University of Alabama
Department of Physics and Astronomy

PH 126 LeClair Fall 2011

Problem Set 3

Instructions:

1. Answer all questions below. Show your work for full credit.
2. All problems are due 12 September 2011 by 11:59pm.
3. You may collaborate, but everyone must turn in their own work.

1. A spherical volume of radius a is filled with charge of uniform density ρ. We want to know the potential energy U of this sphere of charge, that is, the work done in assembling it. Calculate it by building up the sphere up layer by layer, making use of the fact that the field outside a spherical distribution of charge is the same as if all the charge were at the center. Express the result in terms of the total charge Q of the sphere.

2. At the beginning of the 20^{th} century the idea that the rest mass of the electron might have a purely electrical origin was very attractive, especially when the equivalence of energy and mass was revealed by special relativity. Imagine the electron as a ball of charge, of constant volume density ρ out to some maximum radius r_o. Using the result of the previous problem, set the potential energy of the system equal to mc^2 and see what you get for r_o. One defect of this model is rather obvious: nothing is provided to hold the charge together!

3. A spherical conductor A contains two spherical cavities. The total charge on the conductor itself is zero. However, there is a point charge q_b at the center of one cavity and q_c at the center of the other. A considerable distance r away, outside the conductor, is a point charge q_d. What force acts on each of the four objects, A, q_b, q_c, and q_d? Which answers, if any, are only approximate, and depend on r being relatively large?

4. We want to design a spherical vacuum capacitor with a given radius a for the outer sphere, which will be able to store the greatest amount of electrical energy subject to the constraint that the electric field strength on the surface of the inner sphere may not exceed E_o. What radius b should be chosen for the inner spherical conductor, and how much energy can be stored?

5. We have two point charges connected by a rigid rod, forming a dipole. It is placed in an external electric field $\vec{\mathbf{E}}(\vec{\mathbf{r}})$.
(a) Suppose that the electric field is uniform: $\vec{\mathbf{E}}=\vec{\mathbf{E}}_o$ where $\vec{\mathbf{E}}_o$ is a constant vector. What will be the total force on the dipole?

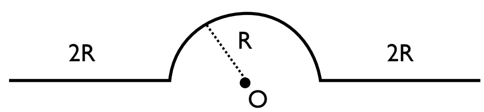
(b) Now suppose the field is not uniform, but that it only changes by a small amount over the distance d. Show that the z-component of the total force on the dipole is approximately

$$F_z = p_z \frac{\partial E_z}{\partial z} \tag{1}$$

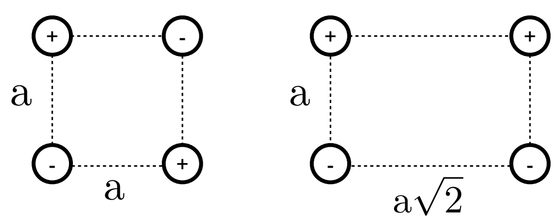
where p_z is the z-component of the dipole moment $\vec{\mathbf{p}}$.[i]

(c) Why is a charged rubber rod able to attract bits of paper without touching them?

6. A wire having uniform linear charge density λ is bent into the shape shown below. Find the electric potential at O.



7. The two figures below show small sections of two different possible surfaces of a NaCl surface. In the left arrangement, the NaCl(100) surface, charges of $+e$ and $-e$ are arranged on a square lattice as shown. In the right arrangement, the NaCl(110) surface, the same charges are arranged in a rectangular lattice. What is the potential energy of each arrangement (symbolic answer)? Which is more stable?



8. A charge Q is located h meters above a conducting plane. How much work is required to bring this charge out to an infinite distance above the plane? *Hint: Consider the method of images.*

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[i]For an arbitrary dipole orientation, this generalizes to $\vec{\mathbf{F}} = \left(\vec{\mathbf{p}} \cdot \vec{\boldsymbol{\nabla}}\right) \vec{\mathbf{E}}$.