University of Alabama
Department of Physics and Astronomy

PH 126 LeClair

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PH126: Final Exam

1. You know for an unperturbed LC circuit the charge as a function of time follows

$$\frac{d^2Q}{dt^2} + \frac{1}{LC_o}Q = 0 \tag{1}$$

for which the solution is $Q = Q_o \sin \omega t$ with $\omega = 1/\sqrt{LC_o}$. If the spacing of the capacitor plates is changing as $d = d_o + \delta d \sin \Omega t$, then the capacitance is now

$$\frac{1}{C(t)} = \frac{d_o + \delta d \sin \Omega t}{\epsilon_o A} = \frac{1}{C_o} + \frac{1}{C_o}\frac{\delta d}{d} \tag{2}$$

where C_o is the equilibrium capacitance. The charge equation then reads

$$\frac{d^2Q}{dt^2} + \frac{1}{LC_o}Q + \frac{\delta d}{d}\frac{1}{LC_o}Q = 0 \tag{3}$$

If $\delta d/d$ is small, then a good approximation would be to plug in the unperturbed solution $Q = Q_o \sin \omega t$ into the last term for Q. Using the identity

$$\sin a \sin b = \frac{1}{2}\left(\cos\left(a-b\right) - \cos\left(a+b\right)\right) \tag{4}$$

you can make the charge equation look approximately like a driven harmonic oscillator, with two driving frequencies, $\Omega \pm \omega$. The condition for maximum amplitude implies a relationship between the driving frequencies and the unperturbed resonance frequency, which tells you at what frequency you should vibrate the plates.

Physically, to get a larger current you would want to expand or contract the plates at just the right points during the cycle. The current scales with capacitance, so just as the current peaks *in either direction* you'd want to increase the capacitance by squishing the plates together at that moment. This gives you a relationship between the mechanical force's frequency and that of the alternating current.

2. When the motor is stationary, it draws current $I = V/R$. At full speed, the applied voltage will be counteracted by the induced voltage in the motor, the so-called "back emf" which we'll call V_b.

The current draw at full speed, which we are told is 1/10 the stationary current, is then

$$I = \frac{V - V_b}{R} = \frac{1}{10}\frac{V}{R} \tag{5}$$

From this you can find V_b and the power at full speed. At half speed, the back emf is half as much!

With a series resistor inserted, you need only replace R with the equivalent resistance to find the current. This current should be the same as half the full speed current, which must be 0.05 times the stationary current with full voltage. This gets you the power in the motor and the total power. With a transistor altering the duty cycle, the idea is that you use full power a fraction f of the time, such that the average power draw is the same as running the motor at half speed. That is, full current times f gives the average current, which must also be the same as 0.05 times the stationary current. Find f, then find the power dissipated in the motor (which must be f times the power required for half speed operation).

3. There are a few ways to go about this, all approximate. We don't need an actual number, just an approximation of the force and what it depends on. The time-varying field from the iron rod + coil will induce a current in the loop, which will then create its own magnetic field.
One way is the following: presume that the field of the iron + coil is approximately that of a magnetic dipole (which you can look up), and find the flux of its field through the ring (only the radial part of the field will contribute to the flux). The time derivative of that flux gets you the current in the ring, and that current loop can also be treated as a magnetic dipole. With the dipole moments of each, the force between two magnetic dipoles pointing at each other can be looked up. This is a rough, but more straightforward method, and it will result in the right dependencies on current, frequency, etc.

4. Presume the electric field lines to be arcs of circles centered on the vertex in the figure. The potential difference is then just the electric field (which you know to be σ/ϵ for a large plate) times the arclength, $r\varphi$. That gives you the surface charge density in terms of the potential, a radial position r along the plate, and the angle φ. Integrate that over the plate to get Q, and using $C = Q/V$ you are done.

Note the (now fixed) typo, the log should have b/a not a/b!

5. Recall our discussion of generators, the flux through the ring is $\Phi = BA\cos\omega t$. Find the induced voltage, which gives you the electrical power dissipated. The mechanical power loss this must correspond to is

$$P_{\text{mech}} = \frac{dK}{dt} = \frac{d}{dt}\left(\frac{1}{2}I\omega^2\right) \tag{6}$$

where I is the moment of inertia (expressible in terms of density and dimensions). Don't forget about the chain rule ... you should come up with a separable differential equation that gives an exponential decay for ω, it is the resulting time constant you're after. It is *of order* 1 s, and depends only on B, the resistivity, and the density.

6. Part a should be easy enough. For part b, integrate the first equation to get an expression for v_x. Plug that expression for v_x into the second, noting that $dv_y/dt = d^2y/dt^2$. The resulting differential equation is one you know and love, plus a constant (in any case, the solution is well-known, and you can look it up). That gets you $y(t)$ and $v_y(t)$. Use your result for v_y in the first equation. Noting the initial conditions, you can just integrate twice.

Physically, you know what should happen: circular motion. Make sure the math bears this out.[i]

7. Just plug in $j = \sigma E$ and note that Gauss' law shows up. We did most of this in class; check section 3.3.3 in the notes on radiation and EM waves for an illustration of this problem in differential form ...

8. Not much to do but integrate from one conductor to the other. Look for suitable places to make approximations ($t \ll b, a$) to get the suggested result.

9. The key here is that the object to image distance is fixed, so $p + q = d$, where d is the object-screen distance, p the object-lens distance, and q the lens-image distance. With this constraint, you know

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{p} + \frac{1}{d-p} = \frac{1}{f} \tag{7}$$

This ends up being a quadratic equation in p; solve it. That gives you the object distance for the two possible images. Call them p_1 and p_2. Since magnification is $|M| = h'/h = q/p$, for the two locations you know

$$h = \frac{ap_1}{q_1} = \frac{bp_2}{q_2} \tag{8}$$

Multiply both of those relationships together to get an expression for h^2. Then use your two solutions, p_1 and p_2, the relationships $d_1 = q_1 + p_1$ and $d_2 = q_2 + p_2$, and grind through some algebra.

10. Larmor formula. You know the acceleration, the energy lost is power times time. Figure the time required to fall a distance of 1 cm. Expect a ridiculously small fraction.

[i] I think I told a couple of you it would result in non-circular motion, but that was due to a math error on my part ...