University of Alabama
Department of Physics and Astronomy

PH 126 LeClair Fall 2011

# Problem Set 5

**Instructions:**

1. Answer all questions below. Show your work for full credit.
2. All problems are due 30 September 2011 by 11:59pm.
3. You may collaborate, but everyone must turn in their own work.

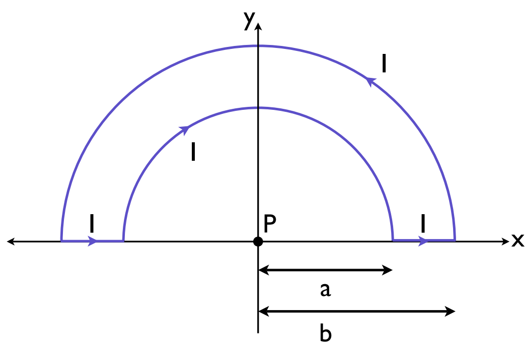
**1.** The electric field of a long, straight line of charge with $\lambda$ coulombs per meter is

$$E = \frac{2k_e\lambda}{r}$$

where $r$ is the distance from the wire. Suppose we move this line of charge parallel to itself at speed $v$. **(a)** The moving line of charge constitutes an electric current. What is the magnitude of this current? **(b)** What is the magnitude of the magnetic field produced by this current? **(c)** Show that the magnitude of the magnetic field is proportional to the magnitude of the electric field, and find the constant of proportionality.

**2.** A flat circular disk with radius $R$ carries a uniform surface charge density $\sigma$. It rotates with an angular velocity $\omega$ about the $z$-axis. Find the magnetic field $B(z)$ at any point $z$ along the rotation axis.

**3.** Find the magnetic field at point $P$ due to the current distribution shown below. *Hint: Break the loop into segments, and use superposition.*



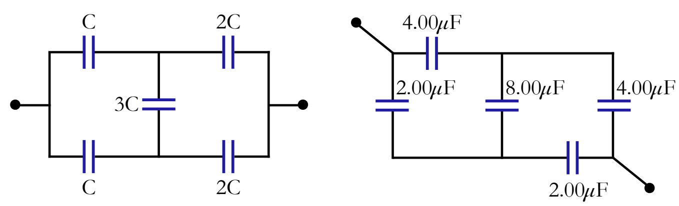

**4.** The dielectric material between the plates of a parallel-plate capacitor has some non-zero conductivity $\sigma = 1/\rho$ (thus it is a "leaky" capacitor). Let $A$ represent the area of each plate, and $d$

their separation. Let $\kappa$ represent the dielectric constant of the material. Show that the resistance $R$ and the capacitance $C$ of the leaky capacitor are related by $RC = \kappa\epsilon_0\sigma$.

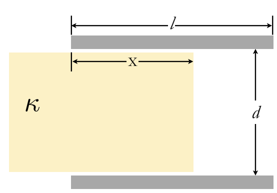
**5.** Two capacitors, one charged and the other uncharged, are connected in parallel. **(a)** Prove that when equilibrium is reached, each carries a fraction of the initial charge equal to the ratio of its capacitance to the sum of the two capacitances. **(b)** Show that the final energy is less than the initial energy, and derive a formula for the difference in terms of the initial charge and the two capacitances.

**6.** Two long, cylindrical conductors of radius $a_1$ and $a_2$ are parallel and separated by a distance $d$ which is large compared with either radius. Find the capacitance per unit length of the two conductors.

**7.** Find the equivalent capacitance for *both* combinations shown below. Be sure to consider the symmetry involved and the relative electric potential at different points in the circuits.




**8.** A capacitor is constructed from two square plates of sides $l$ and separation $d$. A material of dielectric constant $\kappa$ is inserted a distance $x$ into the capacitor, as shown below. **(a)** Find the equivalent capacitance of this device as a function of $x$. **(b)** Calculate the energy stored in the capacitor, letting $\Delta V$ represent the potential difference. **(c)** Find the direction and magnitude of the force exerted on the dielectric, assuming a constant potential difference $\Delta V$. Ignore friction.




**9.** Using the same figure as the previous question, imagine now that the block being inserted is metal, rather than dielectric. Assume that $d \ll l$, and that the plates carries charges $+Q_o$ and $-Q_o$. **(a)** Calculate the stored energy as a function of $x$. **(b)** Find the direction and magnitude of the force acting on the metallic block. *Hint: a metal can be considered a perfect dielectric, $\kappa \to \infty$, which allows no electric field to penetrate it.*