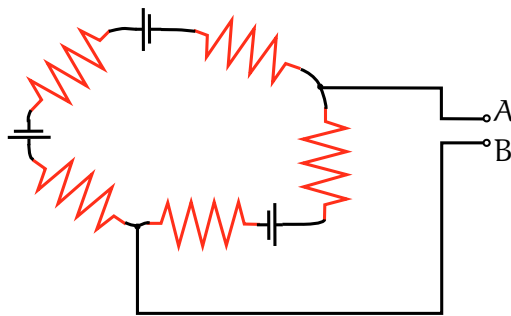


## Problem Set 4

### Instructions:

1. Answer all questions below. Show your work for full credit.
2. All problems are due 23 September 2011 by 11:59pm.
3. You may collaborate, but everyone must turn in their own work.

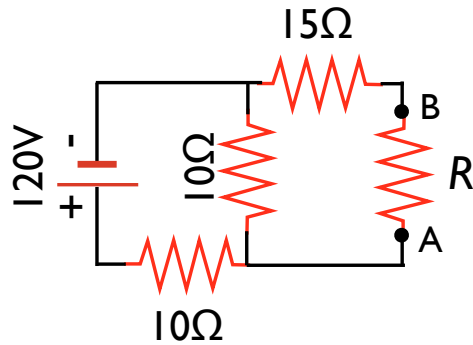
1. A battery has an ideal voltage  $\Delta V$  and an internal resistance  $r$ . A variable load resistance  $R$  is connected to the battery. Determine the value of  $R$  such that the power delivered to the resistor is maximum.



2. In the circuit above, all five resistors have the same value,  $100\ \Omega$ , and each battery has a rated voltage of  $1.5\ \text{V}$  and no internal resistance. Find the open-circuit voltage and the short-circuit current for the terminals A, B. Then find the Thévenin equivalent circuit (i.e., the ideal battery and resistor that could replace this circuit between terminals A, B.)

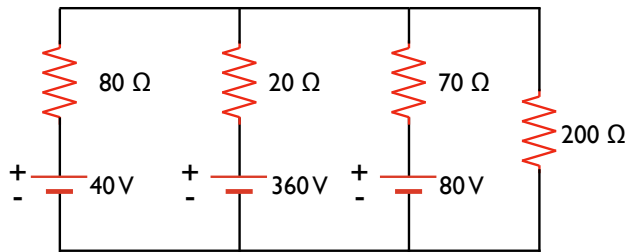
3. Two resistors are connected in parallel, with values  $R_1$  and  $R_2$ . A total current  $I_o$  divides somehow between them. Show that the condition  $I_1 + I_2 = I_o$ , together with the requirement of minimum power dissipation, leads to the same current values that we would calculate with normal circuit formulas. This illustrates a general variational principle that holds for direct current networks: the distribution of currents within the networks, for a given input current  $I_o$ , is always that which gives the least total power dissipation.

4. A resistor  $R$  is to be connected across the terminals A, B of the circuit below. (a) For what value of  $R$  will the power dissipated in the resistor be the greatest? To answer this, construct the Thévenin equivalent circuit and then invoke the result of the first problem. (b) How much power will be dissipated in  $R$ ?

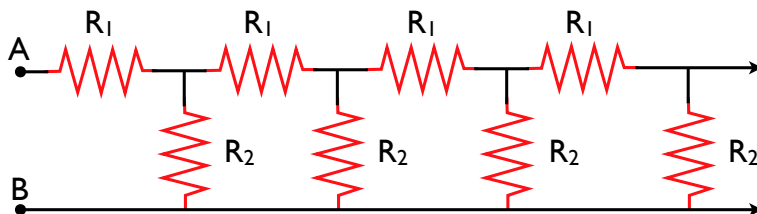


5. A laminated conductor was made by depositing, alternately, layers of silver 10 nm thick and layers of tin 20 nm thick. The composite material, considered on a larger scale, may be considered a homogeneous but anisotropic material with electrical conductivity  $\sigma_{\perp}$  for currents perpendicular to the planes of the layers, and a different conductivity  $\sigma_{\parallel}$  for currents parallel to that plane. Given that the conductivity of silver is 7.2 times that of tin, find the ratio  $\sigma_{\perp}/\sigma_{\parallel}$ .

6. In the circuit below, determine the current in each resistor and the voltage across the 200 Ω resistor.



7. Find the input resistance (between terminals A and B) of the following infinite series of resistors.



Show that, if voltage  $V_0$  is applied at the input to such a chain, the voltage at successive nodes decreases in a geometric series. What ratio is required for the resistors to make the ladder an

attenuator that halves the voltage at every step? Can you suggest a way to terminate the ladder after a few sections without introducing any error in its attenuation? *Hint: If we put another “link” on the left of this infinite chain, we get exactly the same configuration.*

8. For the circuit shown below, with  $V_{in} = 30\text{ V}$  and  $R_1 = R_2 = 10\text{ k}\Omega$ , find (a) the output voltage (between the  $R_1$  and  $R_2$ ) with no load attached; (b) the output voltage with a  $10\text{ k}\Omega$  load resistance; (c) the Thèvenin equivalent circuit (rightmost circuit in the figure below); (d) the power in each resistor with and without a load present.

