University of Alabama
Department of Physics and Astronomy

PH 126 LeClair Fall 2011

Problem Set 6

Instructions:

1. Answer all questions below. Show your work for full credit.
2. All problems are due 14 October 2011 by 11:59pm.
3. You may collaborate, but everyone must turn in their own work.

1. One way to produce a very uniform magnetic field is to use a very long solenoid and work only in the middle section of its interior. This is often convenient, but wasteful of space and power. Can you suggest ways in which two short coils or current rings might be arranged to achieve good uniformity over a limited region? *Hint: consider two coaxial current rings of radius* a *separated axially by a distance* b*. Investigate the uniformity of the field in the vicinity of the point on the axis midway between the two coils. Determine the magnitude of the coil separation* b *which for a given coil radius* a *will make the field in this region as nearly uniform as possible.*

Solution: We first want to find the field at an *arbitrary* point z somewhere between the two coils, then find $\partial B/\partial z$ and $\partial^2 B/\partial z^2$ (a partial derivative, since we are holding everything else constant). For a very uniform field, both spatial derivatives should vanish.

Let $z=0$ at the intersection of the plane of the bottom coil and the z axis. The field from the bottom coil at an arbitrary point a distance z along the axis due to the bottom coil is that of a single current loop centered on the origin, which we have already established:

$$B = \frac{\mu_o I}{2} \frac{R^2}{(z^2+R^2)^{3/2}} \qquad \text{(single loop)}$$

Let the separation of the coils be b. At a position z, since the separation of the coils is b, we are a distance $b-z$ from the upper coil. We need only replace z with $b-z$ in the expression above to find the field from the upper coil at a distance $z<R$ from the bottom coil. Since the currents are in the same directions for both coils, the magnetic fields are in the same direction, and we may just add them together:

$$B_{\text{tot}} = B_{\text{lower}} + B_{\text{upper}} = \frac{\mu_o I}{2} \frac{R^2}{(z^2+R^2)^{3/2}} + \frac{\mu_o I}{2} \frac{R^2}{\left[(b-z)^2+R^2\right]^{3/2}}$$

Now we only need calculate $\partial B/\partial z$:

$$\frac{\partial B_{tot}}{\partial z} = \frac{\mu_o I R^2}{2}\left[\frac{d}{dz}\frac{1}{\left(z^2+R^2\right)^{3/2}} + \frac{d}{dz}\frac{1}{\left((b-z)^2+R^2\right)^{3/2}}\right]$$

$$= \frac{\mu_o I R^2}{2}\left[\frac{-\frac{3}{2}\left(2z\right)}{\left(z^2+R^2\right)^{5/2}} + \frac{-\frac{3}{2}\left(2z-2b\right)}{\left((b-z)^2+R^2\right)^{5/2}}\right]$$

$$= \frac{\mu_o I R^2}{2}\left[\frac{-3z}{\left(z^2+R^2\right)^{5/2}} + \frac{3b-3z}{\left((b-z)^2+R^2\right)^{5/2}}\right]$$

Comparing the two terms, one can see that the first derivative will vanish if $z = z - b$ or $z = b - z$. The first says $b = 0$ - stack the two coils right on top of one another and sit in the middle. The field is indeed uniform then, but over a very tiny volume. The second condition is more interesting, it implies $2z = b$ – whatever the spacing, sit right between the two coils. That much makes sense, and the derivative of the field with respect to position is zero, implying high uniformity near the center of the Helmholtz arrangement. However, this still tells us nothing about how b relates to R

$$\frac{\partial^2 B_{tot}}{\partial z^2} = \frac{\mu_o I R^2}{2}\left[\frac{-3}{\left(z^2+R^2\right)^{5/2}} + \frac{-3z\left(-\frac{5}{2}\right)\left(2z\right)}{\left(z^2+R^2\right)^{7/2}} - \frac{3}{\left((b-z)^2+R^2\right)^{5/2}} + \frac{3\left(b-z\right)\left(-\frac{5}{2}\right)\left(2z-2b\right)}{\left((b-z)^2+R^2\right)^{7/2}}\right]$$

$$= \frac{\mu_o I R^2}{2}\left[\frac{15z^2}{\left(z^2+R^2\right)^{7/2}} - \frac{3}{\left(z^2+R^2\right)^{5/2}} + \frac{15\left(b-z\right)^2}{\left((b-z)^2+R^2\right)^{7/2}} - \frac{3}{\left((b-z)^2+R^2\right)^{5/2}}\right]$$

Now we need to set the part in square brackets equal to zero. This looks bad, but keep in mind that we already know that $z = b/2$ from the first derivative, and that also means everywhere we see $b - z$, we can replace it with $b/2$.

$$0 = \frac{15z^2}{\left(z^2+R^2\right)^{7/2}} - \frac{3}{\left(z^2+R^2\right)^{5/2}} + \frac{15\left(b-z\right)^2}{\left((b-z)^2+R^2\right)^{7/2}} - \frac{3}{\left((b-z)^2+R^2\right)^{5/2}} \tag{1}$$

$$0 = \frac{15b^2/4}{\left(b^2/4+R^2\right)^{7/2}} - \frac{3}{\left(b^2/4+R^2\right)^{5/2}} + \frac{15b^2/4}{\left(b^2/4+R^2\right)^{7/2}} - \frac{3}{\left(b^2/4+R^2\right)^{5/2}} \tag{2}$$

$$0 = \frac{15b^2/4}{\left(b^2/4+R^2\right)^{7/2}} - \frac{3\left(b^2/4+R^2\right)}{\left(b^2/4+R^2\right)^{7/2}} + \frac{15b^2/4}{\left(b^2/4+R^2\right)^{7/2}} - \frac{3\left(b^2/4+R^2\right)}{\left(b^2/4+R^2\right)^{7/2}} \tag{3}$$

$$\tag{4}$$

Now we have a common denominator, so we need only worry about the numerator (since we know the denominator cannot be zero as well).

$$0 = \frac{30}{4}b^2 - 6\left(\frac{b^2}{4} + R^2\right) \tag{5}$$

$$6b^2 = 6R^2 \tag{6}$$

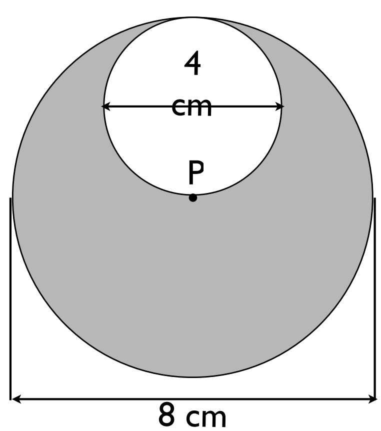
$$b = R \tag{7}$$

Thus, the first two spatial derivatives of the field vanish halfway between the two coils for a coil spacing equal to the coil radius, known as the *Helmholtz* arrangement. One can also show that $\partial^3 B/\partial z^3$ is zero as well (just by symmetry, or by direct calculation), meaning the field is extremely uniform over a reasonably large volume between the two coils. In fact, it is even better than a solenoid of length b. Of course, a very long solenoid is much better, but the field is only really uniform over the middle $\sim\frac{1}{3}$ of the solenoid, meaning you waste a lot of wire and power.

Incidentally, the field midway between the two coils is $B = 8\mu_o I/5^{3/2}R$, about 1.4 times the field one would get directly at the center of a single coil. Not only is the field more uniform, it is larger. How convenient!

You might find the wikipedia article (`http://en.wikipedia.org/wiki/Helmholtz_coil`) interesting, and possibly this comparison of solenoids and Helmholtz coils (`http://www.netdenizen.com/emagnet/helmholtz/idealhelmholtz.htm`). The Maxwell coil is another clever way to generate uniform fields with a minimum of wire (`http://en.wikipedia.org/wiki/Maxwell_coil`).

2. A long copper rod 8 cm in diameter has an off-center cylindrical hole, as shown below. This conductor carries a current of 900 A flowing in the direction into the page. What is the direction and magnitude of the magnetic field at point P on the inner axis of the outer cylinder?



Solution: Here we must use superposition. The field would be equivalent to a whole 8 cm wire with current into the page superimposed with a 4 cm wire carrying current out of the page. The areas of the missing section and the rest have a 1:4 ratio, since the radius of the missing section is half the radius of the rest. That means that a whole 8 cm wire would carry 4/3 the current of the one given, or 1200 A, and the 4 cm wire must carry 300 A in the opposite direction.

By symmetry, the magnetic field due to a whole wire must be zero at the center of the wire.That means that the whole wire carrying 1200 A gives no field at P, and the field is entirely due to the 300 A in the other direction carried by the 4 cm wire.

Ampere's law tells us that the field at some distance from a current-carrying wire depends only on the current enclosed by a closed path surrounding the wire. If we take a 4 cm circle just surrounding the 300 A wire passing through P, that tells us that the field due to the 300 A wire is the same as if all the current were concentrated at the center of the 300 A wire. Recall our discussion of a current-carrying cylinder – outside the cylinder, the field must be the same as a a wire of infinitesimal thickness. Thus, the field at P is simply

$$B(P) = \frac{\mu_o I}{2\pi r} = \frac{\mu_o\,(300\,\mathrm{A})}{2\pi\,(2\times10^{-2}\,\mathrm{m})} \approx 3\,\mathrm{mT} \tag{8}$$

The right-hand rule tells us that the field at P is pointing to the right since the 300 A current must be out of the page. Incidentally, it is remarkable (but easily proved) fact that the field has the same magnitude and direction *anywhere* within the cylindrical hole!

3. Consider the magnetic field of a circular current ring, at points on the axis of the ring (use the exact formula, not your approximate form above). Calculate explicitly the line integral of the magnetic field along the ring axis from $-\infty$ to ∞, and check the general formula

$$\int \vec{\mathbf{B}} \cdot d\vec{\mathbf{l}} = \mu_o I_{\text{encl}} \tag{9}$$

Why may we ignore the "return" part of the path which would be necessary to complete a closed loop?

Solution: The z component of the magnetic field along the z axis of a current ring of radius R lying in the xy plane is

$$B_z = \frac{\mu_o I R^2}{2\,(z^2+R^2)^{3/2}} \tag{10}$$

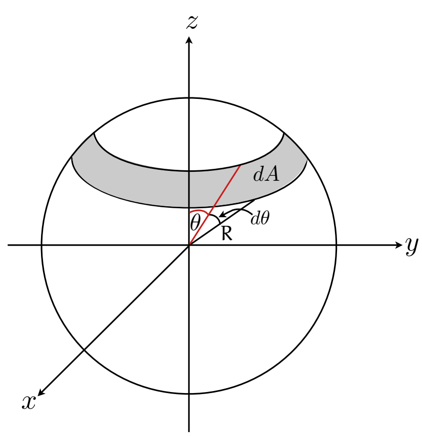
If we take a path along z, then $d\vec{\mathbf{l}} = d\hat{\mathbf{z}}$

$$\int_{-\infty}^{\infty} \vec{\mathbf{B}} \cdot d\vec{\mathbf{l}} = \int_{-\infty}^{\infty} B_z \, dz = \int_{-\infty}^{\infty} \frac{\mu_o I R^2}{2\left(z^2+R^2\right)^{3/2}} dz = \frac{\mu_o I R^2}{2} \left[\frac{z}{R^2\sqrt{z^2+R^2}} \right]_{\infty}^{\infty} = \mu_o I \tag{11}$$

Why does this work without a return path? Because at $z = \infty$ we can take a return path to $z = -\infty$ at an infinite distance from the ring, where the field is zero.

4. A sphere of radius R carries the charge Q which is distributed uniformly over the surface of the sphere with a density $\sigma = 4\pi R^2$. This shell of charge is rotating about an axis of the sphere with angular velocity ω, in radians/sec. Find its magnetic moment. (Divide the sphere into narrow bands of rotating charge, find the current to which each band is equivalent and its dipole moment, and integrate over all bands.)

Solution: Start by dividing the surface into little strips, defined by a subtended angle $d\theta$ as in the figure below.



The surface area of such a strip is its circumference $2\pi R \sin\theta$ times its width, the arclength $R\, d\theta$:

$$da = 2\pi(R\sin\theta)(R\, d\theta)$$

The amount of charge on this strip can be found from the surface charge density $\sigma \equiv Q/4\pi R^2$:

$$dq = \sigma da = \frac{Q}{4\pi R^2} 2\pi R^2 \sin\theta \, d\theta = \frac{1}{2} Q \sin\theta \, d\theta \quad .$$

This bit of charge dq revolves at frequency $f = \omega/2\pi$, so it corresponds to a current $dI = dq/T = f\, dq$:

$$dI = f\, dq = \frac{\omega Q}{4\pi} \sin\theta \, d\theta$$

Each tiny current loop contributes a magnetic moment equal to its current times *the area the loop encloses* A, $d\mu = A\,dI$. The area the *loop encloses* is $\pi(R\sin\theta)^2$, and with that we need only integrate over all such loops, $\theta \in [0, \pi]$.

$$d\mu = A\,dI = \left(\frac{\omega Q}{4\pi}\sin\theta\,d\theta\right)\left(\pi R^2\sin^2\theta\right) = \frac{1}{4}\omega Q R^2 \sin^3\theta\,d\theta \tag{12}$$

$$\mu = \int d\mu = \int_0^\pi \frac{1}{4}\omega Q R^2 \sin^3\theta\,d\theta = \frac{1}{3}\omega Q R^2 \tag{13}$$

If we note that the angular momentum of a hollow sphere of mass m is $L = I\omega = \frac{2}{3}mR^2\omega$, we find that the ratio of the angular momentum and magnetic dipole moment depends only on Q and m - not on ω, the size of the sphere, or even that we *have* a sphere. This ratio is usually called the *gyromagnetic ratio* γ[i]

$$\gamma = \frac{\mu}{L} = \frac{Q}{2m} \tag{14}$$

It turns out this ratio is the same for any classical rotating body – ring, sphere, cube, whatever. This ratio governs the rate (frequency) at which a magnetic dipole will precess when placed in an external (but constant) magnetic field, *Larmor precession.* The torque on the dipole due to an external magnetic field B is given by $\vec{\tau} = \vec{\mu} \times \vec{\mathbf{B}} = \gamma\vec{\mathbf{L}} \times \vec{\mathbf{B}}$, which gives precession[ii] at a frequency $\omega = 2\pi f = \gamma B$.

5. Consider two solenoids, one of which is a tenth-scale model of the other. The larger solenoid is 2 m long, and 1 m in diameter, and is wound with 1 cm-diameter copper wire. When the coil is connected to a 120 V dc generator, the magnetic field at the center is exactly 0.1 T. The scaled-down version is exactly one-tenth the size in every linear dimension, including the diameter of the wire. The number of turns is the same in both coils, and both are designed to provide the same central field.

(a) Show that the voltage required is the same, namely, 120 V
(b) Compare the coils with respect to the power dissipated, and the difficulty of removing this heat by some cooling means.

Solution: This is basically a scaling problem: when everything is shrunk by 10 times, what happens to the required voltage for a given field? First, let's consider the large solenoid. Let's say it has length $L = 2$ m, radius $r = 0.5$ m, contains N turns of wire, and it provides a field $B = 0.1$ T with a current I. We know we can relate the field and the current:

[i]See `http://en.wikipedia.org/wiki/Gyromagnetic_ratio`.
[ii]See `http://en.wikipedia.org/wiki/Larmor_precession`.

$$B = \mu_0 \frac{N}{L} I$$

The solenoid is just a long single strand of wire wrapped around a cylinder. If we say that the total length of wire used to wrap the solenoid is l, and the wire's diameter is d, then we can calculate the resistance of the solenoid:

$$R = \frac{\rho l}{A} = \frac{\rho l}{\pi d^2/4}$$

Here we have used the wire's resistivity ρ, and its cross-sectional area $A = \pi r^2 = \pi d^2/4$. Given the resistance and voltage of $\Delta V = 120\,\text{V}$, we can calculate the current:

$$I = \frac{\Delta V}{R} = \frac{\Delta V \pi d^2/4}{\rho l}$$

Now if we plug that into our first solenoid equation above, we can relate voltage and magnetic field:

$$B = \mu_0 \frac{N}{L} I = \mu_0 \frac{N}{L} \frac{\Delta V \pi d^2/4}{\rho l} = \frac{\mu_0 \pi}{4\rho} \frac{N \Delta V d^2}{L l}$$

Now, what about the small solenoid? Every dimension is a factor of 10 smaller. If *all* the dimensions are 10 times smaller, the number of turns that fit within 1/10 the length is the *same* as the big solenoid if the wire diameter is also 1/10 as large! In other words, both coils will have the same number of turns - the space for the wire is 10 times smaller, but so is the wire.

In order to find the relationship for the small solenoid, we will use the same symbols, but everything for the small solenoid will have a prime $'$. The number of turns in the small solenoid is N', and in for the large solenoid it is just N. The voltage on the little solenoid is $\Delta V'$, and on the large one we have just ΔV. Using the results from above, magnetic field for the small solenoid is then easily found by substitution:

$$B' = \frac{\mu_0 \pi}{4\rho} \frac{N' \Delta V (d')^2}{L' l'} = B$$

We don't have to bother with a prime on the resistivity, both coils have the same sort of wire. Remember, our desired condition is that $B' = B$. We know that $N' = N$, and all the dimensions are 10 times smaller - the length of the solenoid, the wire diameter, and therefore also the length of wire required. We have the same number of *turns* in each coil, but in the smaller coil the circumference of each turn is 10 times smaller, which means overall, the total length of wire required l is 10 times smaller. Thus:

$$
\begin{aligned}
B' &= \frac{\mu_0\pi}{4\rho}\frac{N'\Delta V'(d')^2}{L'l'} \\
&= \frac{\mu_0\pi}{4\rho}\frac{N\Delta V'(d')^2}{L'l'} && \text{note that } N' = N \\
&= \frac{\mu_0\pi}{4\rho}\frac{N\Delta V'(\frac{d}{10})^2}{\frac{L}{10}\frac{l}{10}} && \text{scale all dimensions by } \frac{1}{10} \\
&= \frac{\mu_0\pi}{4\rho}\frac{N\Delta V' d^2}{Ll}
\end{aligned}
$$

Now, we want to enforce the condition that the field is the same in both solenoids:

$$
\begin{aligned}
& B' = B \\
\Longrightarrow \quad & \frac{\mu_0\pi}{4\rho}\frac{N\Delta V' d^2}{Ll} = \frac{\mu_0\pi}{4\rho}\frac{N\Delta V d^2}{Ll} \\
\Longrightarrow \quad & \Delta V' = \Delta V
\end{aligned}
$$

Thus, a solenoid shrunk by 10 times in every dimension will require the same applied voltage for the same magnetic field. What about the power consumption? The current in the large solenoid was

$$
I = \frac{\Delta V}{R} = \frac{\Delta V \pi d^2/4}{\rho l}
$$

In the small solenoid, we now know that the voltage is the same, but the resistance is not, so we should have:

$$
I' = \frac{\Delta V}{R'} = \frac{\Delta V \pi (d')^2/4}{\rho l'} = \frac{\Delta V \pi (\frac{d}{10})^2/4}{\rho \frac{l}{10}} = \frac{1}{10}\frac{\Delta V \pi d^2/4}{\rho l} = \frac{1}{10} I
$$

The current in the little solenoid is 10 times less - sensible, since the total length of wire is 10 times smaller, but the area of the wire is 100 times smaller. The power required for each is the product of current and voltage:

$$
\begin{aligned}
\mathscr{P}_{\text{big}} &= I\Delta V \\
\mathscr{P}_{\text{small}} &= I'\Delta V = \frac{1}{10} I\Delta V = \frac{1}{10}\mathscr{P}_{\text{big}}
\end{aligned}
$$

Not only is the larger solenoid ten times larger, it requires ten times more power, and therefore dissipates ten times more heat. The cooling requirements will be far more formidable for the larger solenoid. For instance, if we decide to use water cooling, the flow rate will need to be at least 10 times larger for the large solenoid to extract a heat load ten times larger. Not to mention the fact that we have to acquire a much larger power supply in the first place - practically speaking, the difference between a 5 A current source and a 50 A current source is significant. Keep in mind that your normal household outlets deliver 120 V at a maximum of ~15 A.

6. You want to confine an electron of kinetic energy 3.0×10^4 eV by making it circle inside a solenoid of radius 0.1 m under the influence of the force exerted by the magnetic field. The solenoid has 12000 turns of wire per meter. What minimum current must you put through the wire if the electron is not to hit the wall of the solenoid?

Solution: If we have a charged particle (charge e) moving with velocity v perpendicular to a magnetic of magnitude B, we know the particle will undergo circular motion with a radius

$$r = \frac{mv}{qB}$$

We want this radius to be equal to or smaller than the radius of the solenoid $R = 0.1$ m, such that the circular orbit fits inside the solenoid. The kinetic energy given tells us the velocity of the electron:

$$\begin{aligned} K &= \frac{1}{2}mv^2 \\ v &= \sqrt{\frac{2K}{m}} \end{aligned}$$

Thus,

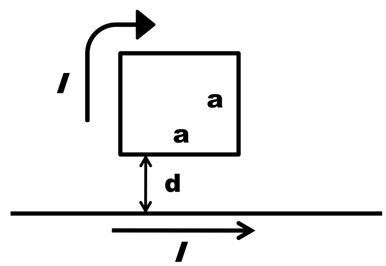
$$r = \frac{m}{eB}\sqrt{\frac{2K}{m}} \leqslant R$$

The larger the B field, the smaller the radius. For a solenoid, we know that the B field produced is proportional to the current I in the solenoid, $B = \mu_o n I$, where n is the number of turns per unit length (given). Substituting in the equation above and solving for I,

$$\begin{aligned} R &\leqslant \frac{m}{eB}\sqrt{\frac{2K}{m}} = r \\ R &\leqslant \frac{m}{e\mu_o n I}\sqrt{\frac{2K}{m}} \\ I &\geqslant \frac{\sqrt{2Km}}{e\mu_o n R} \approx 0.39\,\text{A} \end{aligned}$$

To get the numbers to come out, we have to remember to convert our energy units ($1\,\text{eV} = 1.6 \times 10^{-19}$ J) ...

7. Find the force on a square loop (side a) placed as shown below, near an infinite straight wire. Both loop and wire carry a steady current I.



Solution: The magnetic field above the straight wire is out of the page, so the force on the two vertical segments of the square loop cancels. The top and bottom sections of the loop are just current-carrying segments, and for each one the field due to the current I in the straight wire is constant. The top wire is at a distance $a + d$ and has length a, the bottom is at distance d and has length a. Thus,

$$F_{\text{net}} = F_{\text{bottom}} - F_{\text{top}} = B_{\text{straight}}(d+a)Ia - B_{\text{straight}}(d)Ia = \frac{\mu_o I^2 a}{2\pi}\left(\frac{1}{d+a} - \frac{1}{d}\right) \tag{15}$$

The force on the bottom segment is upward and larger than the downward force on the top segment. The loop experiences a net force upward, but will be squished as well.

8. I have input signals of voltage V_a, V_b, and V_c. I want an a circuit that outputs $V_a+2V_b-3V_c$. Design a circuit that does this, making sure to note all component values and supply voltages. Presume that the signals are essentially constant, i.e., do not worry about frequency response.[iii].

Solution: We know how to build sum and difference circuits with op-amps already, and we know to amplify signals. We can make an inverting amplifier to generate $3V_c$ from V_c, and a non-inverting amplifier to make $2V_b$ from V_b (or, make an inverting amplifier of gain -2 and then follow it with another one of gain -1). Additionally, it would be clever to "buffer" V_a with a gain of ± 1 amplifier to decouple its source from the output, just good practice. I'll choose resistors of $\sim 10\,\text{k}$, that is well above any stray resistances we'll have from the wires, far below the op-amp's input resistance, and leads to low power consumption for signals of a few volts. For an inverting amplifier, the gain is given by R_2/R_1, so for a gain of -3 (three times bigger but inverted) we want $R_2 = 3R_1$. For a non-inverting amplifier, the gain is given by $1+R_2/R_1$, so for a gain of $+2$ we want $R_2 = R_1$. The summation is achieved by making an inverting amplifier of gain -1 and feeding all three signals into it. Since that results in an inverted sum, we'll have to invert again to get $V_a+2V_b-3V_c$.

[iii]If you can add, you can do any other kind of math based on that, so you have in principle the basis for a simple computer

Of course, that isn't the only way to do it – there are many. I could have used inverting amplifiers for V_a and V_c, then a non-inverting amplifier for V_c. Feeding those signals into a summation circuit would then produce the right output without requiring the last inversion step (so we save one op amp that way). Below are example circuits doing it either way.

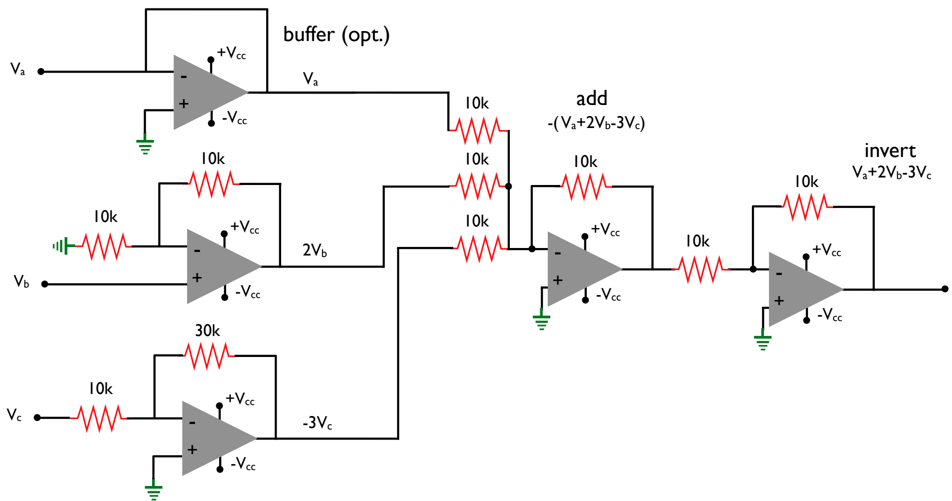


Figure 1: *The little triangles are op-amps.*

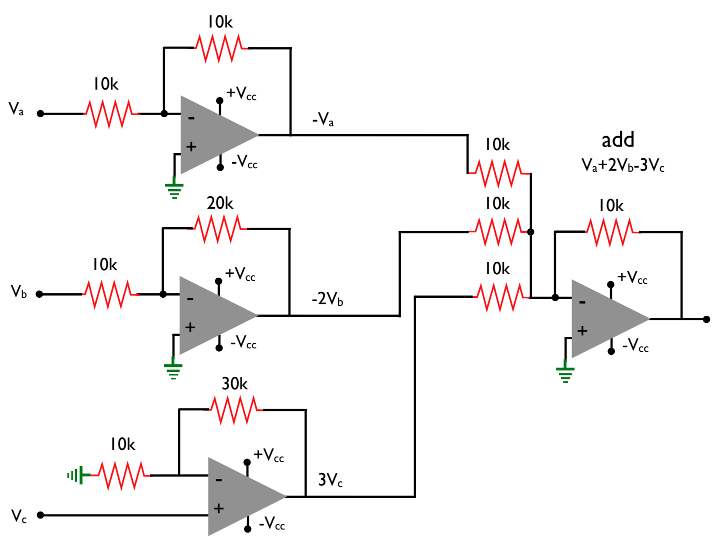


Figure 2: *Same result, one less triangle.*

9. A *pulse height discriminator* (a.k.a., *amplitude discriminator*) is a device one often encounters in physics experiments. It is essentially a circuit that produces a specified output pulse when and *only when* it receives an input pulse whose amplitude exceeds an assigned value. Design on that will produce output pulses if and only if the input is greater than 100 mV. Make sure to note all component values and supply voltages, and presume the frequency of pulses is well within the range of your components. *Hint: You will probably want to use a comparator, though one can do it passively with only a diode, capacitor, and resistor.* [iv]

Solution: All we need is a comparator with our signal as the input and the reference set to 100 mV. We can do that by making a voltage divider from our supply voltage, which we'll set at 5 V (presuming we'll use something like an LM311). This is a nice thing to do since we don't need another power supply - we can use the same 5 V to set the reference and supply our comparator, reducing complexity quite a bit.

For the divider, we want the resistances smaller than the comparator's input resistance (many MΩ or GΩ), but such that the power consumption is minimal. Something in the kΩ as a ballpark is good. From a 5 V supply, to generate 0.1 V we need a ratio of 50 for $R_1/(R_1 + R_2)$, so $R_1/R_2 = 49$. Let's pick 1 k and 49 k (50 k would be close enough). The last thing we need is a "pull-up" resistor on the output – the comparator sets its output to the negative supply voltage (ground; zero). When the positive input is greater than the negative input, the output is *floating*, or undefined. Our 10 k "pull-up" resistor ensures in that case that the output is near 5 V. When the positive input is lower than the negative input, the output is shorted to ground and thus zero. Any time V_{in} exceeds 0.1 V, the output will be nearly 5 V, and when V_{in} is less than 0.1 V, the output is zero.

This means we have about 4.9 V across 49 kΩ for ~0.5 mW dissipated in the larger resistor in the divider, and 10 μW in the smaller one, so we're not wasting too much power in setting the reference voltage. More importantly, when the output is grounded (positive input below threshold) we have 5 V across the 10 k pull-up resistor to ground for 2.5 mW, this is our dominant power consumption. We could make the pull-up resistor a bit larger to reduce that power consumption. Of course, you already built this circuit in the lab, so you know it works!

[iv] You will be building one of these in the next few weeks as part of your mid-semester project ...

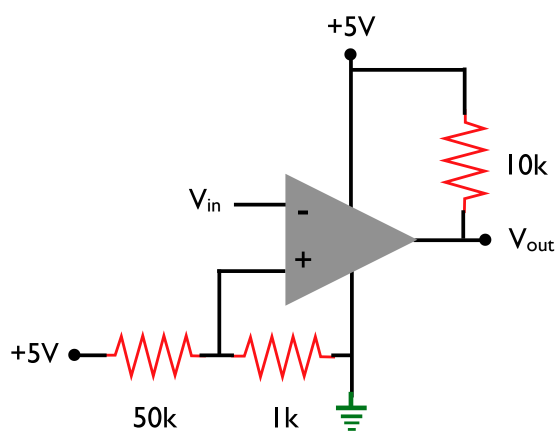


Figure 3: *The little triangles are comparators this time.*