University of Alabama
Department of Physics and Astronomy

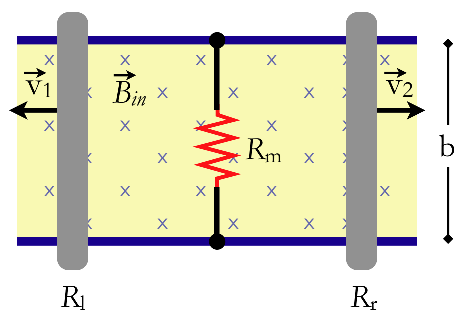
PH 126 LeClair Fall 2011

Problem Set 7

Instructions:

1. Answer all questions below. Show your work for full credit.
2. All problems are due 31 October 2011 by 11:59pm.
3. You may collaborate, but everyone must turn in their own work.

1. Two parallel rails with negligible resistance are a distance d apart and connected by a resistor R_m. The circuit also contains two metal rods, of resistance R_l and R_r, begin pulled away from the resistor at speeds of v_1 and v_2, respectively. A uniform magnetic field B is applied into the plane of the figure. Determine the current in the central resistor R_m.



2. A conducting rod of length l moves with constant velocity $\vec{v}$ in a direction perpendicular to a long, straight wire carrying a current I. The rod remains parallel to the wire. Show that when the rod is a distance r from the wire, the induced potential difference between the ends of the rod is

$$\Delta V = \frac{\mu_o v I l}{2\pi r} \tag{1}$$

3. A wire carrying a current I is bent into the shape of an exponential spiral, $r = e^{\theta}$, from $\theta = 0$ to $\theta = 2\pi$. To complete a loop, the ends of the spiral are connected by a straight wire along the x

axis.[i] Find the magnitude and direction of $\vec{\mathbf{B}}$ at the origin.

Hint: Use Biot-Savart. For a wire following a curve that can be written as $r = f(\theta)$, it takes the following simple form (which I hope to derive for you soon):

$$B = \frac{\mu_o I}{4\pi} \int \frac{d\theta}{r} \tag{2}$$

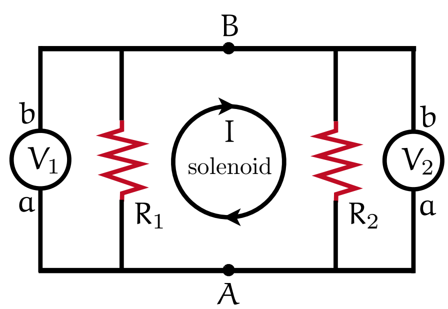
The direction is up to you to find!

4. A thin ring of radius a carries a static charge q. This ring is in a magnetic field of strength B_o, parallel to the ring's axis, and is supported so that it is free to rotate about that axis. If the field is switched off, how much angular momentum will be added to the ring? If the ring has mass m, show that it will acquire an angular velocity $\omega = qB_o/2mc$. *Hint: How is angular momentum being added? Consider the induced electric field and the resulting forces. We will cover this in lecture.*

5. The current in a long solenoid is increasing linearly with time, so that the flux is proportional to t: $\Phi_B = \alpha t$. Two voltmeters are connected to the diametrically opposite points (A and B), together with resistors R_1 and R_2, as shown below. Assume the voltmeters are ideal (infinite input resistance, zero current draw), and that a voltmeter registers the quantity

$$\int_a^b \vec{\mathbf{E}} \cdot d\vec{\mathbf{l}} \tag{3}$$

between the terminals and through the meter. Find the reading on each voltmeter.



6. In a perfect conductor, the conductivity is infinite, so $\vec{\mathbf{E}} = 0$, and any net charge resides on the surface (just as it does for an *imperfect* conductor in electrostatics).

(a) Show that the magnetic field is constant (i.e., $\partial\vec{\mathbf{B}}/\partial t = 0$) inside a perfect conductor.
(b) Show that the magnetic flux through a perfectly conducting loop is constant.

[i] Try going to `wolframalpha.com` and enter `plot of r=e^(theta) from theta=0 to theta=2pi` to see what we mean..

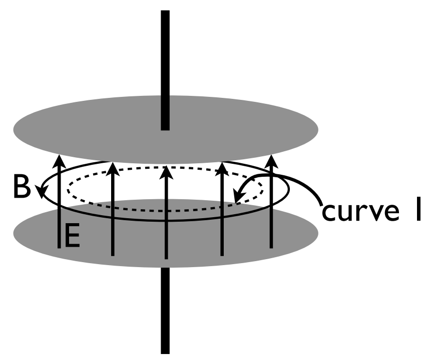
A *superconductor* has infinite conductivity, but is more than a merely perfect conductor: it has the additional property that the (constant) $\vec{\mathbf{B}}$ inside is in fact *zero*. This "flux exclusion" is known as the Meissner effect, perfect diamagnetism.

(c) Show that the there is no volume current density in a superconductor, and therefore any current in a superconductor must be confined to the surface.

7. Suppose we have a parallel plate capacitor connected to an ac generator of relatively low frequency ω. You know that electric field deep inside the capacitor (i.e., ignoring edge effects) can be written as

$$E = E_o e^{i\omega t} \tag{4}$$

where E_o is a constant.



Will this work indefinitely as frequency goes up? Not really; as the electric field oscillates, that creates a time-varying flux through loops like curve 1 shown below, which will produce a magnetic field. We can find that field with Maxwell's equations, using curve 1 as our boundary of integration:

$$c^2 \oint_1 \vec{\mathbf{B}} \cdot d\vec{s} = \frac{\partial}{\partial t} \int_1 \vec{\mathbf{E}} \cdot \vec{\mathbf{n}}\, da \tag{5}$$

$$2\pi r c^2 B = \frac{\partial}{\partial t} \pi r^2 E \tag{6}$$

$$\implies \quad B = \frac{i\omega r}{2c^2} E_o e^{i\omega t} \tag{7}$$

Thus, as you know, there is an oscillating magnetic field in the capacitor as well, meaning at high enough frequencies it has a little bit of inductance. Now we have a problem, however: if there is an oscillating magnetic field inside, as given above, then this must induce an additional *electric* field in addition to the applied electric field. Thus, the total field will be the original one plus that induced by the oscillating magnetic field, which was itself due to the original field! This continues on indefinitely in fact, since the new additional electric field will create a new oscillating magnetic field, and so on.

(a) Using the integral form of Faraday's law, find the first "correction" to the electric field, viz., that due to the oscillating magnetic field given above.

(b) Now that you have the "corrected" electric field, the expression for the magnetic field is not quite right. Find the second term in the magnetic field expression.

(c) Continue on to find the second term in the electric field correction.

(d) Sketch the electric field along the radial direction (i.e., perpendicular to the vertical axis) after the first correction, still ignoring edge effects. You should find that the field is no longer uniform after correcting for the time-variation of the fields.

Incidentally, if you continue correcting the fields indefinitely, until you have a fully self-consistent field (written as an infinite series), you will get the original electric field times what is known as a Bessel function. The first correction term plus the original field is a very good approximation to the real field, however. *Hint: we will do this in lecture.*