University of Alabama
Department of Physics and Astronomy

PH 126 LeClair Fall 2011

PH126: Final Exam

Instructions

1. Solve seven of the problems below. All problems have equal weight.
2. You may use your textbook, notes, and online resources.
3. You may not collaborate, your work must be your own.
4. Show as much work as possible for partial credit. **No work = no credit.**
5. Solve the problems on separate sheets. Staple your sheets to the exam.
6. Return the exam to Dr. LeClair by **11:59pm on Fri 16 Dec 2011**.

1. Explain how the plates of a capacitor in an LC circuit should be vibrated mechanically in order to cause the amplitude of alternating current in the circuit to increase. *Hint: presume the plate spacing changes in time according to* $d = d_o + \delta d \sin \Omega t$.

2. A small permanent-magnet dc electric motor may have its speed varied by inserting a series resistor to decrease the current or by switching the current with a transistor so that the full voltage is applied for a fraction of the time (pulse width modulation). It has been claimed that the latter method gives less resistive power loss.

To determine whether this claim is valid, consider the case where the speed is reduced by a factor of two by either method. Assume that the speed is proportional to the mean torque of the motor and does not vary significantly during the pulse period. Assume also that the resistive loss across the transistor is negligible and that the resistance of the motor is such that the current drawn by the motor at full speed is 0.1 times the current draw when it is held stationary with full voltage applied.

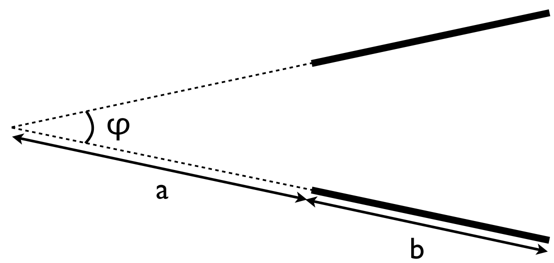
Calculate the power dissipated in the motor for each method. How do these values compare with the dissipation at full speed?

3. A well-known apparatus for demonstrating electromagnetic induction consists of a vertical iron (or laminated iron) post about 50 cm high and 5 cm in diameter, with a coil of wire would around its lower end. If a loose-fitting metal ring is lowered over the post after an ac supply to the coil has been switched on, it will float at a certain height above the coil. Explain why, and derive an expression for the upward force on the ring. What determines the height at which it floats?

4. The diagram below represents a section of a capacitor whose rectangular plates are inclined at an angle φ. The plates are of length z, perpendicular to this section. Show that, ignoring edge effects, the capacitance is

$$C = \frac{\epsilon \epsilon_o z}{\varphi} \ln\left(1 + \frac{b}{a}\right) \tag{1}$$

where ϵ is the relative dielectric constant of the medium between the plates.



5. A thin copper ring rotates freely about a diameter, which is perpendicular to a uniform magnetic field B. Its initial frequency of rotation is ω. Calculate the time taken for the frequency to decrease to $1/e \approx 1/2.71828$ of its original value, under the assumption that the energy goes into Joule heating. Take the resistivity of copper as $1.8 \times 10^{-8}\,\Omega\,\mathrm{m}$, and the density of copper as $8900\,\mathrm{kg\,m^{-3}}$. Take $B = 20\,\mathrm{mT}$.

6. A charged particle of charge e and mass m starts from rest at the origin of a coordinate system and moves under the action of a uniform electric field E directed parallel to the y axis and of a uniform magnetic field B parallel to the z axis.

(a) Show that the equations of motion are

$$m\frac{dv_x}{dt} = eBv_y \tag{2}$$

$$m\frac{dv_y}{dt} = eE - eBv_x \tag{3}$$

$$m\frac{dv_z}{dt} = 0 \tag{4}$$

(b) Find x and y as functions of time from these equations, and show that the particle moves along the x axis with an *average* speed equal to E/B. *Hint: integrate the first equation, using* $v_y = dy/dt$ *to get* v_x *in terms of* y*, then use the second equation.*

7. Starting from the equation of continuity in the form

$$\oint \vec{j} \cdot \hat{n}\, da + \oint \frac{\partial \rho}{\partial t}\, dv = 0 \tag{5}$$

and Gauss' law, show that in a conducting medium obeying Ohm's law ($\vec{\mathbf{j}} = \sigma\vec{\mathbf{E}}$), the following relationship holds in any volume element constant inside the medium:

$$\left(\frac{\partial \rho}{\partial t} + \frac{\sigma}{\epsilon}\rho\right) dv = 0 \tag{6}$$

From this, show that if one has an initial charge density ρ_o at any point inside the conductor, the charge density ρ at any later time t at this point is given by

$$\rho = \rho_o e^{-(\sigma/\epsilon_o)t} \tag{7}$$

8. The inner sphere of a spherical capacitor of inner and outer radii a and b, respectively, is coated with a thin coat of varnish of thickness t and dielectric constant κ. Show that the increase in capacity of the capacitor due to the varnish is given approximately by

$$\Delta C = 4\pi\epsilon_o \left(\frac{b^2(\kappa - 1)}{\kappa(b - a)^2}\right) t \tag{8}$$

9. An image of height a is formed on screen by a thin convex lens. It is found by moving the lens toward the screen that there is a second lens position at which a second sharp image of height b is formed on the screen. Show that the height of the object is $\sqrt{ab}$.

10. An electron is released from rest and falls under the influence of gravity. In the first centimeter, what fraction of the potential energy lost is radiated away?