University of Alabama
Department of Physics and Astronomy

PH 126 LeClair Fall 2011

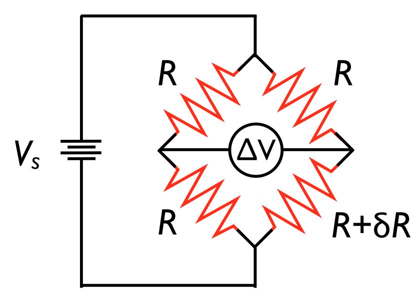
PH126: Exam 1

Instructions:

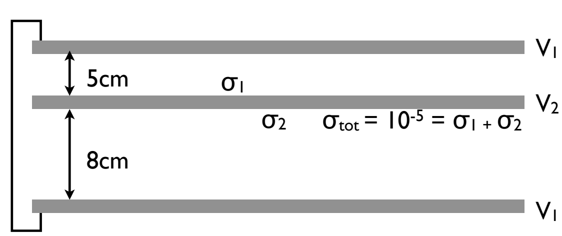
1. Answer four of the five questions below. All problems have equal weight.
2. You must show your work for full credit.

□ **1.** The circuit below is known as a *Wheatstone Bridge*, and it is a useful circuit for measuring small changes in resistance. Perhaps you can figure out why. Three of the four branches on our bridge have identical resistance R, but the fourth has a slightly different resistance, by an amount $\delta\mathsf{R}$ such that its total resistance is $\mathsf{R}+\delta\mathsf{R}$.

In terms of the source voltage V_s, base resistance R and change in resistance $\delta\mathsf{R}$, what is the reading on the voltmeter, ΔV? You may assume the voltmeter and voltage source are perfect (drawing no current and having no internal resistance, respectively).



□ **2.** Three conducting plates are placed parallel to one another as shown below. The outer plates are connected by a wire. The inner plate is isolated and carries a charge amounting to 10^{-5} C per square meter of plate. In what proportion must this charge divide itself into a surface charge σ_1 on one face of the inner plate and a surface charge σ_2 on the other side of the same plate?



□ **3.** Two graphite rods are of equal length. One is a cylinder of radius a. The other is conical, tapering linearly from a radius a at one end to radius b at the other. Show that the end-to-end electrical resistance of the conical rod is a/b times that of the cylindrical rod. *Hint: consider the rod to be made up of thin, disk-like slices, all in series.*

□ **4.** Three protons and three electrons are to be placed at the vertices of a regular octahedron of edge length a. We want to find the potential energy of the system, or the work required to assemble it starting with the particles infinitely far apart. There are essentially two different arrangements possible. What is the energy of each? Symbolic answer, please.

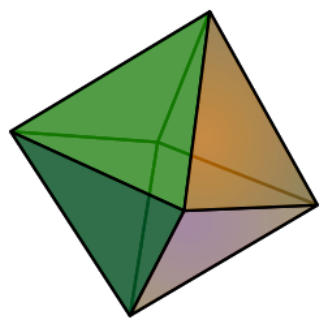


Figure 1: *An octahedron. It has eight faces and six vertices.*

□ **5.** Show that the expression $Q^2/2C$ is the energy stored in a spherical capacitor (two concentric hollow metal spheres) by integrating the energy density $u = \frac{1}{2}\epsilon_o E^2$ over the region between the spheres. Use the volume between two spheres or radius r and $r+dr$ as a volume element.

Constants:

$$\begin{aligned} k_e &\equiv 1/4\pi\epsilon_o = 8.98755\times10^{9}\,\mathrm{N\cdot m^2\cdot C^{-2}} \\ \epsilon_o &= 8.85\times10^{-12}\,\mathrm{C^2/N\cdot m^2} \\ \mu_0 &\equiv 4\pi\times10^{-7}\,\mathrm{T\cdot m/A} \\ c^2 &= 1/\mu_0\epsilon_0 \\ e &= 1.60218\times10^{-19}\,\mathrm{C} \end{aligned}$$

Basic Equations / Mechanics:

$$0 = ax^2 + bx^2 + c \Longrightarrow x = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$\vec{\mathbf{F}}_{\mathrm{centr}} = -\frac{mv^2}{r}\hat{\mathbf{r}}\ \text{Centripetal}$$

Electric Force & Field (static case):

$$\vec{\mathbf{F}}_{12} = k_e\frac{q_1q_2}{r_{12}^2}\hat{\mathbf{r}}_{12} = q_2\vec{\mathbf{E}}_1 \qquad \vec{\mathbf{r}}_{12}=\vec{\mathbf{r}}_1-\vec{\mathbf{r}}_2$$

$$\vec{\mathbf{E}}_1 = \vec{\mathbf{F}}_{12}/q_2 = k_e\frac{q_1}{r_{12}^2}\hat{\mathbf{r}}_{12}$$

$$\vec{\mathbf{E}} = k_e\sum_i\frac{q_i}{r_i^2}\hat{\mathbf{r}}_i \rightarrow k_e\int\frac{dq}{r^2}\hat{\mathbf{r}} = k_e\int_V\frac{\rho\hat{\mathbf{r}}}{r^2}\,d\tau$$

$$\rho\,d\tau \rightarrow \sigma\,da \rightarrow \lambda\,dl$$

$$\vec{\nabla}\cdot\vec{\mathbf{E}} = \rho/\epsilon_o \qquad \vec{\mathbf{E}} = -\vec{\nabla}V$$

Capacitors:

$$\begin{aligned} Q_{\mathrm{capacitor}} &= C\Delta V \\ C_{\mathrm{parallel\ plate}} &= \frac{\epsilon_0 A}{d} \\ E_{\mathrm{capacitor}} &= \frac{1}{2}Q\Delta V = \frac{Q^2}{2C} \\ C_{\mathrm{eq,\ par}} &= C_1 + C_2 \\ C_{\mathrm{eq,\ series}} &= \frac{C_1C_2}{C_1+C_2} \\ C_{\mathrm{with\ dielectric}} &= \kappa C_{\mathrm{without}} \end{aligned}$$

Current & Resistance:

$$I = \int_S \vec{\mathbf{J}}\cdot d\vec{\mathbf{A}} \xrightarrow{\text{uniform J}} I = \frac{dQ}{dt} = nqAv_d$$

$$J = \sum_k n_kq_kv_k \xrightarrow{\text{uniform J}} J = \frac{I}{A} = nqv_d$$

$$\int_S \vec{\mathbf{J}}\cdot d\vec{\mathbf{A}} = -\frac{d}{dt}\int_V \rho\,dV_{ol}$$

$$R = \frac{\rho l}{A} \qquad \rho = 1/\sigma$$

$$\vec{\mathbf{v}}_d = \frac{q\tau}{m}\vec{\mathbf{E}} \qquad \tau = \text{scattering time}$$

$$\rho = 1/\sigma = \frac{m}{nq^2\tau}$$

$$R = \Delta V/I \quad \text{Ohm}$$

$$E = \rho J \quad \text{or} J = \sigma E \quad \text{Ohm}$$

$$\mathscr{P} = dU/dt = I\Delta V \quad \text{power}$$

$$R_{\mathrm{eq}} = R_1 + R_2 + \ldots \quad \text{series}$$

$$1/R_{\mathrm{eq}} = 1/R_1 + 1/R_2 + \ldots \quad \text{parallel}$$

$$\sum I_{\mathrm{in}} = \sum I_{\mathrm{out}} \quad \text{junction}$$

$$\sum_{\text{closed path}} \Delta V = 0 \quad \text{loop}$$

Electric Potential (static case):

$$\Delta V = V_B - V_A = \frac{\Delta U}{q} = -\int_A^B \vec{\mathbf{E}}\cdot d\vec{\mathbf{l}}$$

$$V_{\mathrm{point}} = k_e\frac{q}{r} \rightarrow V_{\mathrm{continuous}} = k_e\int\frac{dq}{r} = k_e\int\frac{\rho}{r}\,d\tau$$

$$U_{\mathrm{pair\ of\ point\ charges}} = k_e\frac{q_1q_2}{r_{12}} = V_1q_2 = V_2q_1$$

$$U_{\mathrm{system}} = \text{sum over unique pairs} = \sum_{\text{pairs } ij}\frac{k_eq_iq_j}{r_{ij}} \rightarrow \frac{1}{2}\int\rho V\,d\tau$$

$$U_{\mathrm{field}} = \frac{\epsilon_o}{2}\int E^2\,d\tau = \frac{1}{2}\int\rho V\,d\tau$$

Calculus of possible utility:

$$\int\frac{1}{x}\,dx = \ln x + c$$

$$\int u\,dv = uv - \int v\,du$$

$$\int\frac{1}{1+x^2}\,dx = \tan^{-1}x + c$$

$$\int\frac{x}{a^2+x^2}\,dx = \frac{1}{2}\ln|a^2+x^2| + c$$

$$\int\frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\frac{x}{a} + C$$

$$\int\frac{1}{\sqrt{x^2\pm a^2}}\,dx = \ln\left|x+\sqrt{x^2\pm a^2}\right| + C$$

$$\int\frac{dx}{(a^2+x^2)^{3/2}} = \frac{x}{a^2\sqrt{a^2+x^2}} + C$$

$$\frac{d}{dx}\tan x = \sec^2 x = \frac{1}{\cos^2 x}$$

$$\frac{d}{dx}\sin x = \cos x \qquad \frac{d}{dx}\cos x = -\sin x$$

$$\frac{d}{dx}\frac{1}{u} = \frac{-1}{u^2}\frac{du}{dx}$$

Unit	Symbol	equivalent to
newton	N	$\mathrm{kg\cdot m/s^2}$
joule	J	$\mathrm{kg\cdot m^2/s^2 = N\cdot m}$
watt	W	$\mathrm{J/s = m^2\cdot kg/s^3}$
coulomb	C	$\mathrm{A\cdot s}$
amp	A	$\mathrm{C/s}$
volt	V	$\mathrm{W/A = m^2\cdot kg/\cdot s^3\cdot A}$
farad	F	$\mathrm{C/V = A^2\cdot s^4/m^2\cdot kg}$
ohm	Ω	$\mathrm{V/A = m^2\cdot kg/s^3\cdot A^2}$
-	1 N/C	1 V/m