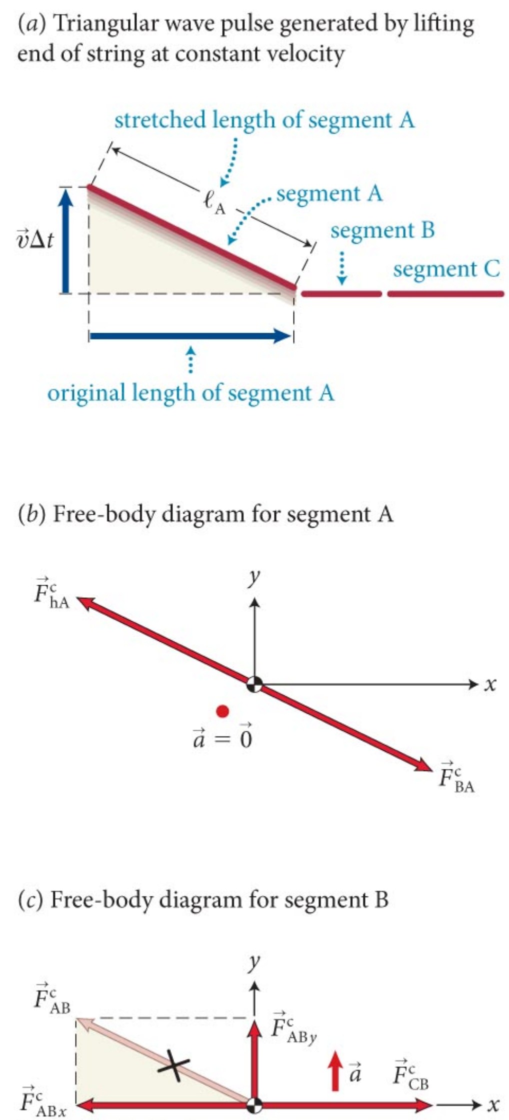
$$P_{av} \equiv \frac{\Delta E}{\Delta t} = \frac{E_\lambda}{T}$$

- Using the figure, or from simple harmonic motion:

$$E_\lambda = \frac{1}{2} m\omega^2 A^2 = \frac{1}{2}(\mu\lambda)\,\omega^2 A^2$$

- And the average power is

$$P_{av} = \frac{1}{2}\mu\lambda A^2\omega^2/T = \frac{1}{2}\mu A^2\omega^2 c$$



Example 16.5 Delivering energy

A wire with linear mass density $\mu = 0.0500$ kg/m is held taut with a tension of 100 N. At what rate must energy be supplied to the wire to generate a traveling harmonic wave that has a frequency of 500 Hz and an amplitude of 5.00 mm?

Example 16.5 Delivering energy (cont.)

❶ GETTING STARTED The rate at which energy must be supplied to the wire is the power. Because none of the quantities given varies with time, the (instantaneous) power is equal to the average power, which is given by Eq. 16.42.

Example 16.5 Delivering energy (cont.)

❷ DEVISE PLAN To calculate the average power, I need to know μ and A, both of which are given, as well as ω and c. The angular frequency ω is related to the frequency f by Eq. 15.4, $\omega = 2\pi f$, and I can obtain the wave speed c from Eq. 16.30.

$$c = \sqrt{\frac{\mathcal{T}}{\mu}}$$

$$P_{av} = \frac{1}{2}\mu\lambda A^2\omega^2/T = \frac{1}{2}\mu A^2\omega^2 c$$

Example 16.5 Delivering energy (cont.)

❸ EXECUTE PLAN From Eq. 16.30 I obtain $c = \sqrt{\mathcal{T}/\mu} = \sqrt{(100\text{ N})/(0.0500\text{ kg/m})} = 44.7\text{ m/s}$. Equation 15.4 yields $\omega = 2\pi f = 2\pi(500\text{ Hz}) = 3.14 \times 10^3\text{ s}^{-1}$.

Substituting these values into Eq. 16.42 gives

$$P_{\text{av}} = \tfrac{1}{2}(0.0500\text{ kg/m})(0.00500\text{ m})^2(3.14\times10^3\text{s}^{-1})^2(44.7\text{ m/s})$$

$$= 275\text{ W}.$$

Example 16.5 Delivering energy (cont.)

❹ EVALUATE RESULT The answer I obtain is a fairly large power—comparable to the power delivered by a 250-W light bulb or by a person exercising. The wave travels very fast, though, and the frequency is high, so the answer is not unreasonable.

Section 16.9: The wave equation

Section Goals

You will learn to

- Derive the **general equation** for the time- and space-evolution of a wave using differential calculus.
- Relate the **speed of wave propagation**, c, to physical quantities for the case of a wave on a string.

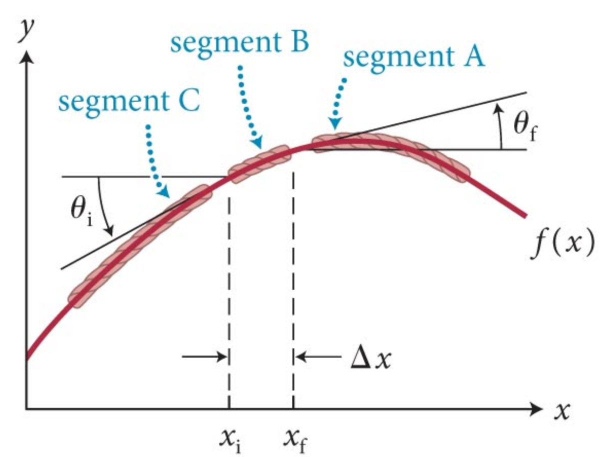
Section 16.9: The wave equation

- The figure shows a piece of a string that has been displaced by a passing wave.
- With the use of this figure, we can show that the wave function $f(x, t)$ that represents a wave is a solution of the **wave equation** given by

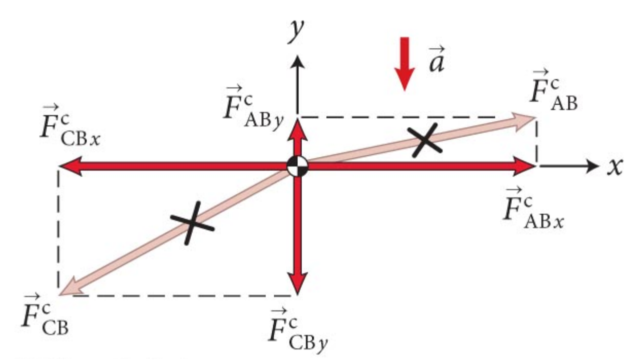
$$\frac{\partial^2 f}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 f}{\partial t^2},$$

(Read ∂ as d …)

(*a*) String transmitting wave pulse



(*b*) Free-body diagram for segment B



Exercise 16.6 Sinusoidal solution to the wave equation

Show that a sinusoidal traveling wave of the form $f(x, t) = A \sin(kx - \omega t)$ satisfies the wave equation for any value of k and ω.

Exercise 16.6 Sinusoidal solution to the wave equation (cont.)

SOLUTION The first partial derivative of $f(x, t)$ with respect to x is

$$\frac{\partial f}{\partial x} = kA\cos(kx - \omega t),$$

and so the second partial derivative with respect to x is

$$\frac{\partial^2 f}{\partial x^2} = -k^2 A \sin(kx - \omega t). \qquad (1)$$

Exercise 16.6 Sinusoidal solution to the wave equation (cont.)

The first partial derivative of $f(x, t)$ with respect to t is

$$\frac{\partial f}{\partial t} = -\omega A \cos(kx - \omega t).$$

Differentiating again gives

$$\frac{\partial^2 f}{\partial t^2} = -\omega^2 A \sin(kx - \omega t).$$

Exercise 16.6 Sinusoidal solution to the wave equation (cont.)

Multiplying both sides of this equation by $1/c^2$ and using $k = \omega/c$ (Eq. 16.11), I obtain

$$\frac{1}{c^2}\frac{\partial^2 f}{\partial t^2} = -\frac{\omega^2}{c^2}A\ \sin(kx - \omega t) = -k^2 A\ \sin(kx - \omega t). \qquad (2)$$

Because the right sides of Eqs. 1 and 2 are equal, the left sides must also be equal, so

$$\frac{\partial^2 f}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 f}{\partial t^2},$$

which is Eq. 16.51, and so the wave equation is satisfied. ✔

Exercise 16.6 Sinusoidal solution to the wave equation (cont.)

Note that this works whether you start with

$$f(x, t) = A \sin(kx - \omega t)$$

Or

$$f(x, t) = A \cos(kx - \omega t)$$

What matters, as with SHM, is that

$$\frac{d^2 f}{dt^2} = -(\text{const})^2 f(x, t)$$

$$\frac{d^2 f}{dx^2} = -(\text{const})^2 f(x, t)$$

Both sines and cosines work for this (complex exponentials FTW)

Section 16.9
Question 9

A wave pulse moves along a stretched string. Describe the relationship among the speed at which the pulse moves, the pulse curvature, and the acceleration of small segments of the string.

1. Pulse speed $\propto$ pulse curvature $\propto$ acceleration
2. Pulse speed $\propto$ pulse curvature $\propto$ 1/acceleration
3. Pulse speed $\propto$ 1/pulse curvature $\propto$ acceleration
4. 1/Pulse speed $\propto$ pulse curvature $\propto$ acceleration
5. 1/Pulse speed $\propto$ 1/pulse curvature $\propto$ 1/acceleration

Section 16.9
Question 9

A wave pulse moves along a stretched string. Describe the relationship among the speed at which the pulse moves, the pulse curvature, and the acceleration of small segments of the string.

4. 1/Pulse speed $\propto$ pulse curvature $\propto$ acceleration

Curvature ~ second derivative w.r.t. position

Acceleration ~ second derivative w.r.t. time

wave equation → curvature ~ acceleration

Established that lower speed waves have higher curvature

Concepts: Representing waves

- A **wave** is a *disturbance* that propagates through material (the medium) or through empty space.
- A **wave pulse** is a single isolated propagating disturbance.
- The **wave function** represents the shape of a wave at any given instant and changes with time as the wave travels.

Chapter 16: Summary

Concepts: Representing waves

- The **wave speed** *c* is the speed at which a wave propagates. For a mechanical wave, *c* is different from the speed *v* of the particles of the medium and is determined by the properties of the medium.
- The **displacement** $\vec{D}$ of any particle of a medium through which a mechanical wave travels is a vector that points from the equilibrium position of the particle to its actual position.

Concepts: Representing waves

- In a **transverse** mechanical wave, the particles of the medium move perpendicular to the direction of the pulse movement.
- In a **longitudinal** mechanical wave, these particles move parallel to the direction of the pulse movement.
- In a **periodic wave**, the displacement at any location in the medium is a periodic function of time.
  A periodic wave is **harmonic** when the particle displacement can be represented by a sinusoidally varying function of space and time.

Quantitative Tools: Representing waves

- If a wave travels in the x direction with speed c and $f(x)$ describes the form (shape) of the wave, then the y component D_y of the displacement of a particle of the medium is

$$D_y = f(x - ct)$$

 if the wave travels in the positive x direction and

$$D_y = f(x + ct)$$

 if the wave travels in the negative x direction.
- The **wavelength** λ of a periodic wave is the minimum distance over which the wave repeats itself.

Quantitative Tools: Representing waves

- For a periodic wave of period T, frequency f, and speed c, the **wave number** k is

$$k = \frac{2\pi}{\lambda}.$$

- The wavelength λ is

$$\lambda = cT,$$

- The angular frequency ω is

$$\omega = \frac{2\pi}{T},$$

- The wave speed is

$$c = \lambda f.$$

Quantitative Tools: Representing waves

- For a transverse harmonic wave of amplitude A and initial phase ϕ_i traveling in the positive x direction, the y component D_y of the displacement of a particle of the medium is

$$D_y = f(x, t) = A \sin(kx - \omega t + \phi_i).$$

Concepts: Combining waves

- **Superposition of waves:** The resultant displacement of two or more overlapping waves is the algebraic sum of the displacements of the individual waves.
- **Interference** occurs when two waves overlap. The interference is *constructive* when the displacements due to the two waves are in the same direction and *destructive* when the displacements are in opposite directions.
- If the displacement at a point in space remains zero as a wave travels through, that point is a **node.** The displacement at other points typically varies with time. If the displacement at a point in space varies over the greatest range as a wave travels through, that point is an **antinode.**

Concepts: Combining waves

- When a wave pulse (the *incident* wave) reaches a boundary where the transmitting medium ends, the pulse is *reflected*, which means it reverses its direction.
- When a wave pulse is reflected from a fixed boundary, the reflected pulse is *inverted* relative to the incident pulse. When the reflection is from a boundary that is free to move, the reflected pulse is not inverted.
- A **standing wave** is a pulsating stationary pattern caused by the interference of harmonic waves of equal amplitude and wavelengths traveling in opposite directions.

Quantitative Tools: Combining waves

- If two harmonic waves of equal wavelength λ and equal amplitude A travel in opposite directions on a string, they produce a standing wave. The y component D_y of the string displacement at any position x along the string is given by

$$D_y = 2A \sin kx \cos \omega t.$$

- The nodes occur at

$$x = 0, \pm\frac{\lambda}{2}, \pm\lambda, \pm\frac{3\lambda}{2}, \ldots$$

- The antinodes occur at

$$x = \pm\frac{\lambda}{4}, \pm\frac{3\lambda}{4}, \pm\frac{5\lambda}{4}, \ldots$$

Chapter 16: Summary

Quantitative Tools: Waves on a string

- For a uniform string of mass m and length ℓ, the **linear mass density** μ (mass per unit length) is

$$\mu = \frac{m}{\ell}.$$

- The speed of a wave on a string under tension $\mathcal{T}$ is

$$c = \sqrt{\frac{\mathcal{T}}{\mu}}.$$

Quantitative Tools: Waves on a string

- The average power P_{av} that must be supplied to generate a wave of period T is

$$P_{av} = \tfrac{1}{2}\mu\lambda A^2\omega^2/T = \tfrac{1}{2}\mu A^2\omega^2 c.$$

- Any function f of the form $f(x - ct)$ or $f(x + ct)$ that represents a wave traveling with speed c is a solution of the wave equation:

$$\frac{\partial^2 f}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 f}{\partial t^2}.$$