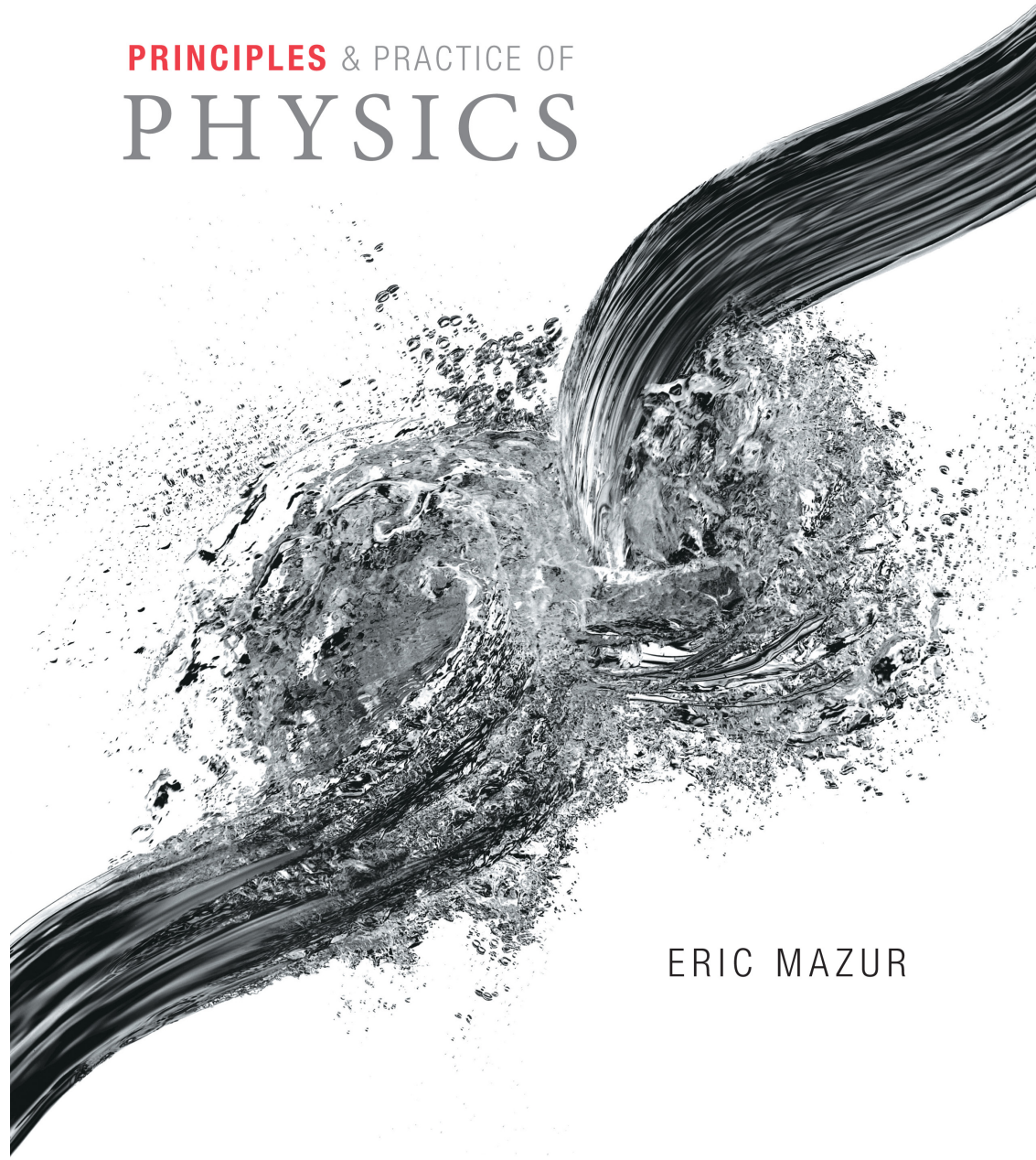


PRINCIPLES & PRACTICE OF  
PHYSICS

**Chapter 2**  
**Motion in One**  
**Dimension**



ERIC MAZUR

# MasteringPhysics, PackBack Answers

- You should be on both by now.
- MasteringPhysics – had first reading quiz today
  - One more on Thursday, HW on Friday
  - if you couldn't get on in time I'll extend deadline
- PackBack – should have email & be signed up

# PackBack Answers

- Try to ask questions you are curious about
- Don't just use book discussion questions, ideally
  - unless that is what you are curious about ...
- Can be about physics we don't cover
- the volume of questions can be high at first
  - try to browse and see what has already been asked
  - provide answers & up/down vote as you browse

# Labs

- first procedure – link on MasteringPhysics
- read ahead of time
- can print there (one per group!)

# Reading quiz

- normally we'd go over the reading quiz now ...
- we'll wait until Thursday so everyone has a chance to do it

# Chapter 2 Motion in One Dimension

## Concepts

# Section 2.1: From reality to model

- Visualize the motion of objects in one dimension in several situations using the “frame sequence” diagram and by using motion graphs.
- Correlate the information about motion contained in “frame sequence” diagrams and motion graphs.
- Understand how a reference axis allows for the precise determination of physical information about motion.

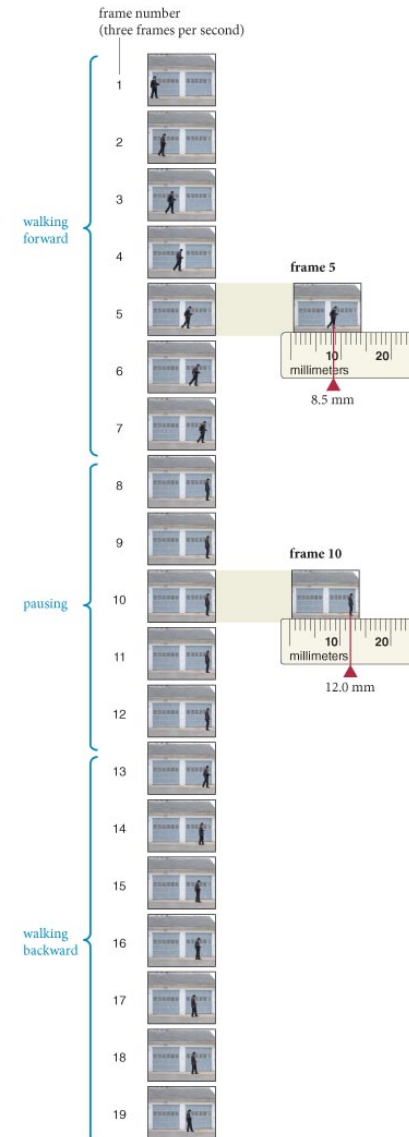
## Section 2.1: From reality to model

- The branch of physics that deals with the quantitative representation of motion is called **kinematics**.
- Kinematics provides a mathematical description of motion.
- It does not however, consider cause and effect of motion.



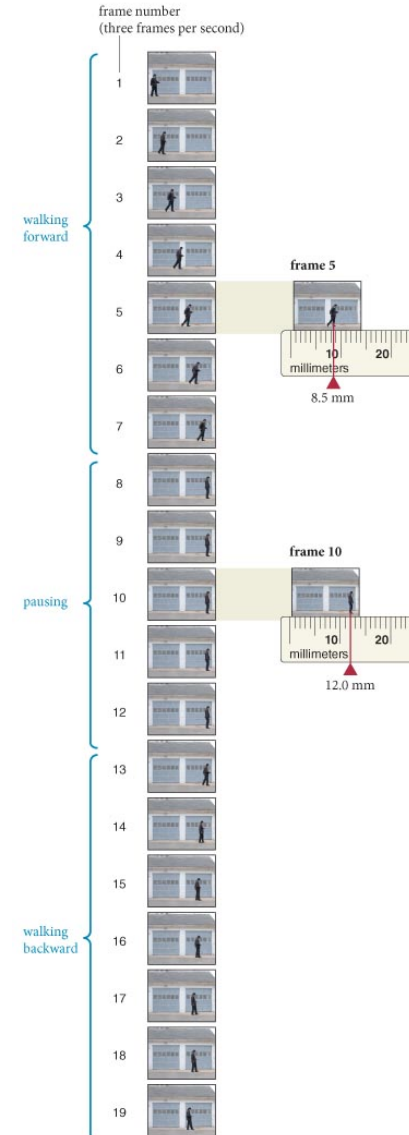
# Section 2.1: From reality to model

- To analyze the motion of an object, we need to keep track of the object's position at different instants:
  - If the position does not change the object is at rest.
  - If the position changes the object is moving.



# Section 2.1: From reality to model

- analyze the motion of a man walking by studying a film clip
- first need to establish
  - A **reference axis**—an imaginary straight line along the ground .
  - An **origin**—an arbitrarily chosen reference point: Choose the left edge of the frame in the figure.
- can now determine the position of the man in each frame measure from the origin.
- *general problem solving*: pick axes and origin



# Section 2.1: From reality to model

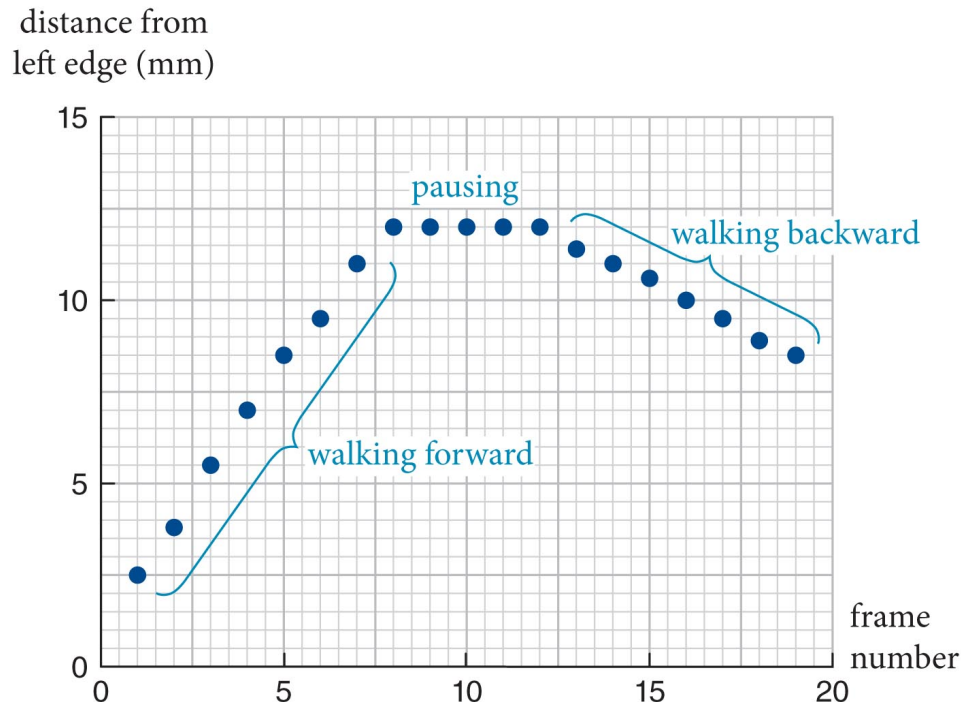
- Do this for every frame

**Table 2.1** Position versus frame number from the film sequence in Figure 2.1

Frame number	Distance from left edge (mm)
1	2.5
2	3.8
3	5.5
4	7.0
5	8.5
6	9.5
7	11.0
8	12.0
9	12.0
10	12.0
11	12.0
12	12.0
13	11.4
14	11.0
15	10.6
16	10.0
17	9.5
18	8.9
19	8.5

# Section 2.1: From reality to model

- The same information can be shown graphically:
  - The man's position relative to the origin is plotted along the vertical axis.
  - The horizontal axis represents the frame numbers.

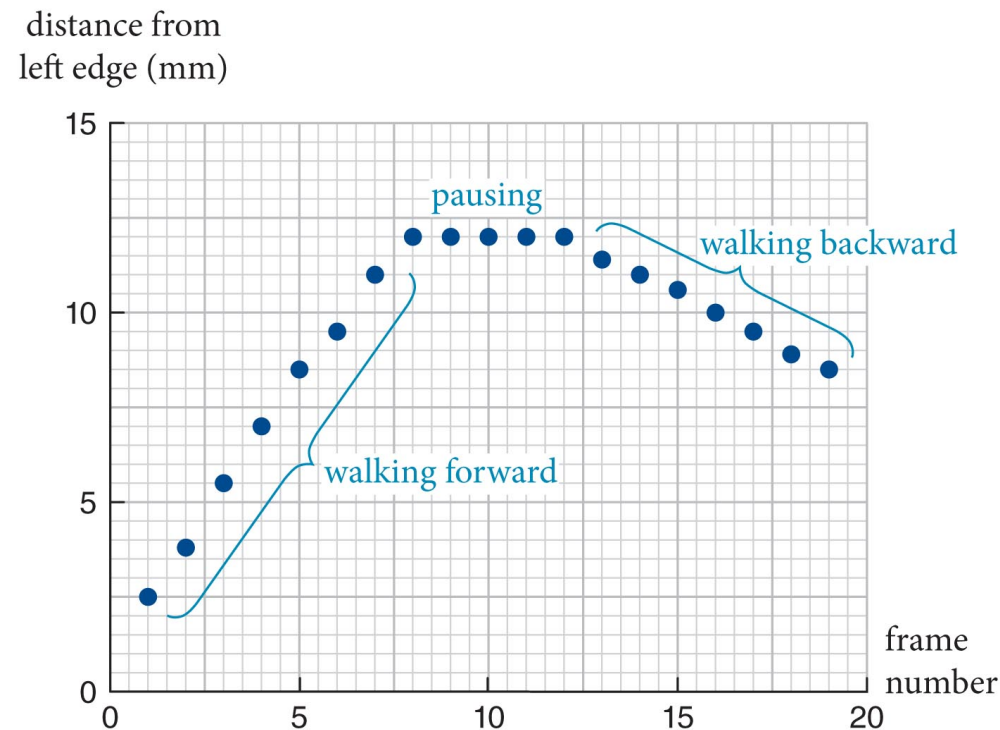


always  
label  
axes!

# Checkpoint 2.2



- (a) The data points for frames 8–12 are all at position 12.0 mm.  
What must be happening?
- (b) The data points for frames 7 and 14 are aligned horizontally.  
What must be happening?
- (c) What if two data points were aligned vertically?  
Could this happen?

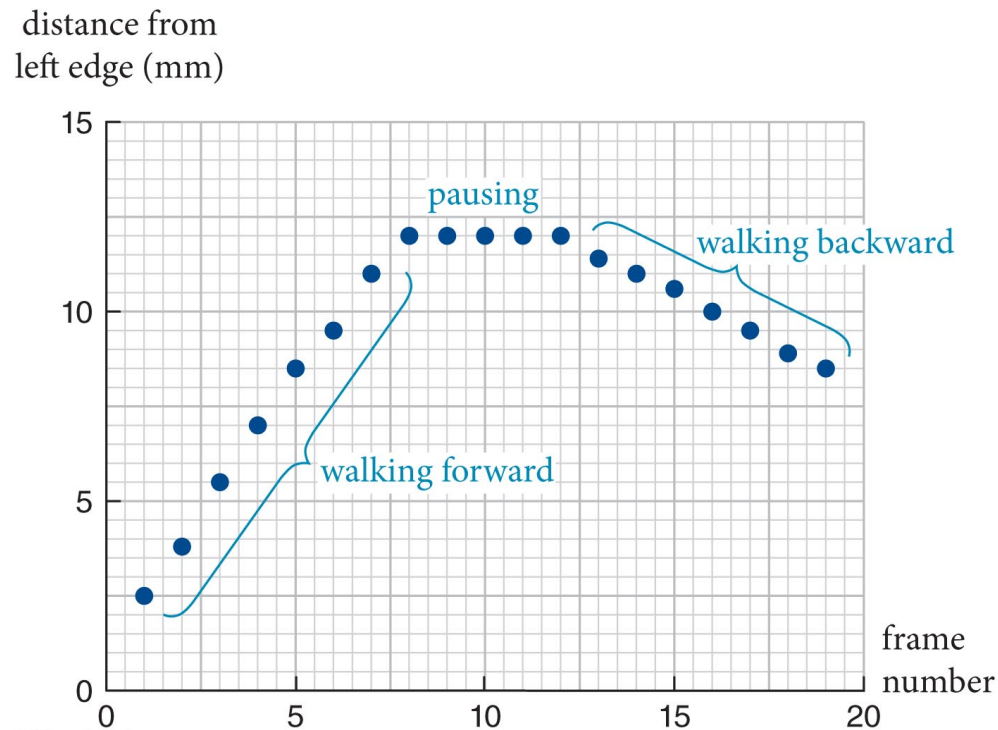


# Checkpoint 2.2



(a) The data points for frames 8–12 are all at position 12.0 mm.

The man is not moving – position isn't changing

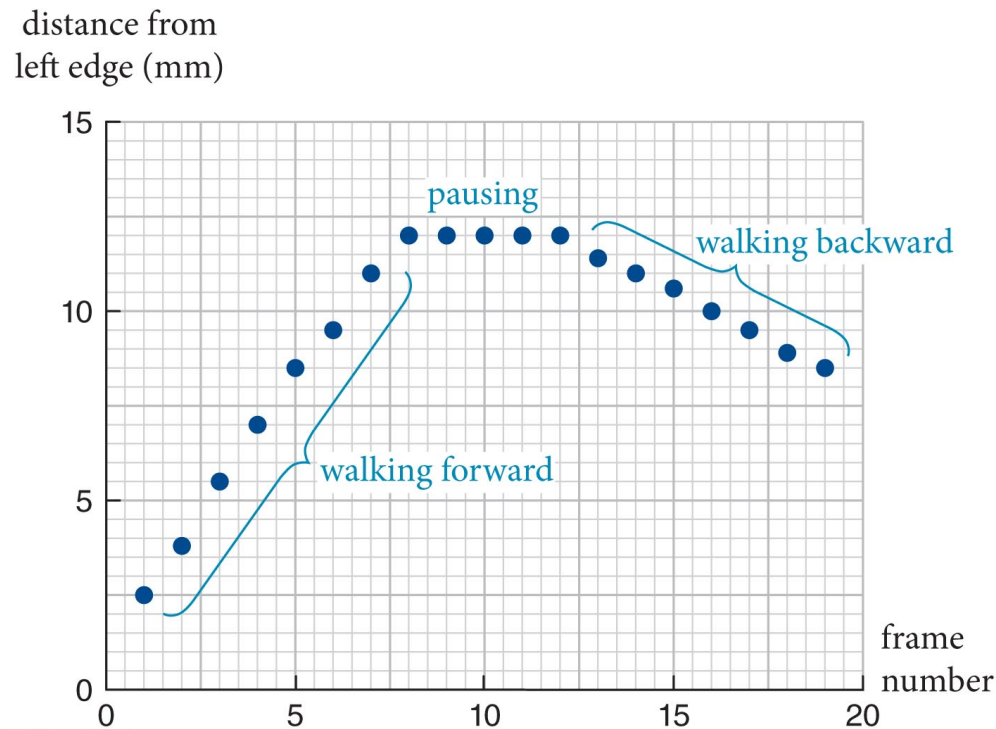


# Checkpoint 2.2



(b) The data points for frames 7 and 14 are aligned horizontally.

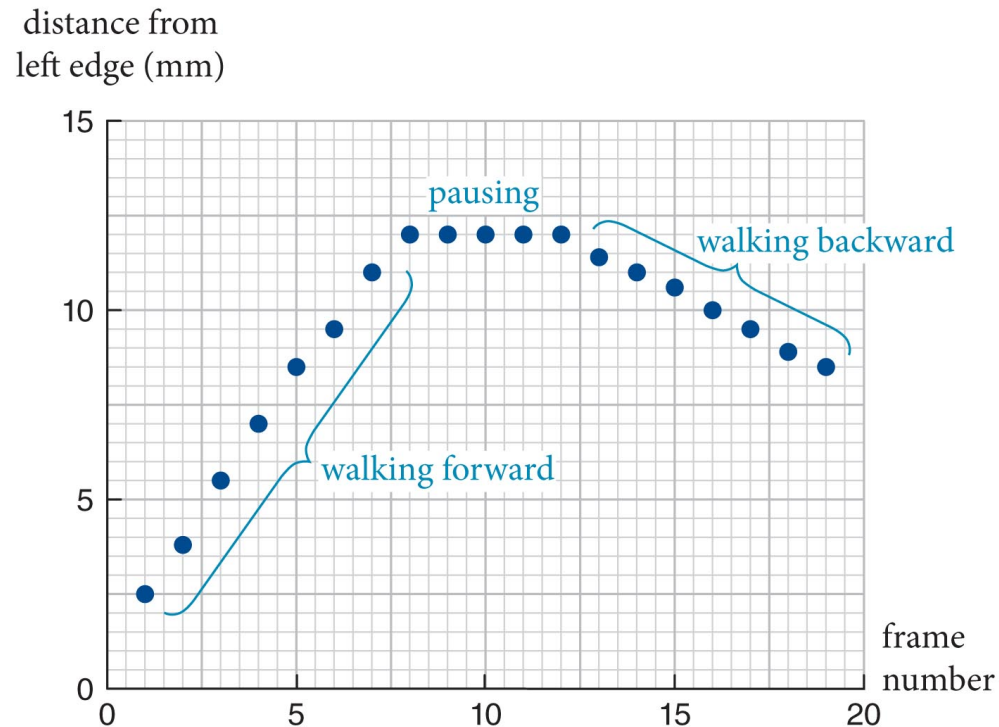
In frame 14 the man returned to the same position he had in frame 7



# Checkpoint 2.2

 (c) What if two data points were aligned vertically?

That would imply an object has two different positions at a single instant in time. This is not a thing that happens.





# Section 2.2: Position and displacement

You will learn to

- Differentiate the concepts of position and displacement for motion in one dimension.
- Recognize the physical significance of the algebraic signs for position and displacement.
- Distinguish between the distance traveled by an object in one dimensional motion and its displacement.

## Section 2.2: Position and displacement

- These are initial steps towards creating our first **model**:
  - A model is a simplified abstract representation of a real-world phenomenon.
  - What is really important in the video?
  - Developing models is one of the most important skills in physics.
- To make this model useful: calibrate the data points
  - need real-world distances and time intervals.

## Section 2.2: Position and displacement

- To calibrate the axes in our video:
  - Put markers on the ground and a clock in the camera's field of view

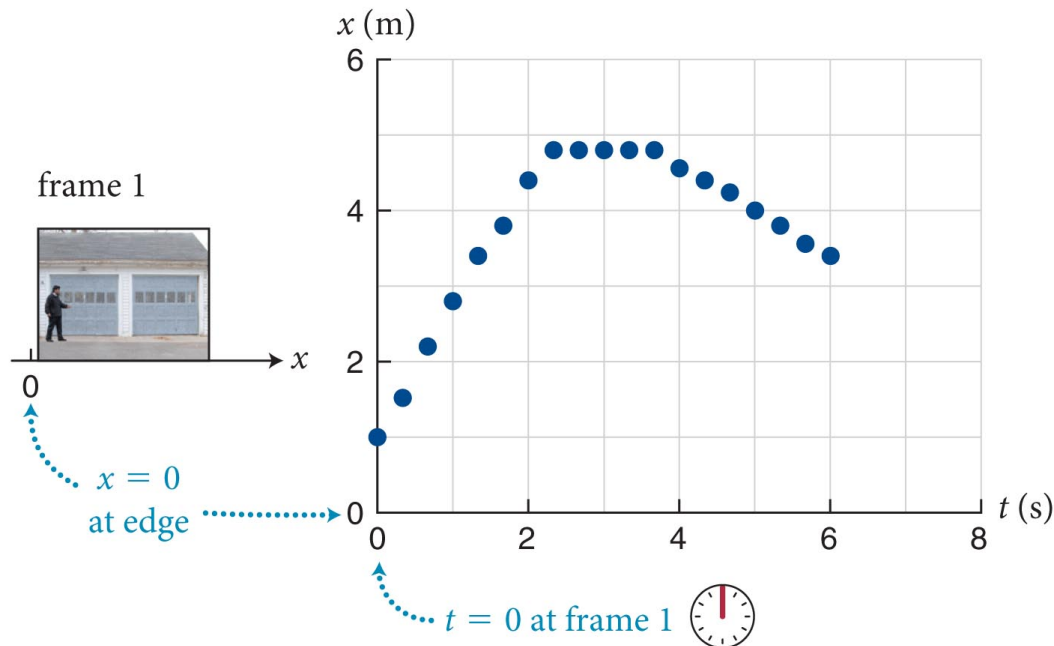
Or



- Use an object of known height already in the video and the frame rate of the camera.
  - E.g., 1.8 m for the height of the man and 30 frames per second for the frame rate

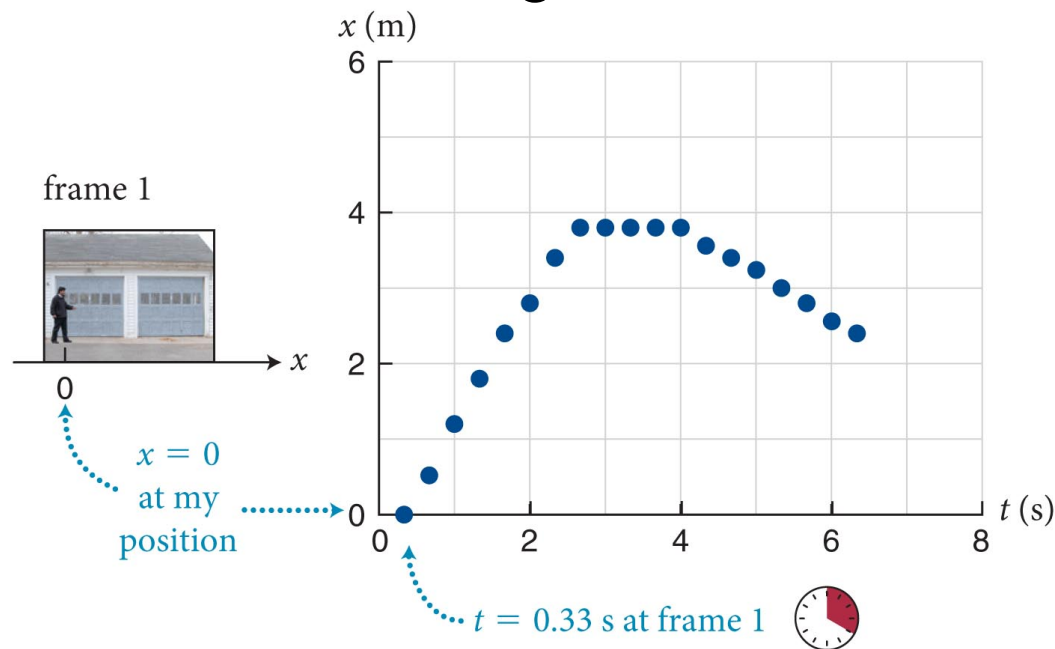
# Section 2.2: Position and displacement

- The resulting calibrated data as a *position-versus-time* graph.
  - The vertical axis is position ( $x$ ) in meters
  - The horizontal axis is time ( $t$ ) in seconds.
  - The choice of origin is arbitrary (for  $x$  or  $t$ ).



# Section 2.2: Position and displacement

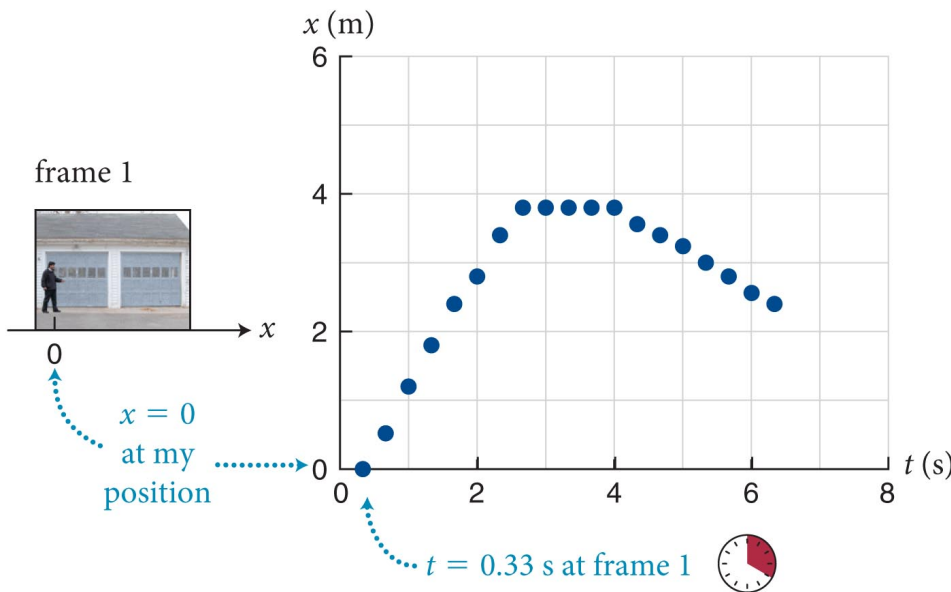
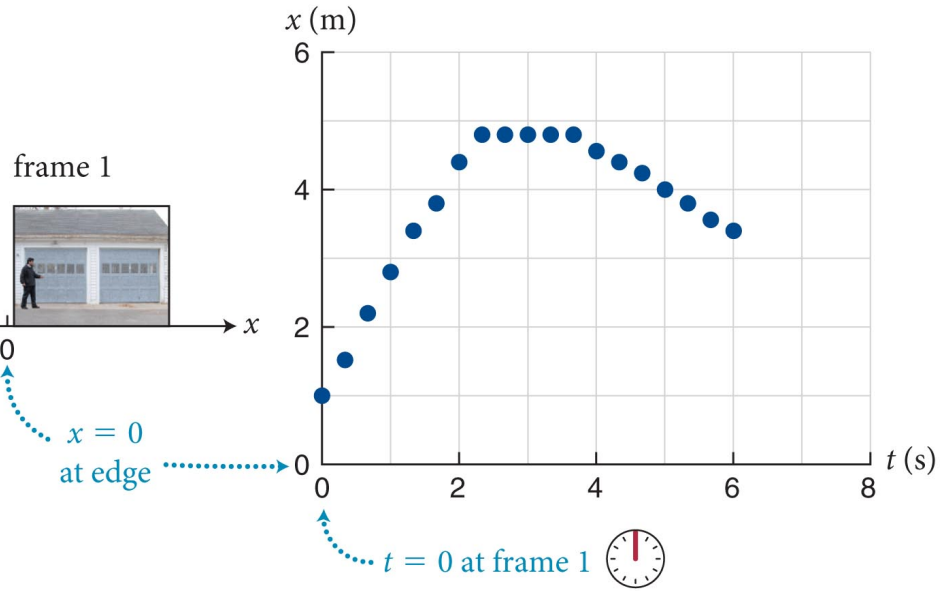
- Origin was arbitrarily chosen to be the left edge of frame.
- Suppose, now we choose the man's position in frame 1 to be the origin and the clock reading at frame 1 to be  $t = 0.33$  s.
  - Doing so *does not* affect relative positions of data points as seen in the figure
  - It just shifts the whole thing



# Checkpoint 2.4



The *absolute positions* are completely different in each graph.  
The *relative positions* from one time to another are the same  
Which do you think really matters?



*the physics can't depend on arbitrary choices we make*

# QUESTION ....

An object goes from one point in space to another. After it arrives at its destination, its displacement is

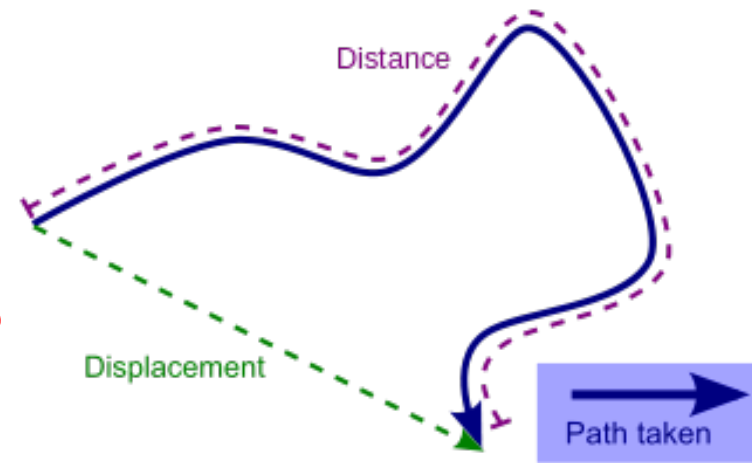
1. either greater than or equal to
2. always greater than
3. always equal to
4. either smaller than or equal to
5. always smaller than
6. either smaller than or larger than

the distance it traveled.

# We try to use words in very specific ways

An object goes from one point in space to another.  
After it arrives at its destination, its displacement is

1. either greater than or equal to
2. always greater than
3. always equal to
- ✓ 4. **either smaller than or equal to**
5. always smaller than
6. either smaller than or larger than

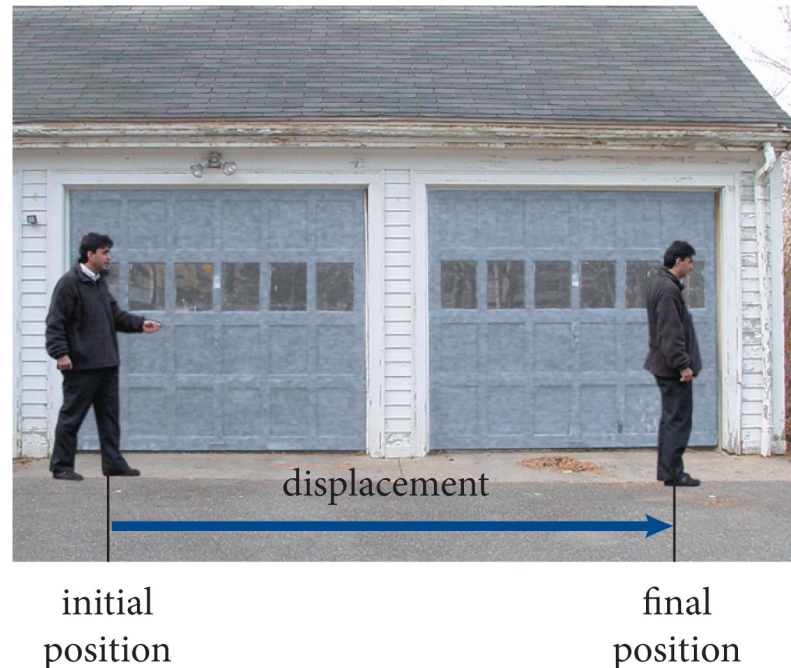


the distance it traveled.



# Displacement

- The **arrow** pointing from the initial position of an object to its final position represents a physical quantity called the **displacement** of the object.
- Displacement *does not* depend on the choice of axis or origin.
- This is how you give directions – series of displacements



## Section 2.2: Position and displacement

- The  $x$  component of an object's displacement is the change in its  $x$  coordinate.
- Find by subtracting the **initial**  $x$  coordinate from the **final**  $x$  coordinate:  $\Delta x = x_f - x_i$
- Independent of the choice of origin:
  - The  $x$  component of displacement is measured along some specific  $x$  axis.
  - The  $x$  component of displacement can be negative or positive. (You can go forward or backward!)

# Checkpoint 2.5



**2.5** Suppose you walk in a straight line from a point P to a point Q, 2 m away from P, and then walk back along the same line to P.

- (a) What is the  $x$  component of your displacement for the round trip?
- (b) What distance did you travel during the round trip? Is *distance traveled* the same thing as the  *$x$  component of the displacement*?
- (c) Is *distance traveled* the same thing as the  *$x$  component of the displacement*?

# Checkpoint 2.5



**2.5** Suppose you walk in a straight line from a point P to a point Q, 2 m away from P, and then walk back along the same line to P.

(a) What is the  $x$  component of your displacement for the round trip?

Displacement is how far you went *in net*, not the length of your journey.

You didn't go anywhere in the end – displacement is 0.

# Checkpoint 2.5



**2.5** Suppose you walk in a straight line from a point P to a point Q, 2 m away from P, and then walk back along the same line to P.

(b) What distance did you travel during the round trip?

You went nowhere in the end, but walked 4m to do it.

# Checkpoint 2.5



**2.5** Suppose you walk in a straight line from a point P to a point Q, 2 m away from P, and then walk back along the same line to P.

(c) Is *distance traveled* the same thing as the *x component of the displacement*?

**No – displacement doesn't care about backtracking, but distance does!**

# Section 2.2: Position and displacement

## Distance

- The **distance** traveled is the length covered by a moving object along the path of its motion.
- In contrast to the  $x$  component of displacement, distance traveled is always positive.
- Your *displacement* to drive to class might be 1mi, but you might drive 2mi **distance** to accomplish it.
  - Given that you'll probably just drive back home for lack of parking

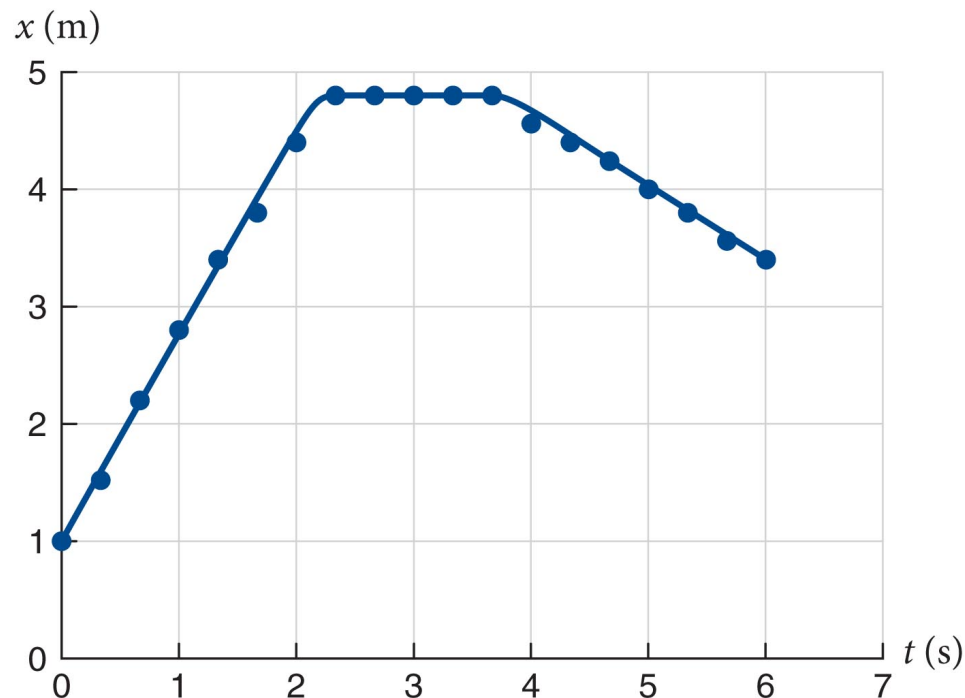
## Section 2.3: Representing motion

- Represent the motion of an object in several graphical and mathematical ways.
- Correlate these various representations.
- Determine the important kinematic quantities of position, displacement, speed, and velocity from graphical and mathematical representations.

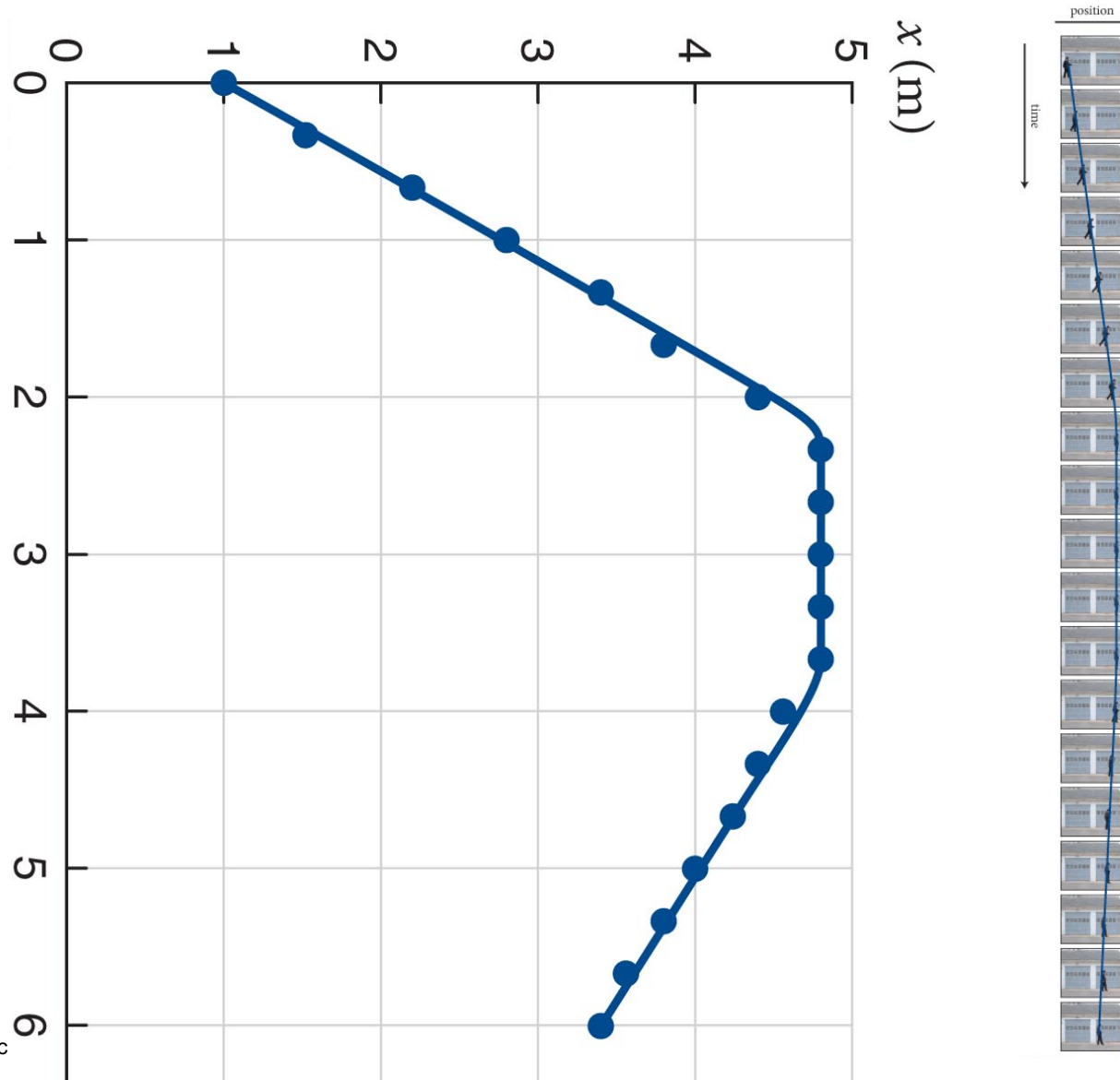


## Section 2.3: Representing motion

- A complete representation of the motion of the man should allow us to determine his positions between frames.
- This can be achieved by **interpolating** data points in the *position-versus-time* graph

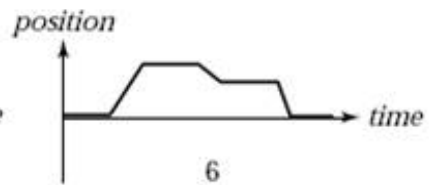
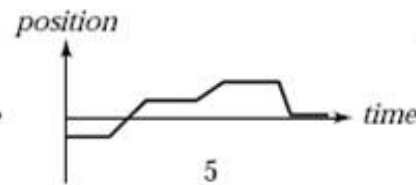
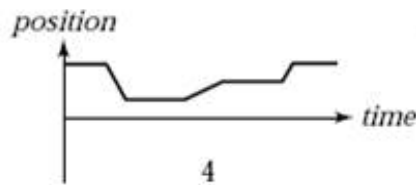
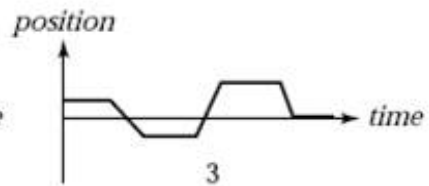
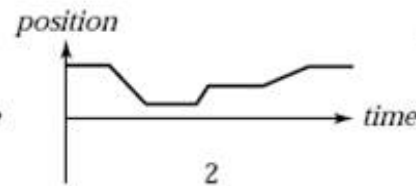
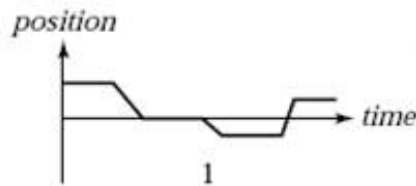
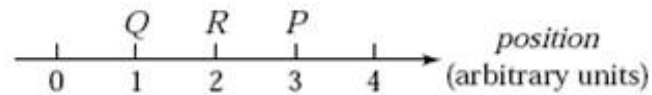


# How the *position-versus-time* graph relates to the film clip



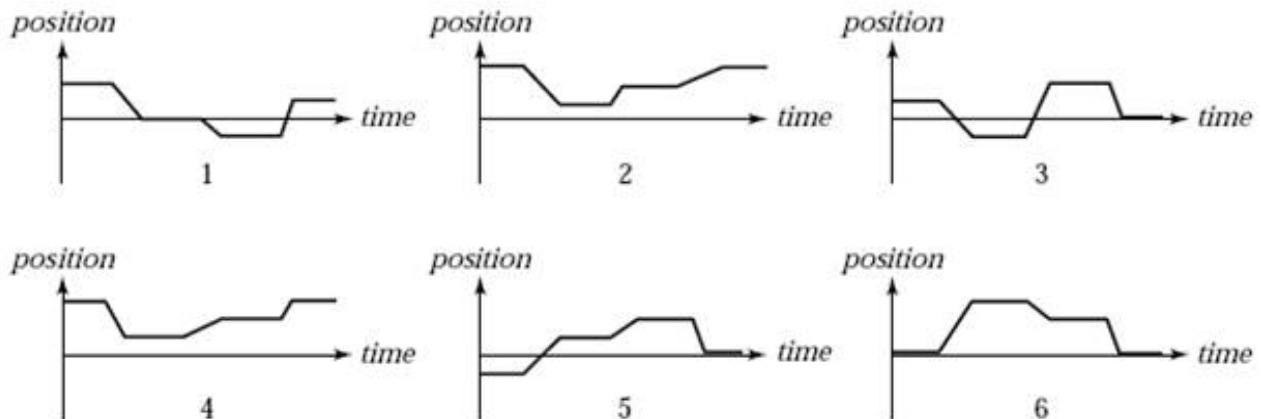
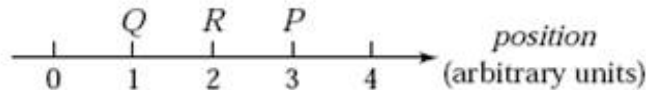
# Section 2.3

A person initially at point  $P$  in the illustration stays there a moment. She then moves along the axis to  $Q$  and stays there a moment. She then runs quickly to  $R$ , stays there a moment, and then strolls slowly back to  $P$ . Which of the position-versus-time graphs below correctly represents this motion?



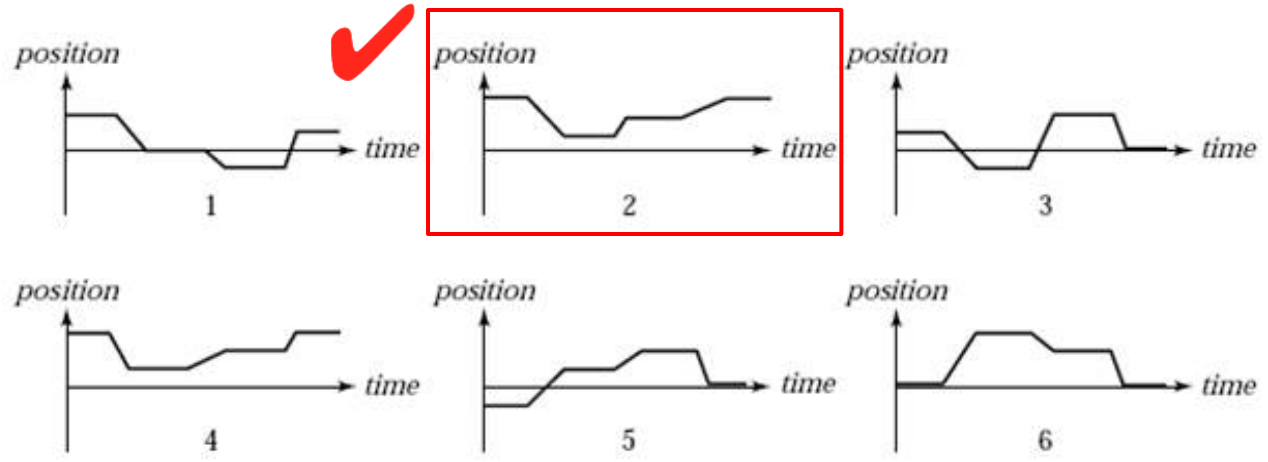
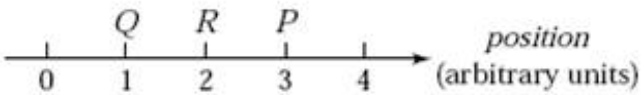
# Section 2.3

- P=3 to Q=2 (**backward, slowly = small negative slope**)
- Stay at Q=1 a bit (**flat**)
- Q to R=2 (**forward, quickly = larger positive slope**)
- Stay at R (**flat**)
- R=2 to P=3 (**forward, slowly = small positive slope**)



# Section 2.3

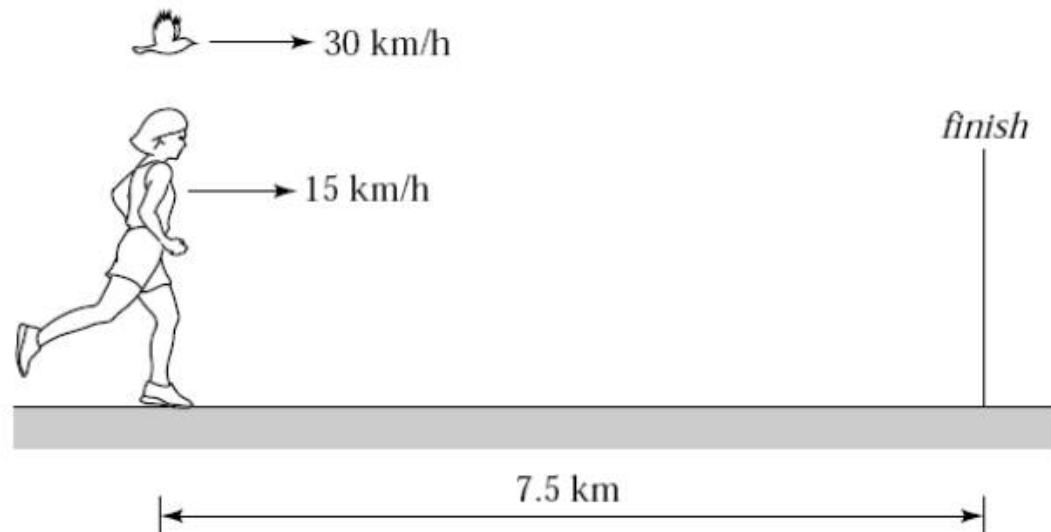
A person initially at point  $P$  in the illustration stays there a moment and then moves along the axis to  $Q$  and stays there a moment. She then runs quickly to  $R$ , stays there a moment, and then strolls slowly back to  $P$ . Which of the position-versus-time graphs below correctly represents this motion?



## Section 2.3

A marathon runner runs at a steady 15 km/hr. When the runner is 7.5 km from the finish, a bird begins flying from the runner to the finish at 30 km/hr. When the bird reaches the finish line, it turns around and flies back to the runner, and then turns around again, repeating the back-and-forth trips until the runner reaches the finish line. How many kilometers does the bird travel?

1. 10 km
2. 15 km
3. 20 km
4. 30 km





# Section 2.3

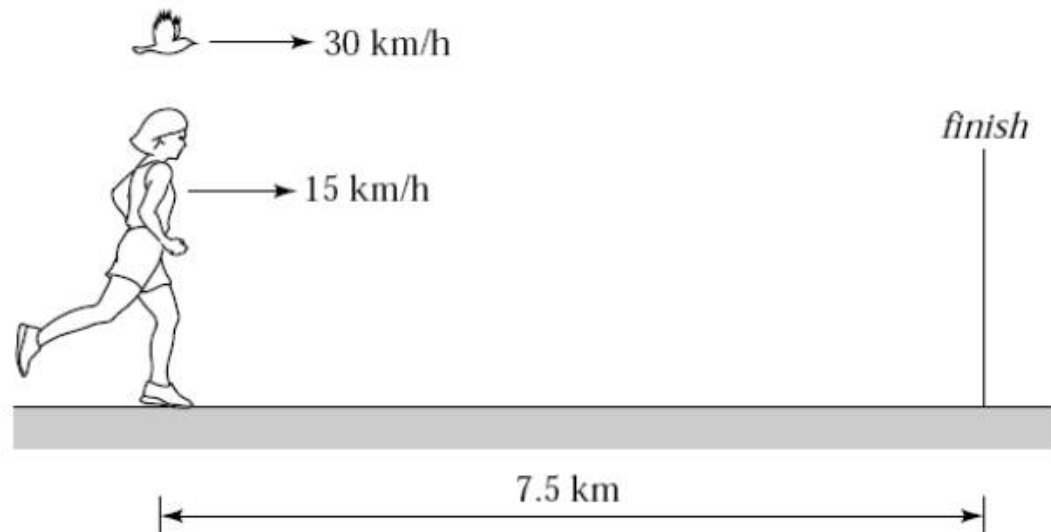
## Question 3

The bird flies as long as the runner is running.

The runner takes half an hour to run 7.5km.

The bird flies  $\frac{1}{2}$  hr, covers  $(\frac{1}{2} \text{ h})(30 \text{ km/h}) = 15 \text{ km}$

1. 10 km
- ✓ 2. 15 km
3. 20 km
4. 30 km





# Section 2.4: Average speed and average velocity

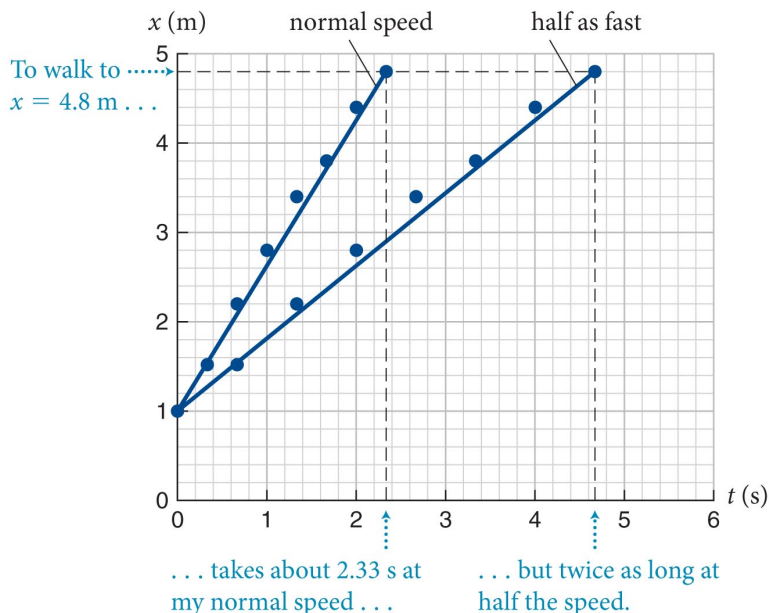
You will learn to

- Define the average speed and average velocity of an object in one-dimensional motion.
- Compute the average speed and the average velocity of an object in one-dimensional motion from motion graphs.
- Correlate the geometry of motion graphs with the average speed and the average velocity of an object in one-dimensional motion.

# Section 2.4: Average speed and average velocity

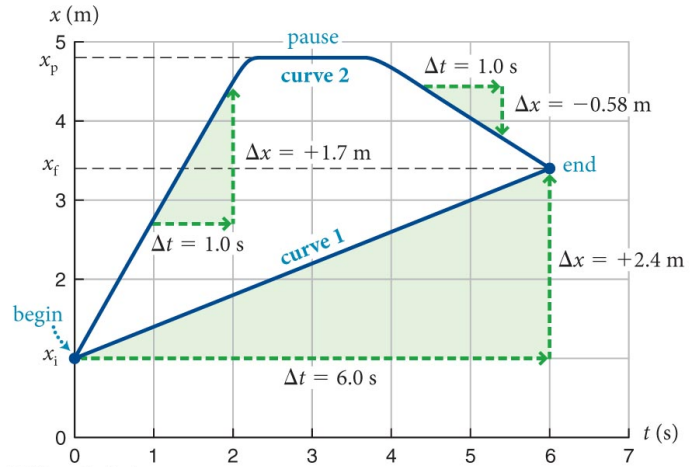
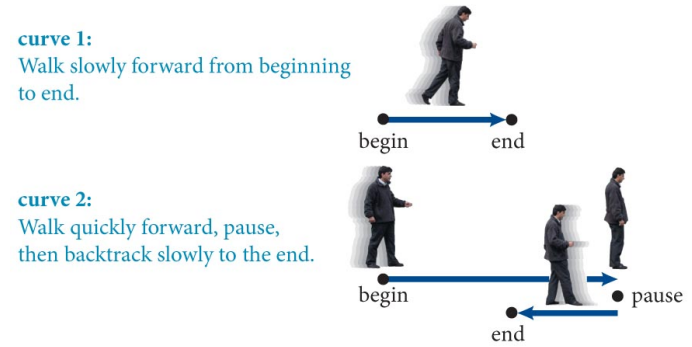
## Average Distance

- The next step in describing the motion of the man is to determine the **average speed** of the motion.
- The rate at which the *x-versus-t* curve rises with increasing time is called the slope of the curve:
  - Notice how the slope becomes less steep when the speed decreases.



# Section 2.4: Average speed and average velocity

- An object's average speed is the **distance traveled** divided by the time interval required to travel that distance.
- Curve 2 shows that the man travels a total distance of 5.2 m in 6.0 s:
  - Average speed =  $(5.2 \text{ m}) / (6.0 \text{ s}) = 0.87 \text{ m/s}$  in both cases.

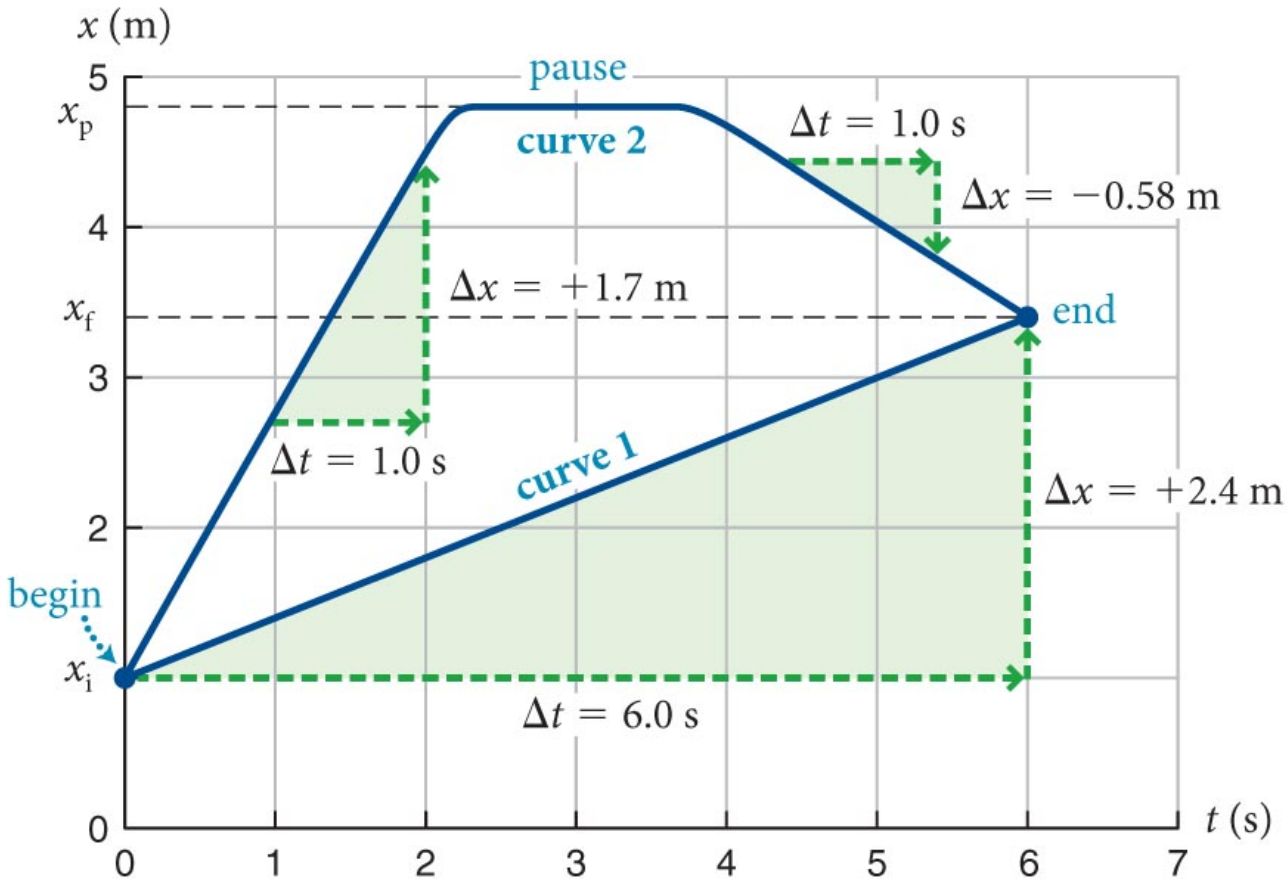


# Average speed


- Average speed would be dividing your odometer reading by how long the trip took
- This is *not* the same as your speed at any given instant
- If position vs time is a straight line:
  - constant slope  $\rightarrow$  constant speed
  - all intervals along the line have same average speed
- If position vs time is a flat line:
  - object is not moving
- If position vs time is neither:
  - object's speed is changing

# Checkpoint 2.8

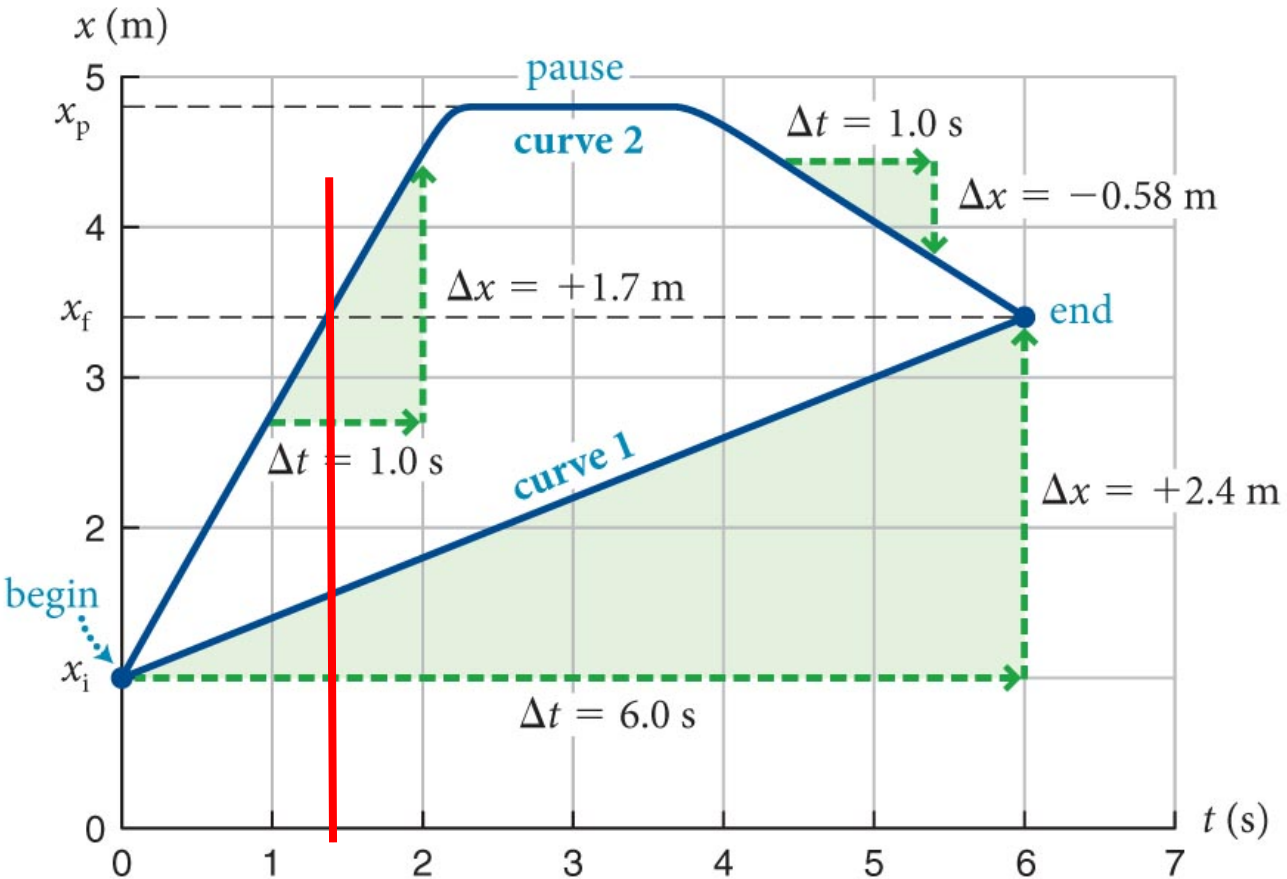
- 2.8 (a) Using curve 2, at what instant did the man first pass the position  $x_f = +3.4$  m
- (b) What was his average speed up to that instant?



# Checkpoint 2.8

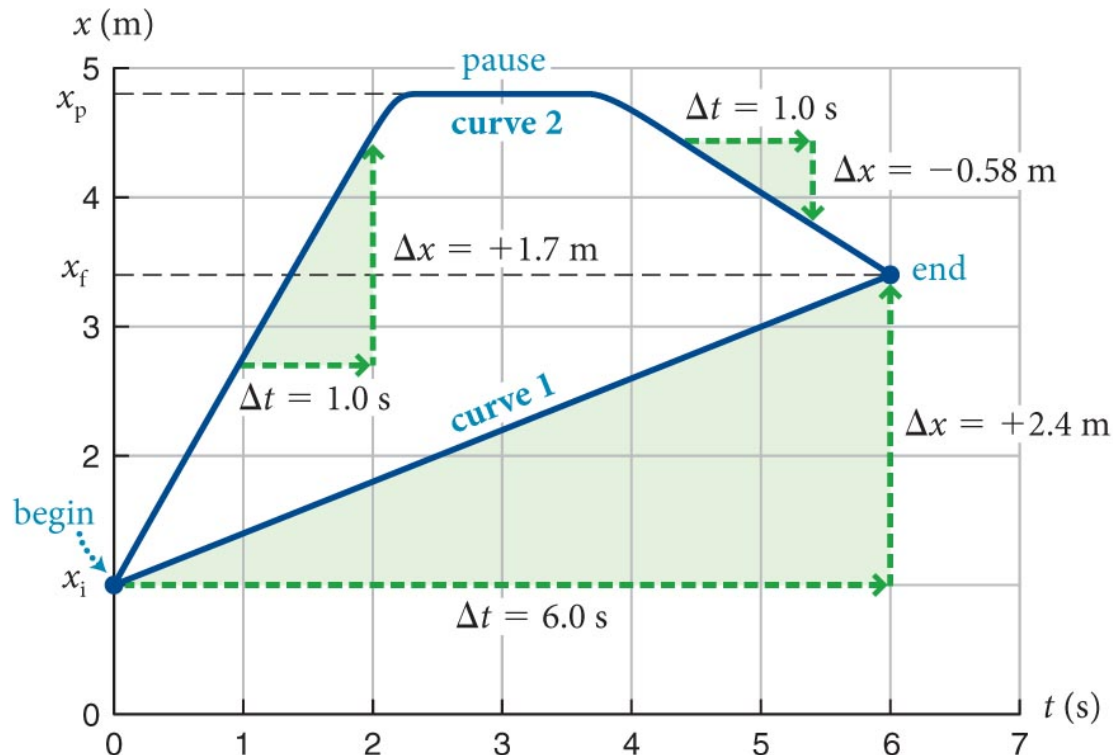
 **2.8** First passes  $x_f$  at around 1.4 seconds or so

Average speed at that point is the slope of the line, 1.7 m/s



# Section 2.4: Average speed and average velocity

- In a graph of position versus time, the steeper the slope of the curve, the higher the speed.
- Compare the average speeds during the first and third segments of the motion represented by curve 2 in the figure.



# Section 2.4: Average speed and average velocity

## Velocity

- The quantity that gives both the speed and the direction of travel is **velocity**.
- The  $x$  component of an object's average velocity is the  $x$  component of its displacement divided by the length of the time interval taken to travel that displacement.
- speed: “The car is going 100 km/h”
- velocity: “The car is going 100 km/h due west”



# Section 2.4: Average speed and average velocity

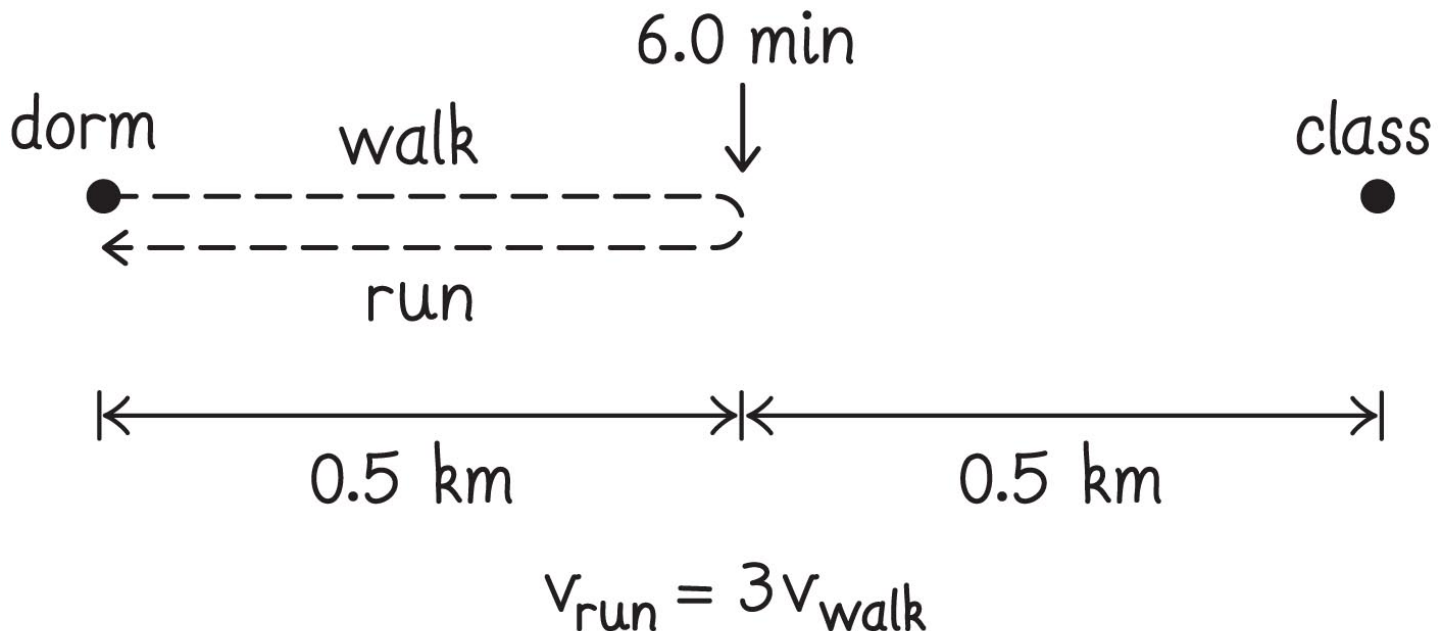
## Example 2.3 Speed and velocity

- With about 20 min to spare, you walk leisurely from your dorm to class, which is 1.0 km away.
- Halfway there, you realize you have forgotten your notebook and run back to your dorm.
- You walked 6.0 min before turning around, and you travel three times as far per unit of time running as you do when walking.
- What are (a) the  $x$  component of your average velocity and (b) your average speed over the entire trip?

# Section 2.4: Average speed and average velocity

## Example 2.3 Speed and velocity (cont.)

**1** GETTING STARTED To begin this problem, it helps to make a sketch to visualize the situation



# Section 2.4: Average speed and average velocity

## Example 2.3 Speed and velocity (cont.)

② **DEVISE PLAN** To find the  $x$  component of your average velocity, I need to divide the  $x$  component of your displacement by the time interval over which this displacement took place. To find your average speed, I must divide the distance traveled by the time interval.

# Section 2.4: Average speed and average velocity

## Example 2.3 Speed and velocity (cont.)

### ③ EXECUTE PLAN

(a) When you are back at the dorm, the  $x$  component of your displacement is zero because your initial and final positions are the same. Therefore the  $x$  component of your average velocity is zero too. ✓

average velocity =  $(x \text{ component of the displ})/(\text{time})$

## Section 2.4: Average speed and average velocity

### Example 2.3 Speed and velocity (cont.)

③ (b) You traveled halfway to class and back, and the distance between your dorm and the classroom is 1 km. You traveled 0.5 km twice for a total of 1.0 km.

You walked for 6.0 min. Because you run three times as fast as you walk, it only takes 2.0 min to return. The total round-trip time is 8.0 min, or 480 s. Your average speed therefore is

$$(\text{distance})/(\text{time interval}) = (1000 \text{ m})/(480 \text{ s}) = 2.1 \text{ m/s. } \checkmark$$

# Section 2.4: Average speed and average velocity

## Example 2.3 Speed and velocity (cont.)

### ④ EVALUATE RESULT

If you start and end at the same place, your displacement is zero and so your average velocity is zero no matter what.

For (b): average speed is  $2.1\text{m/s} \sim 4.7\text{ mi/h}$

In the right ballpark for a person doing a combination of walking and running.

# Section 2.4: Average speed and average velocity

## Guided Problem 2.2 City driving

You need to drive to a store that is 1.0 mi west of your house on the same street on which you live. There are 5 traffic lights between your house and the store, and on your trip you hit all five of them just as they change to red. While you are moving, your average speed is 20 mi/h, but you have to wait 1 min at each light.

(a) How long does it take you to reach the store? (b) What is your average velocity for the trip? (c) What is your average speed?

# Section 2.4: Average speed and average velocity

## Guided Problem 2.2 City driving (cont.)

### 1 GETTING STARTED

1. Draw a diagram that helps you visualize all the driving and stopped segments and your speed in each segment. Does it matter where the traffic lights are located?
2. You start and stop, speed up and slow down. What does the average speed signify in this case, and how is it related to your displacement in each segment?



# Section 2.4: Average speed and average velocity

## Guided Problem 2.2 City driving (cont.)

### ② DEVISE PLAN

3. During how long a time interval are you moving? During how long a time interval are you stopped at the lights?
4. What is your displacement for the trip?
5. How can you apply the answers to questions 3 and 4 to obtain your average velocity?
6. What is the distance traveled, and how is it related to your average speed?
7. How are average velocity and average speed related in this case?

# Section 2.4: Average speed and average velocity

## Guided Problem 2.2 City driving (cont.)

### ③ EXECUTE PLAN

Total time is 3 min traveling (1 mi at 20mi/hr = 1/20 hr = 3 min) plus 5 min waiting. Total distance is 1 mi.

$$\text{average speed} = (1 \text{ mi}) / (8 \text{ min}) = 7.5 \text{ mi/h}$$

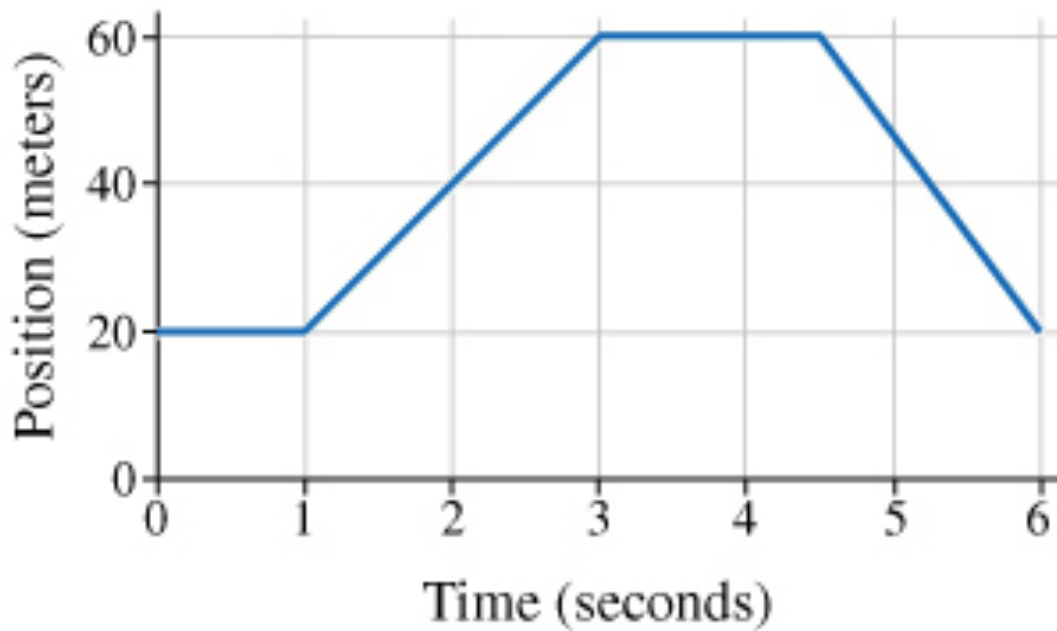
! Neglected speeding up/slowing down times.  
reasonable?

### ④ EVALUATE RESULT

8. Are your answers plausible, and is your result within the range of your expectations?

# Average velocity

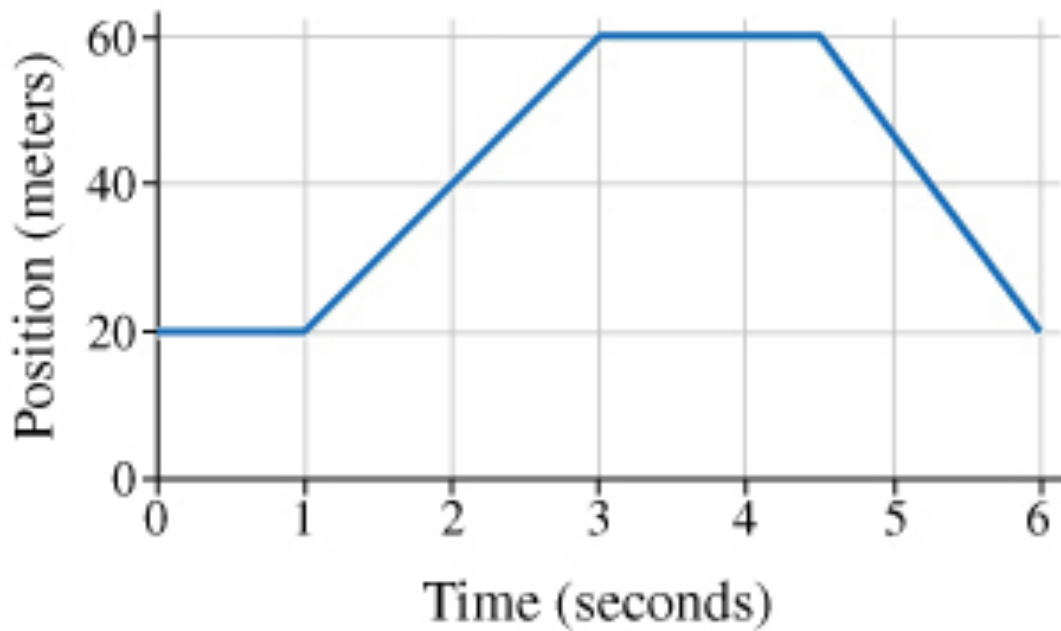
## Average velocity from a graph



- avg velocity over  $[0,1\text{s}]$ ?
- over  $[1,3\text{s}]$ ?
- over  $[0,3\text{s}]$ ?
- over  $[3,6\text{s}]$ ?
- whole graph?

# Average velocity

## Average velocity from a graph



- avg velocity over [0,1s]?  
**0 m/s**
- over [1,3s]?  
 **$40\text{m}/2\text{s} = 20\text{m/s}$**
- over [0,3s]?  
 **$40\text{m}/3\text{s} = 13.3\text{m/s}$**
- over [3,6s]?  
 **$-40\text{m}/3\text{s} = -13.3\text{m/s}$**
- whole graph?  
**0 m/s**

# Choice of origin

## What depends on choice of origin?

- Average velocity
- Position
- Displacement
- Speed
- Instantaneous velocity

# Choice of origin

## What depends on choice of origin?

- Average velocity
- **Position**
- Displacement
- Speed
- Instantaneous velocity

The rest only depend on differences in position

# Chapter 2: Motion in One Dimension

## Quantitative Tools

# PackBack

**How would you demonstrate simple harmonic motion as an example to someone who doesn't have a background in physics?**

Basically how would you develop a model of it in layman's terms?

- Originally asked by a PH 105 student from The University of Alabama

10:03 AM, 8/6/2017 ⚙ Options ▾

41 Curiosity Points



**simple harmonic motion is the repetition of a back-and-forth movement passing through a central position so that the distance and time on each side of the position is the same**

Simply put, simple harmonic motion is the repetition of a back-and-forth movement passing through a central position so that the distance and time on each side of the position is the same.

To develop a simple model, we can think about a metronome, an instrument that keeps time in music. The metronome clicks each time it returns to the central position, and both sides of the central position are the same distance and take the same time to travel. Thus, musicians can use this to keep a steady beat in a piece of music.

I have attached a link to a SparkNotes article on the subject, which I think is a good source to use when looking for a simplified explanation of a topic.

 **Source:**

<http://www.sparknotes.com/physics/oscillations/oscillationsandsimpleharmonicmotion/section2.rhtml>

# RQ1

In the study of physics, what distinguishes a scalar from a vector?

- A scalar is a dimensionless number, while vectors are numbers that have dimensions.
- Scalars have both a magnitude and a direction, but vectors have only a magnitude.
- A scalar is specified with a single number, but a vector is specified using both a magnitude and a direction.
- Nothing—the terms "vector" and "scalar" are different names for the same thing.
- A scalar must always be positive, but vectors can be positive, negative, or zero.

# RQ1

## Prelecture Concept Question 2.02

---

### Part A

Which of the following is a vector?

- Speed
- Mass
- Displacement
- Temperature
- Distance

# RQ1

## Problem 2.6

An object's initial position is  $x_i = +6.6$  m and its final position is  $x_f = +0.18$  m .

### Part A

What is the  $x$  component of its displacement?

Express your answer with the appropriate units.

□μÅ↶↷↻⌨?

$\Delta x =$

Submit[My Answers](#)[Give Up](#)

$$\Delta x = x_f - x_i = 0.18\text{m} - 6.6\text{m} = -6.4\text{m}$$

# RQ1

## Problem 2.8

A 2000-m race is held on a 400-m oval track.

---

### Part A

From start to finish, what is the displacement of the winner?

0m. Didn't go anywhere!

# RQ2

## Problem 2.21

After a rocket is launched at  $t = 0$ , its position is given by  $x(t) = qt^3$ , where  $q$  is some positive constant.

---

### Part A

What is the rocket's displacement during the interval from  $t = T$  to  $t = 3T$ ?

Express your answer in terms of  $q$  and  $T$ .

$$\Delta x = x_f - x_i = x(3T) - x(T) = q(3T)^3 - qT^3 = 26qT^3$$

## Problem 2.26

---

# RQ2

### Part A

Can two cars traveling in opposite directions on a highway have the same speed?

Yes

No

Submit

[My Answers](#) Give Up

**Correct**

---

### Part B

Can they have the same velocity?

Yes

No

Submit

[My Answers](#) Give Up

**Correct**

## Problem 2.50

You drive due east at  $30 \text{ km/h}$  for  $2.0 \text{ h}$  and then stop.

### Part A

Is your speed during the trip a scalar or a vector?

- Speed is a scalar.  
 Speed is a vector.

Submit

[My Answers](#) Give Up

Correct

### Part B

What is your speed during the trip?

Express your answer to two significant figures and include the appropriate units.

$v = 30 \text{ km/h}$

Submit

[My Answers](#) Give Up

Correct

### Part C

How far have you gone?

Express your answer to two significant figures and include the appropriate units.

$d = 60 \text{ km}$

Submit

[My Answers](#) Give Up



# RQ2

## Part D

Is distance a scalar or a vector?

- Distance is a vector.
- Distance is a scalar.

Submit

[My Answers](#) Give Up

Correct

## Part E

Select the correct expression for your position after you stop (in the form "N units of distance in the Q direction")

- 60 km in the west direction.
- 30 km in the east direction.
- 30 km in the wet direction.
- 60 km in the east direction.

Submit

[My Answers](#) Give Up

Correct

## Part F

Write a vector expression for your velocity during the trip, assuming east is the  $+x$  direction.

Express your answer using two significant figures.

$$\vec{v} = 30 \text{ km/h} \cdot \hat{i}$$


Submit

[My Answers](#) Give Up

## Section 2.5: Scalars and vectors

- **Scalars:** just numbers. Can be + or -, may have units.
  - Examples: Temperature, volume
  - Scalars follow ordinary arithmetic
- **Vectors:** Physical quantities that are completely specified by a magnitude and a direction in space.
  - vectors are things that need arrows to be properly represented
  - they have their own arithmetic

# Checkpoint 2.11

 **2.11** Are the following quantities vectors or scalars:

- (i) the price of a movie ticket,
- (ii) the average velocity of a ball launched vertically upward,
- (iii) the position of the corner of a rectangle
- (iv) the length of a side of that rectangle?

# Checkpoint 2.11

 **2.11** Are the following quantities vectors or scalars:

(i) the price of a movie ticket,

scalar – just a number (with units)

(ii) the average velocity of a ball launched vertically upward,

vector – needs direction

(iii) the position of the corner of a rectangle

vector – needs a direction (which corner?)

(iv) the length of a side of that rectangle?

scalar – just a number (with units)

## Section 2.5: Scalars and vectors

- For now we will only study one-dimensional (1-D) motion.
  - only move forward or backward in 1-D.
  - direction indicated by sign (+ or -)
- To distinguish vector from scalars: notation

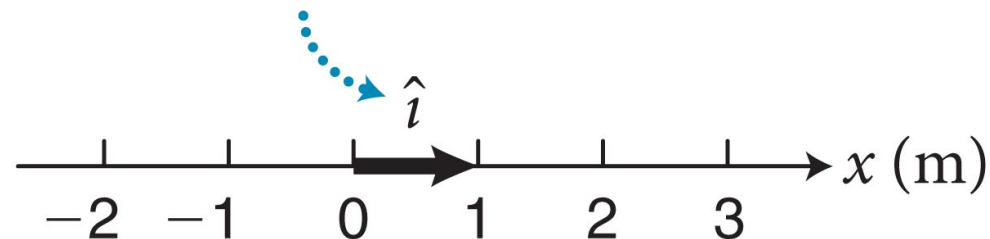
$\vec{a}$  vector

$b$  scalar

## Section 2.5: Scalars and vectors

- The **magnitude of a vector** is how long it is
  - magnitude is always positive.
  - magnitude is denoted by  $|\vec{\mathbf{b}}|$  or  $\mathbf{b}$
- To specify mathematically, we introduce **unit vectors**:
  - a unit vector pointing in the  $+x$  direction is denoted by  $\hat{\mathbf{i}}$
  - unit vectors have a magnitude of 1.  $|\hat{\mathbf{i}}| \equiv 1$

(a) Unit vector  $\hat{\mathbf{i}}$  defines axis.

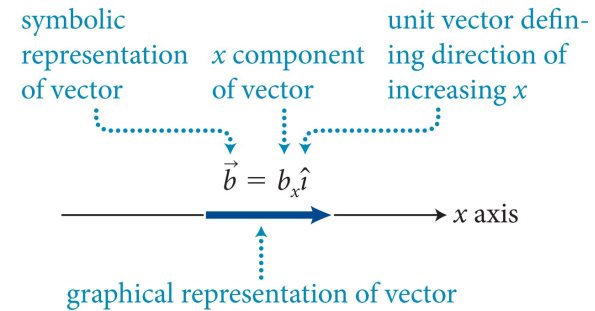
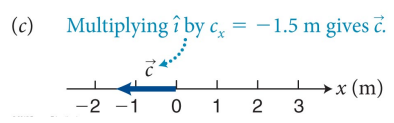
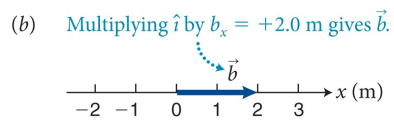
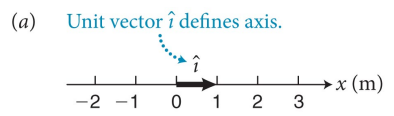


# Section 2.5: Scalars and vectors

- Any vector along the  $x$  axis can be written in unit vector notation.

$$\vec{b} = b_x \hat{i}$$

- $b_x$  is a scalar called the  $x$  **component** of the vector
- $b_x$  is negative if the vector points in the  $-x$  direction.
- this notation says “move  $b_x$  units along  $x$ ”
- in 1-D, the magnitude of  $\vec{b}$  is the absolute value of  $b_x$ .



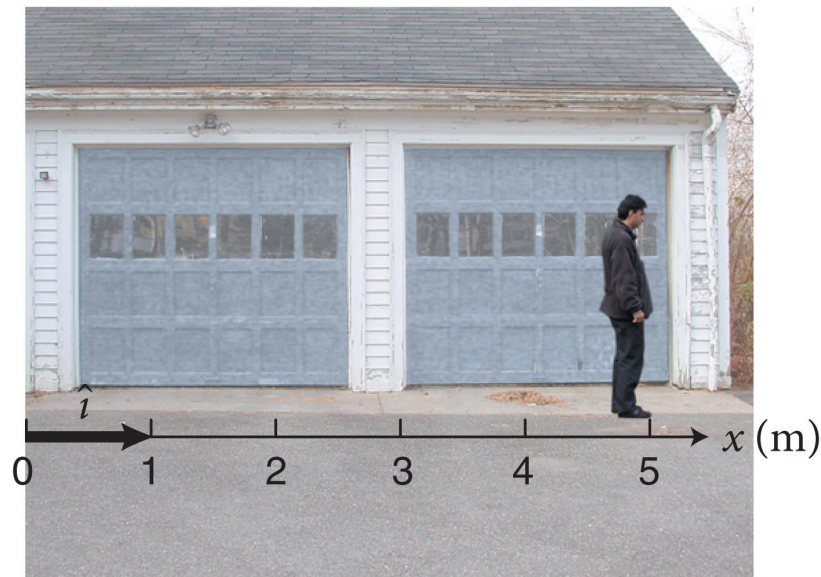
- magnitude of vector:  $b = |b_x|$
- If  $b_x > 0$ :  $\vec{b}$  is in same direction as  $\hat{i}$
- If  $b_x < 0$ :  $\vec{b}$  is in opposite direction to  $\hat{i}$

# Checkpoint 2.12



**2.12** Consider the axis and unit vector below.

- Define a vector pointing from the origin to tip of the man's feet
- Write it in terms of an  $x$  component and unit vector
- What does the vector represent?





# Checkpoint 2.12



**2.12** Consider the axis and unit vector below.

- Define a vector pointing from the origin to tip of the man's feet
- Write it in terms of an x component and unit vector

$$\vec{r} = (5\text{m}) \hat{i}$$

- What does the vector represent? **His position!**

# Section 2.6: Position and displacement vectors

## Section Goals

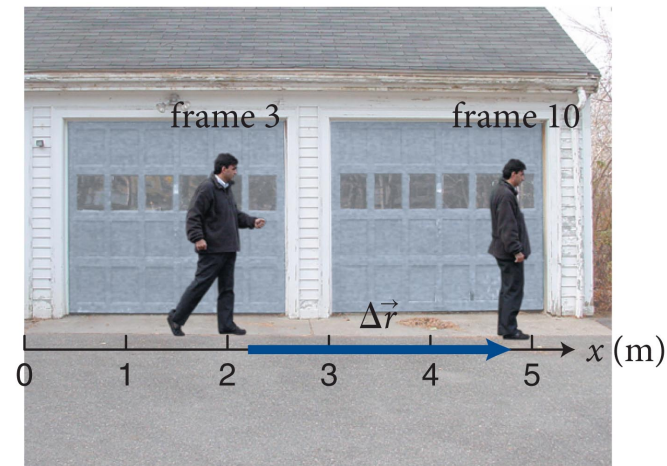
You will learn to

- Use “delta” notation to represent the change in a physical quantity.
- Correlate the distance traveled and displacement for an object moving in one dimension.
- Represent the distance traveled and displacement of an object mathematically.


# Section 2.6: Position and displacement vectors

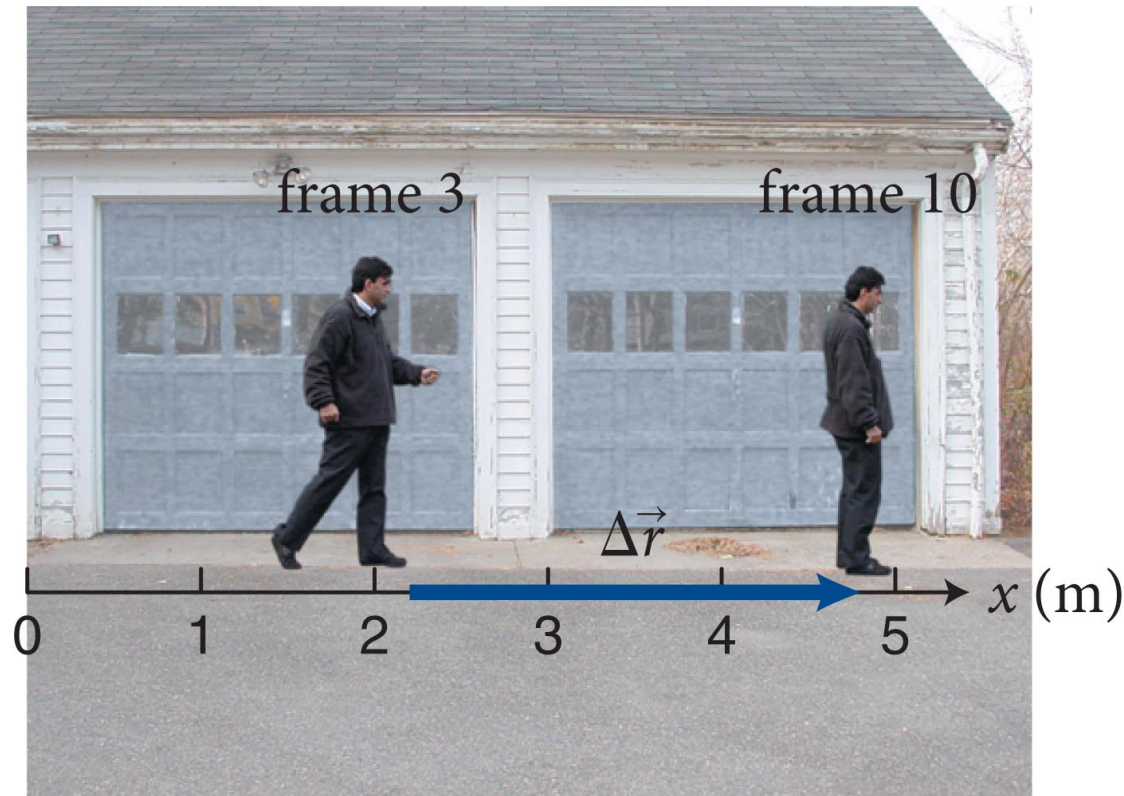
- **Displacement** can be represented graphically by an arrow that points from an object's initial to final position.
  - We denote displacement by  $\Delta\vec{r}$
  - $\Delta$  means 'change in'
  - The  $x$  component of displacement is  $\Delta r_x \equiv \Delta x$
  - $\Delta x = x_f - x_i$ , where  $x_i$  and  $x_f$  are initial and final positions.
  - The **distance** between initial and final points is

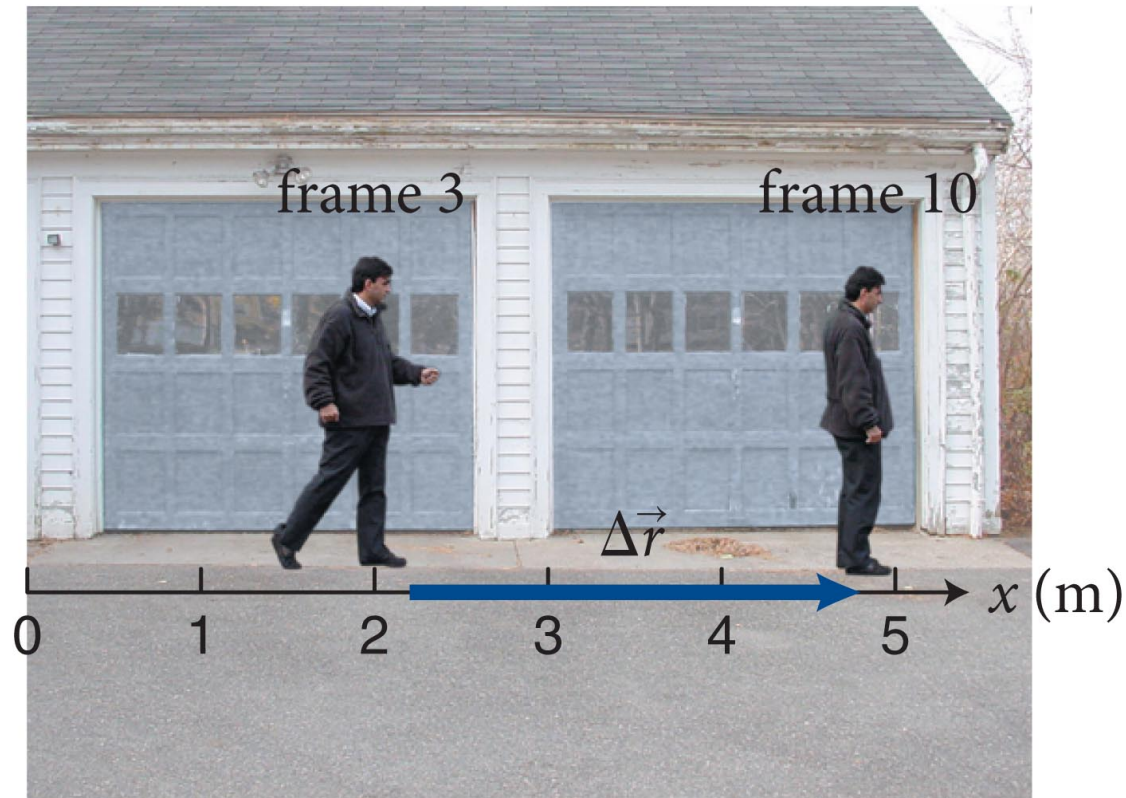
$$d \equiv |x_f - x_i| \text{ (1D only)}$$



# Checkpoint 2.13

 **2.13** (a) What is the  $x$  component of the displacement in the figure below? (b) Write the displacement in terms of its  $x$  component and the unit vector.





(a)  $x$  component is 3m

(b) Displacement is along  $+x$ , so  $+3\hat{i}$

# Section 2.6: Position and displacement vectors

- Since displacement is a vector,

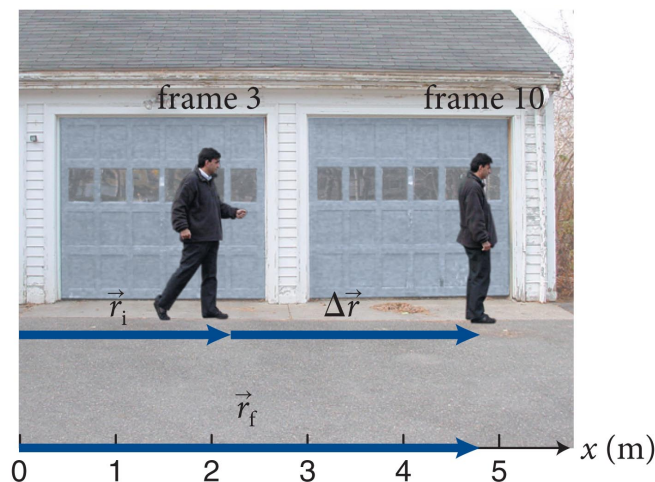
$$\Delta x \hat{i} = (x_f - x_i) \hat{i} = x_f \hat{i} - x_i \hat{i}, \text{ where } \Delta x \hat{i} = \Delta r_x \hat{i} = \Delta \vec{r}.$$

- Using vector notation displacement can be written as

$$\Delta \vec{r} = \vec{r}_f - \vec{r}_i$$

- Rearranging we get the final position vector:

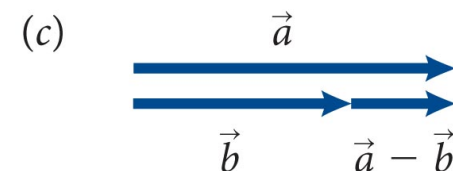
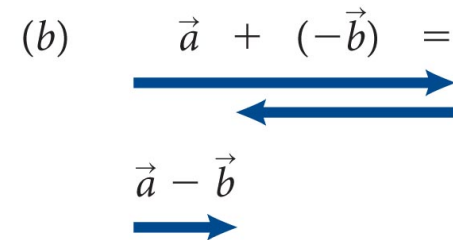
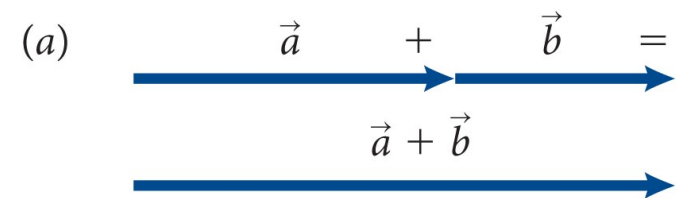
$$\vec{r}_f = \vec{r}_i + \Delta \vec{r}$$



# Section 2.6: Position and displacement vectors

## Graphical prescription of adding and subtracting vectors

- To add vectors, connect them head to tail.
- To subtract a vector from another vector, reverse the direction of the vector being subtracted and connect head to tail.



# Section 2.7: Velocity as a vector

## Section Goals

You will learn to

- Define average speed and average velocity mathematically.
- Correlate the graphical and mathematical representations for average speed and average velocity.



## Section 2.7: Velocity as a vector

- The  $x$  component of average velocity is

$$v_{x, \text{av}} \equiv \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{t_f - t_i}$$

- Since  $\Delta x \hat{i} = \Delta r_x \hat{i} = \Delta \vec{r}$ , average velocity can also be written using vector notation as

$$\vec{v}_{\text{av}} \equiv v_{x, \text{av}} \hat{i} = \frac{\Delta \vec{r}}{\Delta t}$$

- subtract final & initial position vectors

# Section 2.8: Motion at constant velocity

## Section Goals

You will learn to

- Represent motion at constant velocity using motion graphs and mathematics.
- Construct position-versus-time and velocity-versus-time graphs for motion at constant velocity.
- Adapt the mathematical definition of average velocity into an equation that allows the future motion of a moving object to be predicted from its present motion.

# Section 2.8: Motion at constant velocity

- If an object moves at a constant velocity, then the  $x(t)$  graph is a straight line.

- b/c slope is velocity ...

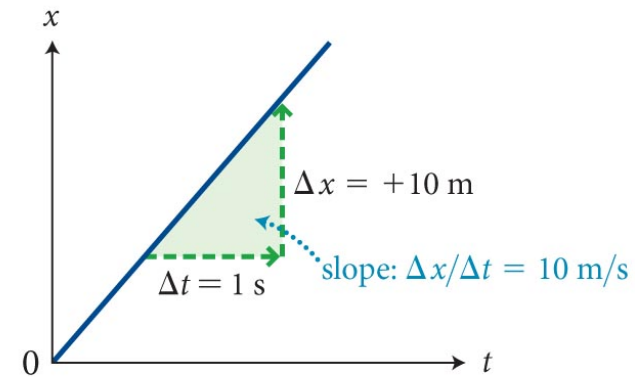
$$\Delta x = v_x \Delta t \quad (\text{constant velocity})$$

- equivalently:

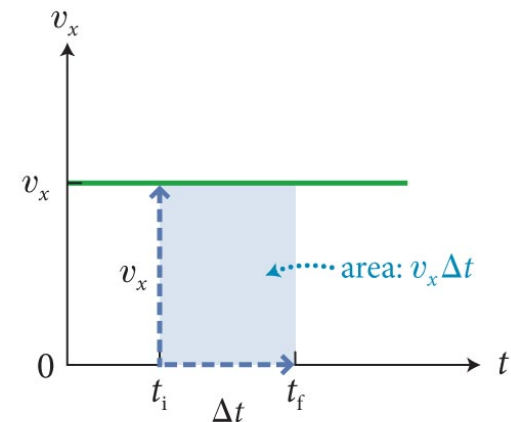
$$x_f = x_i + v_x \Delta t \quad (\text{constant velocity})$$

- $\Delta x$  during any interval is the **area under the  $v_x(t)$  curve** over that interval.

(a) Position versus time: straight line with nonzero slope



(b) Velocity versus time: horizontal line



## Section 2.8: Motion at constant velocity

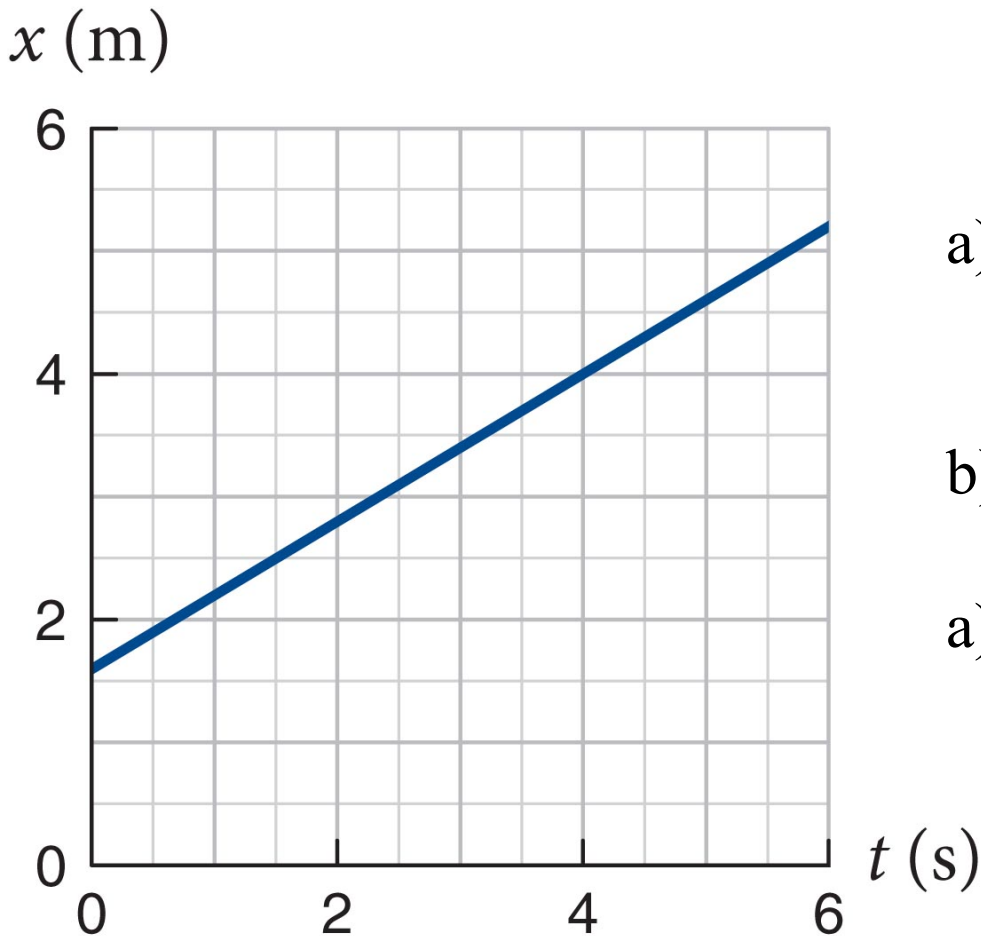
### Example 2.10 Motion at constant velocity

I'll show you a position-versus-time graph for part of the motion of an object moving at constant velocity.

- (a) What is the  $x$  component of the object's velocity?
- (b) Write an expression for  $x(t)$ , the  $x$  coordinate of the position of the object at an arbitrary time  $t$ .
- (c) What is the object's position at  $t = 25$  s?

# Section 2.8: Motion at constant velocity

## Example 2.10 Motion at constant velocity



- a) What is the  $x$  component of the object's velocity?
- b) Write an expression for  $x(t)$
- a) What is the  $x$  coordinate of the object at  $t = 25$  s?

# Section 2.8: Motion at constant velocity

## Example 2.10 Motion at constant velocity (cont.)

**1** GETTING STARTED – Clearly  $x(t)$  is a straight line.

I need a the slope to get velocity.

I need the y-intercept to write  $x(t)$ .

## Section 2.8: Motion at constant velocity

### Example 2.10 Motion at constant velocity (cont.)

② DEVISE PLAN The slope is the  $x$  component of the object's velocity,  $\Delta x/\Delta t$ . Since  $x(t)$  is a line, this is the same everywhere.

I can use  $x_f = x_i + v_x \Delta t$  to write  $x(t)$

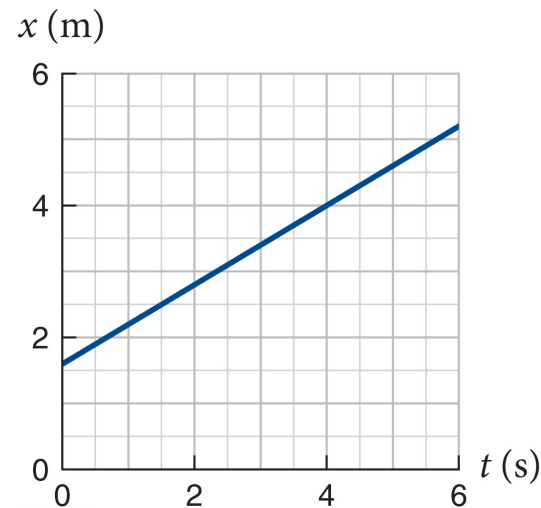
I can use the  $x(t)$  equation to find position at  $t = 25$  s, presuming the motion does not change

# Section 2.8: Motion at constant velocity

## Example 2.10 Motion at constant velocity (cont.)

**3** EXECUTE PLAN (*a*) read the graph. We can use positions at 0 and 6s

$$v_x = \frac{x_f - x_i}{t_f - t_i} = \frac{+5.2 \text{ m} - (+1.6 \text{ m})}{6.0 \text{ s} - 0} = \frac{+3.6 \text{ m}}{6.0 \text{ s}}$$
$$= +0.60 \text{ m/s. } \checkmark$$





## Section 2.8: Motion at constant velocity

### ③ EXECUTE PLAN

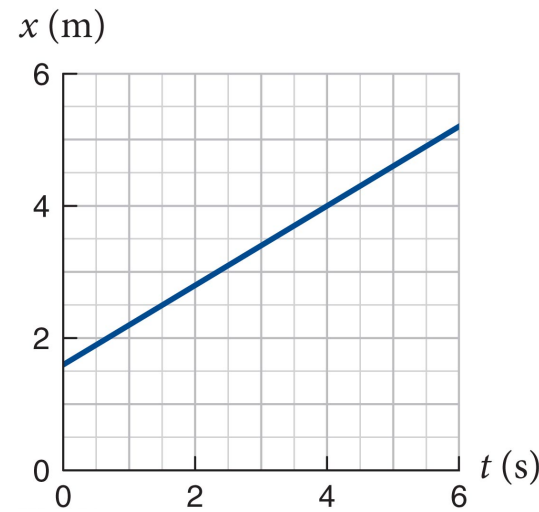
(b) Take the object's position at  $t = 0$  as the initial position.

With  $t_i = 0$ ,  $x_i = +1.6$  m, and  $v_x = +0.60$  m/s:

$$\begin{aligned}x_f &= x_i + v_x \Delta t = +1.6 \text{ m} + (+0.60 \text{ m/s})(t_f - 0) \\ &= +1.6 \text{ m} + (+0.60 \text{ m/s})t_f.\end{aligned}$$

$t_f$  is arbitrary, so this works for any time:

$$x(t) = +1.6 \text{ m} + (+0.60 \text{ m/s})t. \quad \checkmark$$



## Section 2.8: Motion at constant velocity

### Example 2.10 Motion at constant velocity (cont.)

③ EXECUTE PLAN (*c*) at  $t = 25$  s

$$\begin{aligned}x(t = 25 \text{ s}) &= +1.6 \text{ m} + (+0.60 \text{ m/s})(25 \text{ s}) \\ &= +1.6 \text{ m} + 15 \text{ m} = +17 \text{ m.} \quad \checkmark\end{aligned}$$

- *17 b/c significant digits*
- *assumes particle is still moving at constant velocity then!*

# Section 2.8: Motion at constant velocity

## Example 2.10 Motion at constant velocity (cont.)

### ④ EVALUATE RESULT

- The  $x$  component of the object's velocity is positive, as I would expect from the positive slope of the  $x(t)$  curve.
- I can double-check the value (0.60 m/s) by reading off the increase in  $x$  over a 1-s interval in the graph.
- I see that the object starts near  $x = 2$  m and the  $x$  coordinate increases by 3 m every 5 s. In 25 s, the object displacement should therefore be  $5(3 \text{ m}) = 15 \text{ m}$ , and so the object's final position should be near  $(2 \text{ m}) + (15 \text{ m}) = 17 \text{ m}$ , which is what I got.

# Section 2.9: Instantaneous velocity

## Section Goals

You will learn to

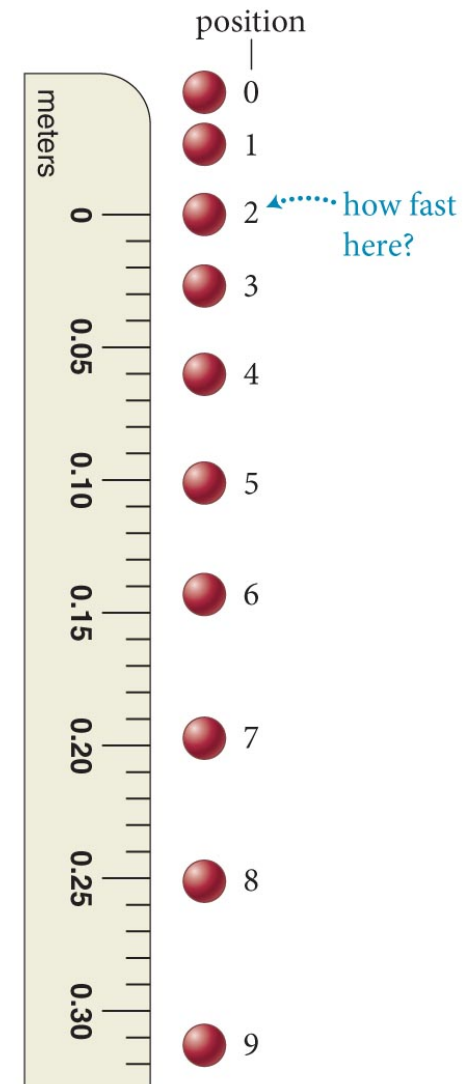
- Represent motion with changing velocity using motion graphs and mathematics.
- Relate the concept of a tangent line with instantaneous velocity on a position-versus-time graph.
- Generalize mathematically the definition of the average velocity of a moving object to its instantaneous velocity by the use of a limiting process.

## Section 2.9: Instantaneous velocity

The figure shows the successive position of a ball falling at 0.0300-s intervals.

- We want to be able to find the velocity as the ball passes any position in its motion, that is the instantaneous velocity  $\vec{v}$ .
- Let us start by first finding the average velocity between positions 2 and 9:

$$v_{x,29} = \frac{x_9 - x_2}{t_9 - t_2}$$



## Section 2.9: Instantaneous velocity

By reducing the time interval  $\Delta t$  you can get closer to the velocity at point 2.

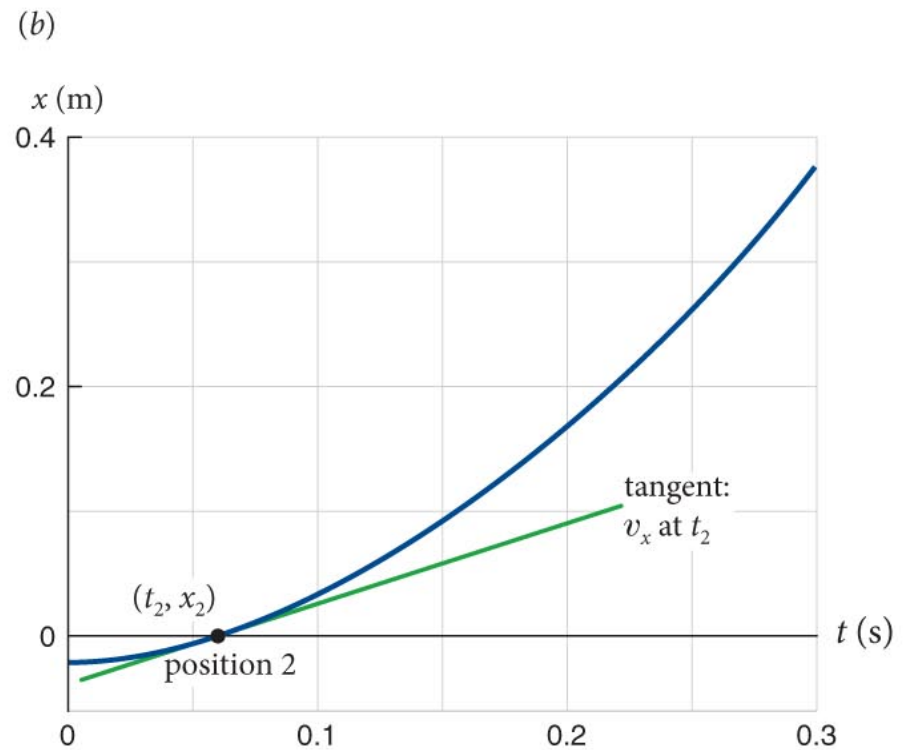
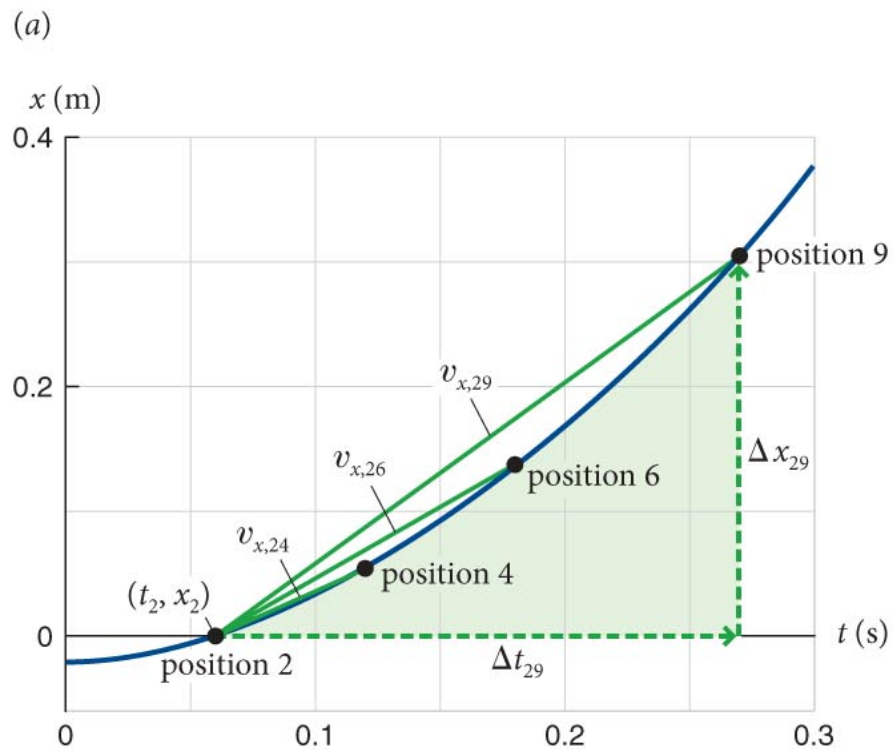
- Letting  $\Delta t$  approach zero, we obtain the  $x$  component of the velocity at instant  $t$ :

$$v_x \equiv \lim_{\Delta t \rightarrow 0} \frac{\Delta x}{\Delta t}$$

- The right side of the equation is the definition for the derivative, so

$$v_x \equiv \frac{dx}{dt}$$

# Section 2.9: Instantaneous velocity



## Section 2.9

### Quick check

The slope of the curve in the position vs. time graph for a particle's motion gives

1. the particle's speed.
2. the particle's acceleration.
3. the particle's average velocity.
4. the particle's instantaneous velocity.



# Section 2.9

## Question 4

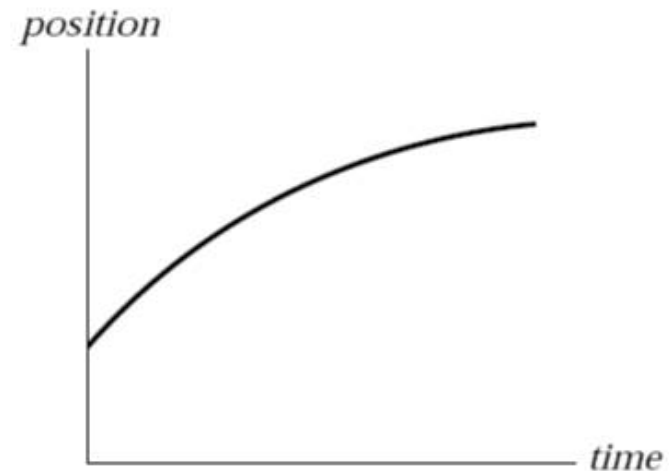
The slope of the curve in the position vs. time graph for a particle's motion gives

1. the particle's speed.
2. the particle's acceleration.
3. the particle's average velocity.
- ✓ 4. the particle's instantaneous velocity.
  - (noting the sign gives you direction)

## Section 2.9

### Question 5

A train car moves along a long straight track. The graph shows the position as a function of time for this train. The graph shows that the train

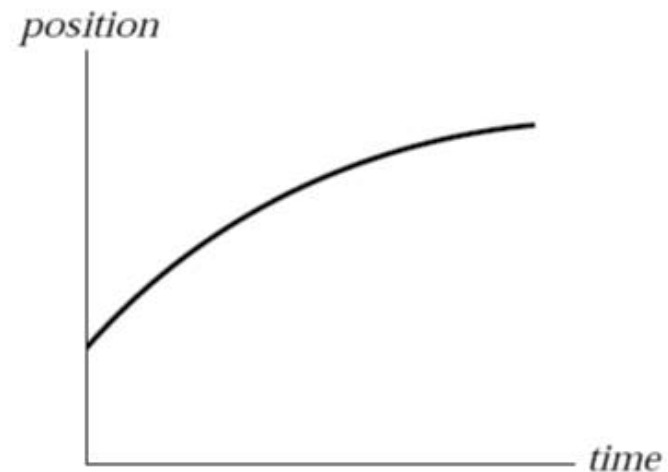


1. speeds up all the time.
2. slows down all the time.
3. speeds up part of the time and slows down part of the time.
4. moves at a constant velocity.

## Section 2.9

### Question 5

A train car moves along a long straight track. The graph shows the position as a function of time for this train. The graph shows that the train

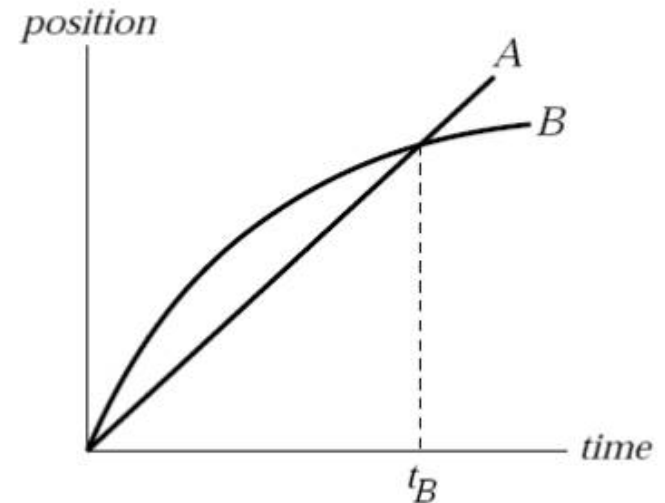


1. speeds up all the time.
- ✓ 2. slows down all the time.
3. speeds up part of the time and slows down part of the time.
4. moves at a constant velocity.

## Section 2.9

### Question 6

The graph shows position as a function of time for two trains running on parallel tracks. Which is true?

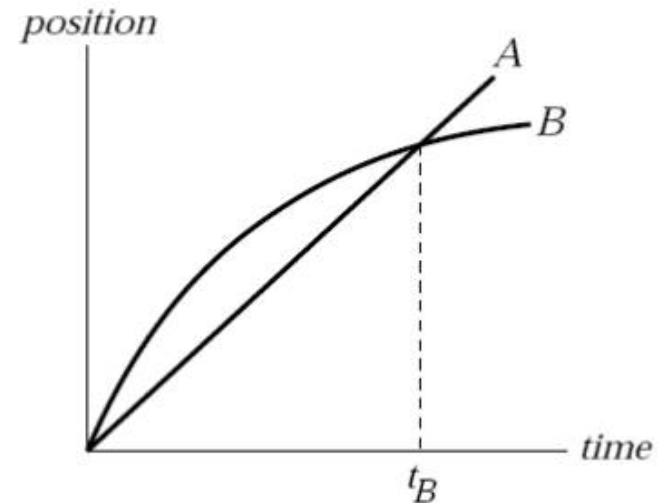


1. At time  $t_B$ , both trains have the same velocity.
2. Both trains speed up all the time.
3. Both trains have the same velocity at some time before  $t_B$ .
4. Somewhere on the graph, both trains have the same acceleration.

## Section 2.9

### Question 6

The graph shows position as a function of time for two trains running on parallel tracks. Which is true?



1. At time  $t_B$ , both trains have the same velocity.
2. Both trains speed up all the time.
- ✓ 3. Both trains have the same velocity at some time before  $t_B$ .
4. Somewhere on the graph, both trains have the same acceleration.

# Homework

*can work in groups outside class*

2.22

2.33

2.56

2.69

(more tomorrow)

# Chapter 2: Summary

## Concepts: Average speed and average velocity

- The **distance traveled** is the accumulated distance covered by an object along the path of its motion, without regard to direction. The  $x$  component of an object's **displacement** is the change in its  $x$  coordinate.
- The **average speed** of an object is the distance it travels divided by the time interval it takes to travel that distance.

# Chapter 2: Summary

## Concepts: Average speed and average velocity

- The  $x$  component of an object's **average velocity** is the  $x$  component of its displacement divided by the time interval taken to travel that displacement. The magnitude of the average velocity is not necessarily equal to the average speed.



# Chapter 2: Summary

## Quantitative Tools: Average speed and average velocity

- The **distance**  $d$  between two points  $x_1$  and  $x_2$  is

$$d = |x_1 - x_2|$$

- The  $x$  component of the **displacement**  $\Delta x$  of an object that moves from a point  $x_i$  to a point  $x_f$  is

$$\Delta x = x_f - x_i$$

- The  $x$  component of the object's **average velocity** is

$$v_{x, \text{av}} = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{t_f - t_i}$$

# Chapter 2: Summary

## Concepts: Vectors and scalars

- A **scalar** is a physical quantity that is completely specified by a number and a unit of measure.
- A **vector** is a physical quantity that is completely specified by a number, a unit of measure, and a direction.
- The number and unit of measure together are called the **magnitude** of the vector.
- A **unit vector** has magnitude one and no units.

# Chapter 2: Summary

## Concepts: Position, displacement, and velocity as vectors

- The **position vector**, or **position**, of a point is drawn from the origin of a coordinate system to the point.
- The **displacement vector** of an object is the change in its position vector. It is the vector that points from the tip of the original position vector to the tip of the final position vector.

# Chapter 2: Summary

## Quantitative Tools: Position, displacement, and velocity as vectors

- The position  $\vec{r}$  of a point that has  $x$  coordinate  $x$  is

$$\vec{r} = x\hat{i}$$

- The displacement vector  $\Delta\vec{r}$  of an object moving along the  $x$  axis is

$$\Delta\vec{r} = \vec{r}_f - \vec{r}_i = (x_f - x_i)\hat{i}$$

- The average velocity of an object moving along the  $x$  axis is

$$\vec{v}_{\text{av}} = \frac{\Delta\vec{r}}{\Delta t} = \frac{x_f - x_i}{t_f - t_i}\hat{i}$$

# Chapter 2: Summary

## Concepts: Additional properties of velocity

- When the velocity of an object is constant, its **position-versus-time graph** is a straight line, **velocity versus-time graph** is a horizontal line.
- The **instantaneous velocity** of an object is its velocity at any particular instant.
- The slope of an object's  $x(t)$  curve at a given instant is numerically equal to the  $x$  component of the object's velocity at that instant.
- The area under the object's  $v_x(t)$  curve during a time interval is the  $x$  component of the object's displacement during that interval.

# Chapter 2: Summary

## Quantitative Tools: Additional properties of velocity

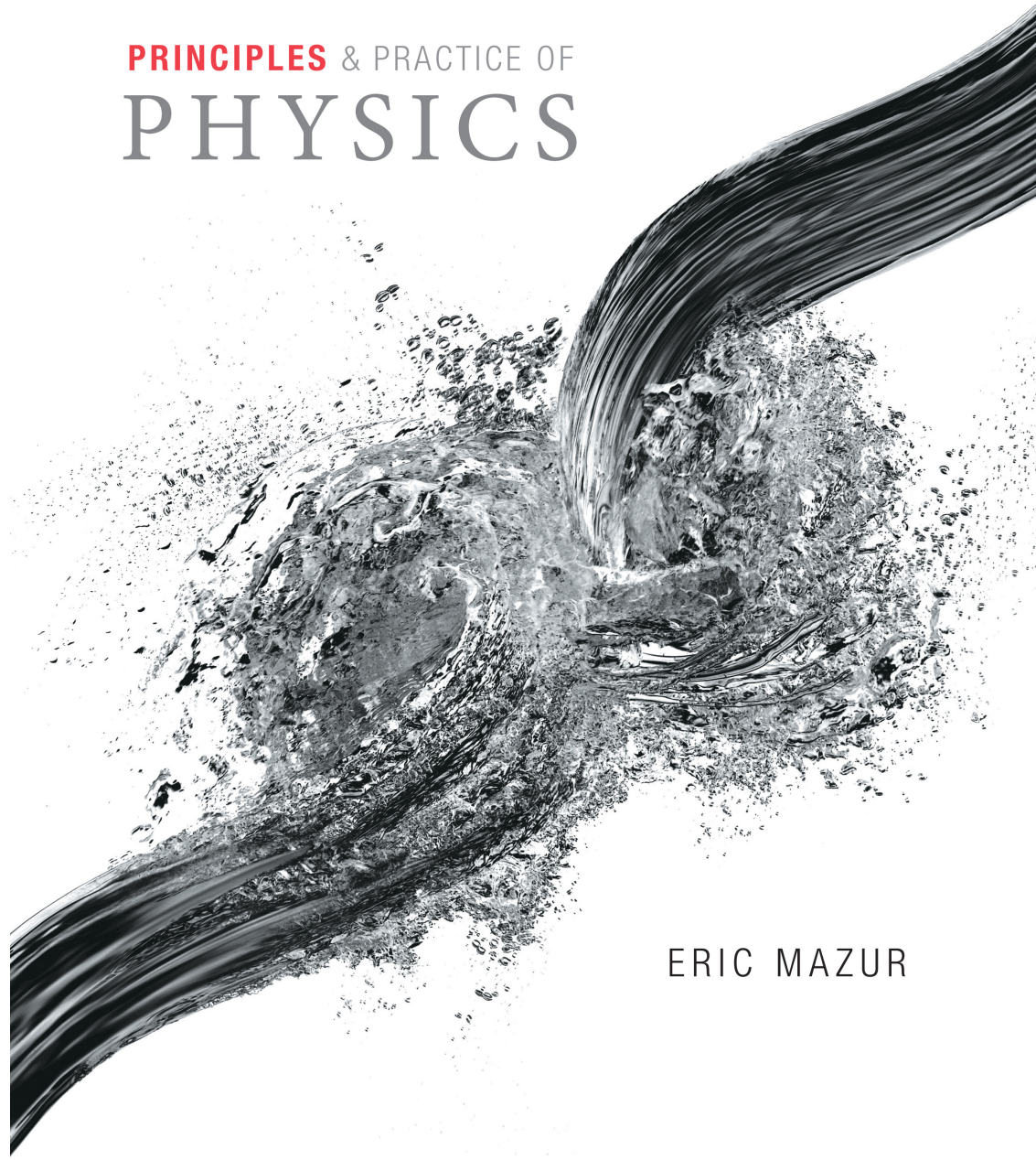
- The  $x$  component of an object's instantaneous velocity is the derivative of the object's  $x$  coordinate with respect to time:

$$v_x = \frac{dx}{dt}$$

PRINCIPLES & PRACTICE OF  
PHYSICS

Chapter 3  
Acceleration

ERIC MAZUR



# Chapter 3: Acceleration

## Concepts



# Section 3.1: Changes in velocity

## Section Goals

- Define acceleration from velocity.
- Identify if an object is accelerating from several different graphical representations of motion.
- Understand the vector relationships between velocity and acceleration.

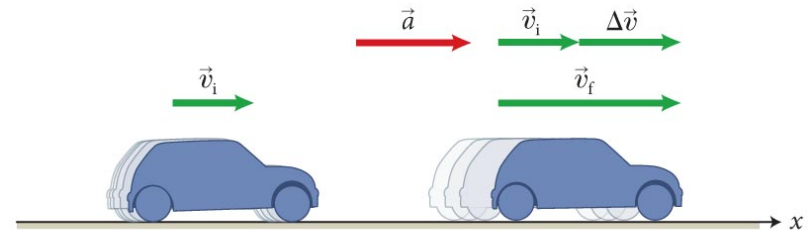
## Section 3.1: Changes in velocity

- If an object's velocity is changing, the object is accelerating.
- The  $x$  component of the average acceleration of an object is the change in the  $x$  component of the velocity divided by the time interval during which this change took place.
- The SI unit of acceleration is  $\text{m/s}^2$ .

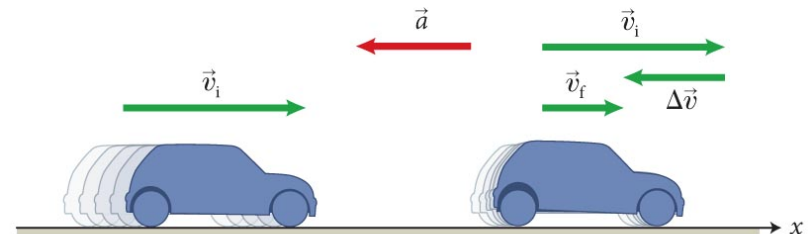
# Section 3.1: Changes in velocity

- Whenever an object's velocity vector  $\vec{v}$  and acceleration vector  $\vec{a}$  point in the same direction, the object speeds up.
- If  $\vec{v}$  and  $\vec{a}$  point in the opposite direction, the object slows down.

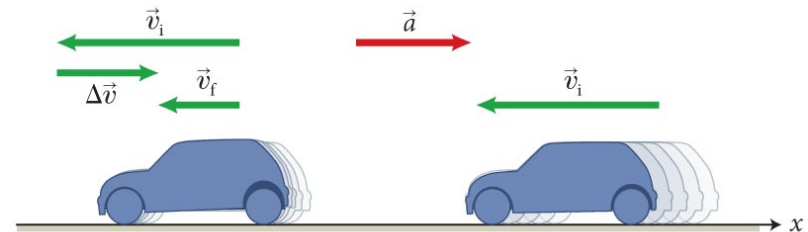
(a) Car speeds up in positive  $x$  direction



(b) Car slows down in positive  $x$  direction

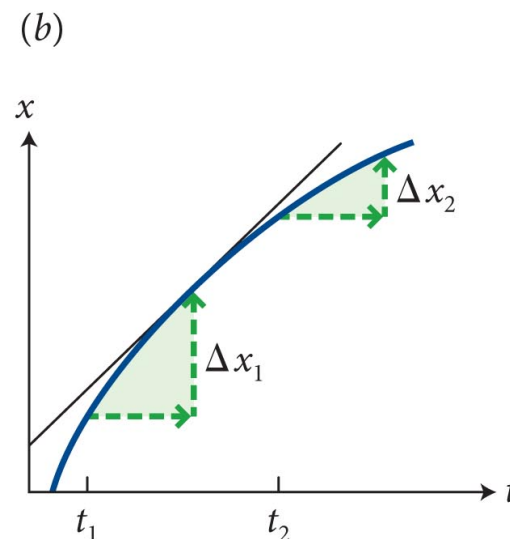
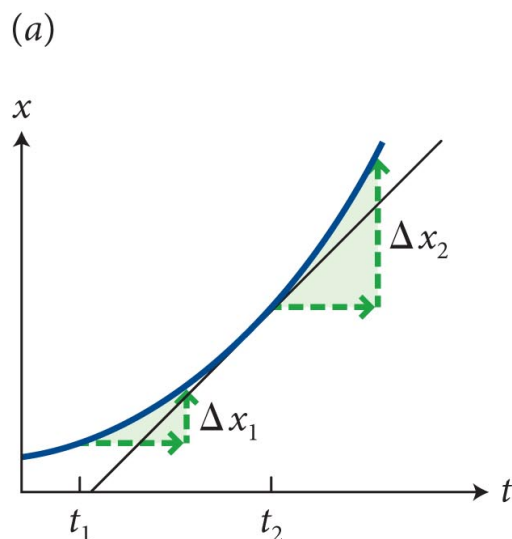


(c) Car slows down in negative  $x$  direction



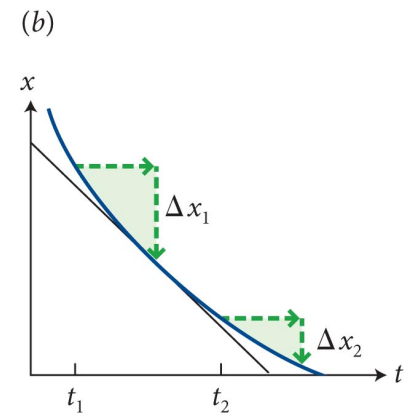
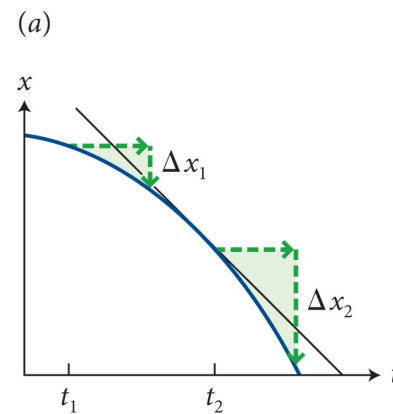
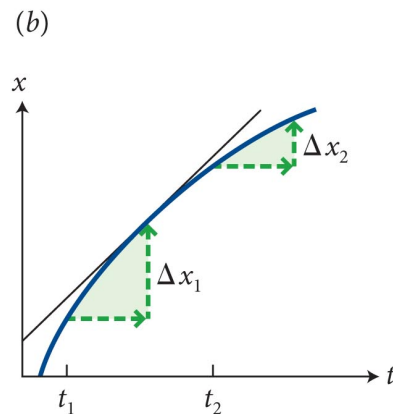
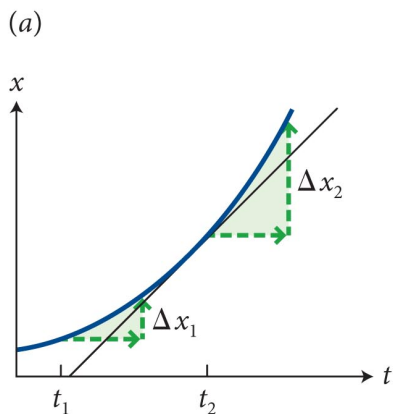
# Section 3.1: Changes in velocity

- For accelerating objects, the  $x(t)$  curve is a *not* a straight line.
- The figure shows the  $x(t)$  curve for two accelerating objects:
  - For each object, consider the displacements  $\Delta x_1$  and  $\Delta x_2$  during two equal time intervals ( $\Delta t$ ) at two different times.
  - If the displacement increases with time then the velocity is increasing (for example,  $\Delta x_2 > \Delta x_1$ ).
  - If the displacement decreases then velocity is decreasing.



# Section 3.1: Changes in velocity

- The curvature of the  $x(t)$  curve is a measure of the  $x$  component of acceleration ( $a_x$ ).
- An upward curvature corresponds to a positive  $a_x$ :
  - The curve lies above the tangent; **faster than linear**
- A downward curvature corresponds to a negative  $a_x$ :
  - The curve lies below the tangent; **slower than linear**



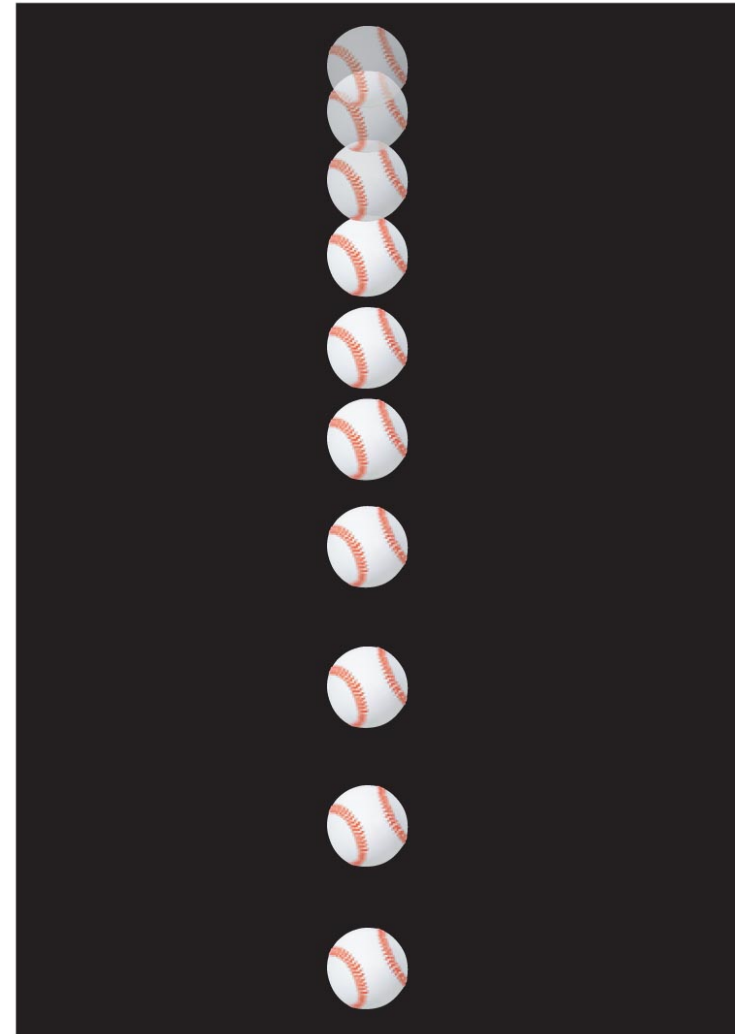
# Section 3.2: Acceleration due to gravity

## Section Goals

- Identify gravity as the cause of the vertical acceleration of an object moving near the surface of the Earth.
- Display the effects of gravity of an object using motion diagrams.
- Model the idealized case of vertical motion in the absence of other influences, such as air resistance, using the concept of free fall.

## Section 3.2: Acceleration due to gravity

- An object falling in a straight line toward the Earth's surface: **accelerated motion**.
- A falling ball recorded at equal time intervals of 0.05 s.
- The increasing displacements tells us that the speed increases as it falls: the ball accelerates.



## Checkpoint Question 3.5

- Does the acceleration of an object as it falls depend on the physical characteristics of the object?



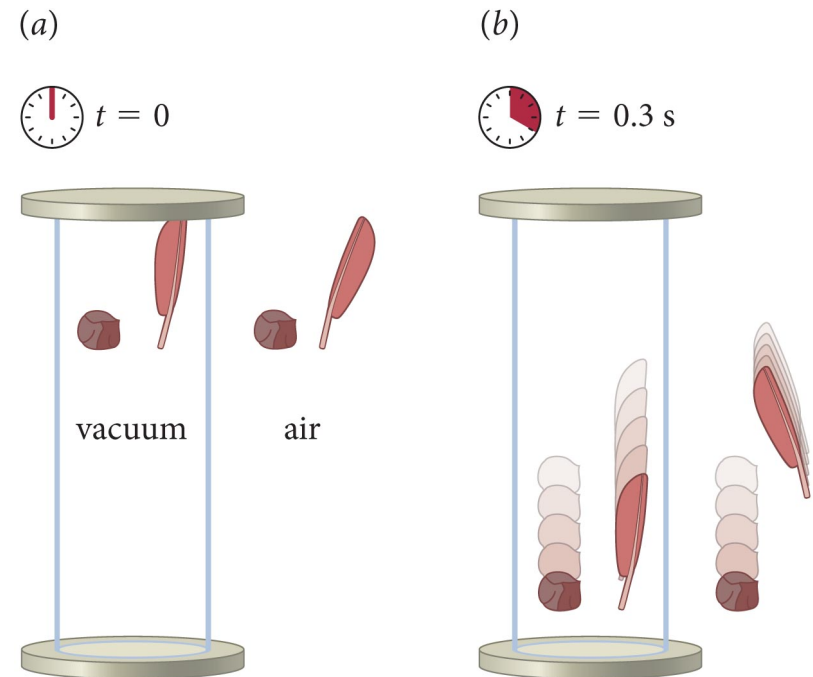
**3.5** Hold a book and a sheet of paper, cut to the same size as the book, side by side 1 m above the floor. Hold the paper parallel to the floor and the book with its covers parallel to the floor, and release them at the same instant. Which hits the floor first? Now put the paper on top of the book and repeat the experiment. What do you notice?

We need to separate *extrinsic* effects like air resistance from *intrinsic* effects like gravity. The latter is universal, the former depends on the details ...



# Section 3.2: Acceleration due to gravity

- What is the magnitude of acceleration due to gravity as an object falls?
  - feather vs stone
  - in a vacuum, no air resistance.
  - the acceleration **due to gravity** *does not* depend on the physical characteristics of object
    - The motion of an object under the influence of gravity only is called **free fall**.
    - (No air resistance either)
- what properties are most fundamental?
- which are circumstantial?

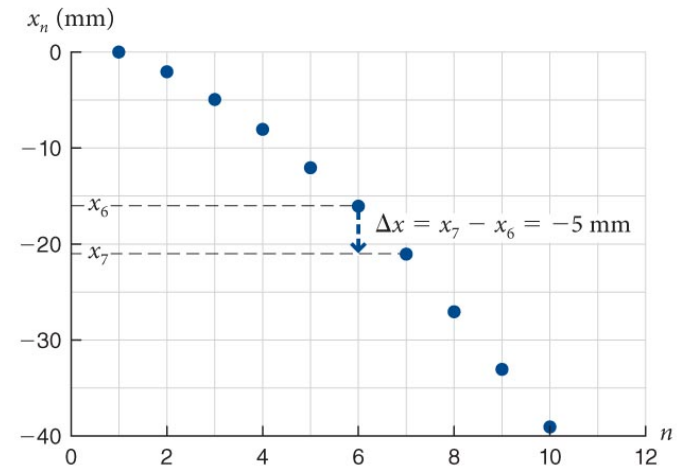


[https://youtu.be/frZ9dN\\_ATew](https://youtu.be/frZ9dN_ATew)

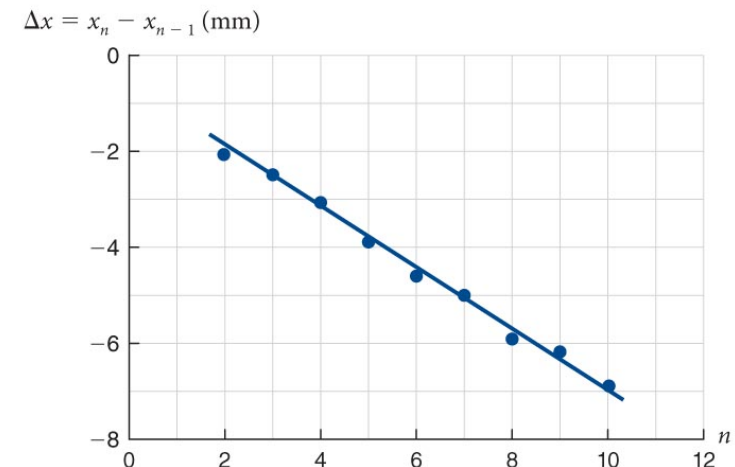
# Section 3.2: Acceleration due to gravity

- What is the magnitude of acceleration due to gravity as an object falls?
  - like last time, position vs time changes faster than linearly
  - *displacement* between adjacent times is linear in time
    - therefore, the velocity increases at a constant rate.

(a)



(b)



## Section 3.2: Acceleration due to gravity

- What is the magnitude of acceleration due to gravity as an object falls?
  - without air resistance, the magnitude of the acceleration of all falling objects is  $9.8 \text{ m/s}^2$ .
- Means? The amount of time it takes to fall from a certain height is the same for all falling objects.