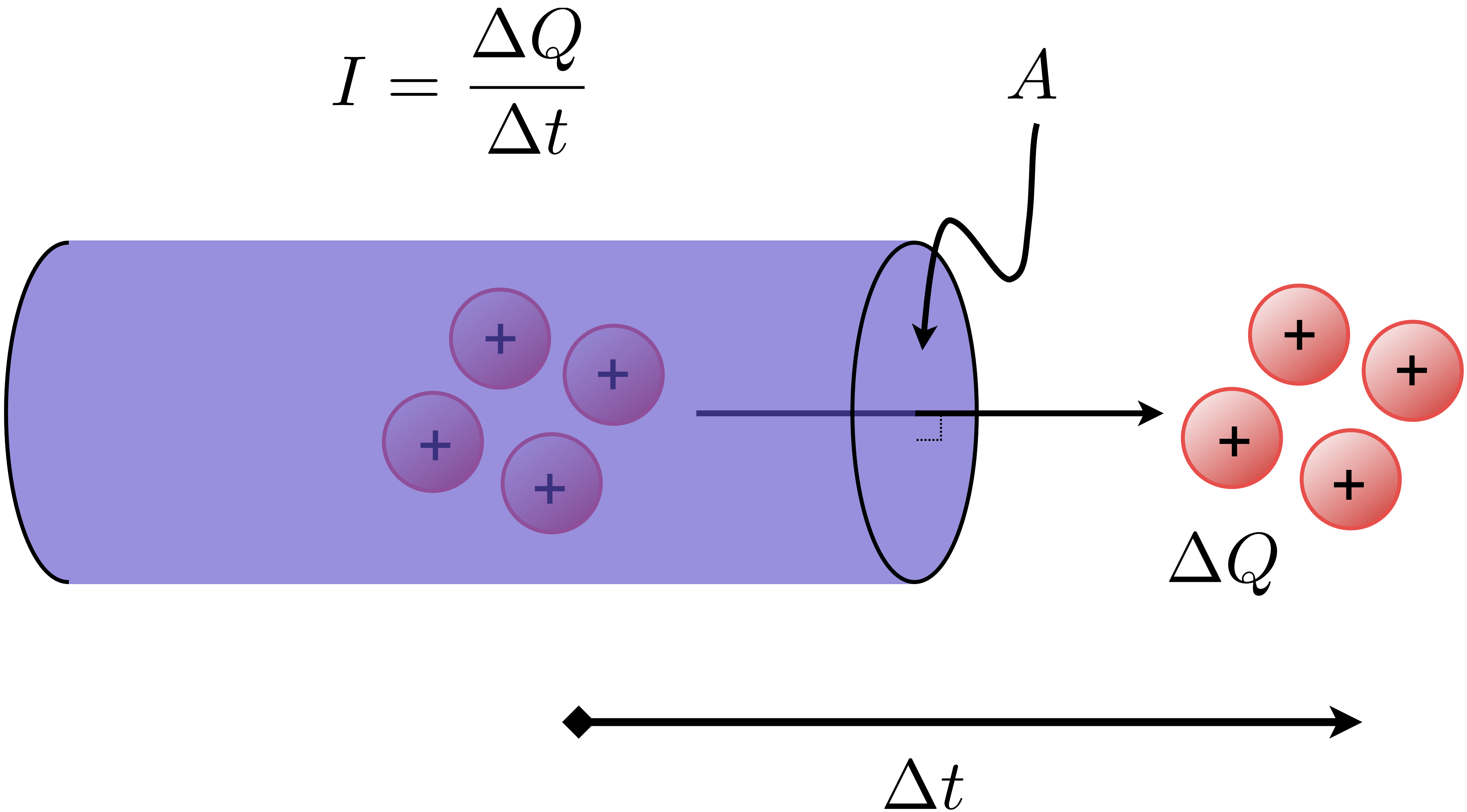
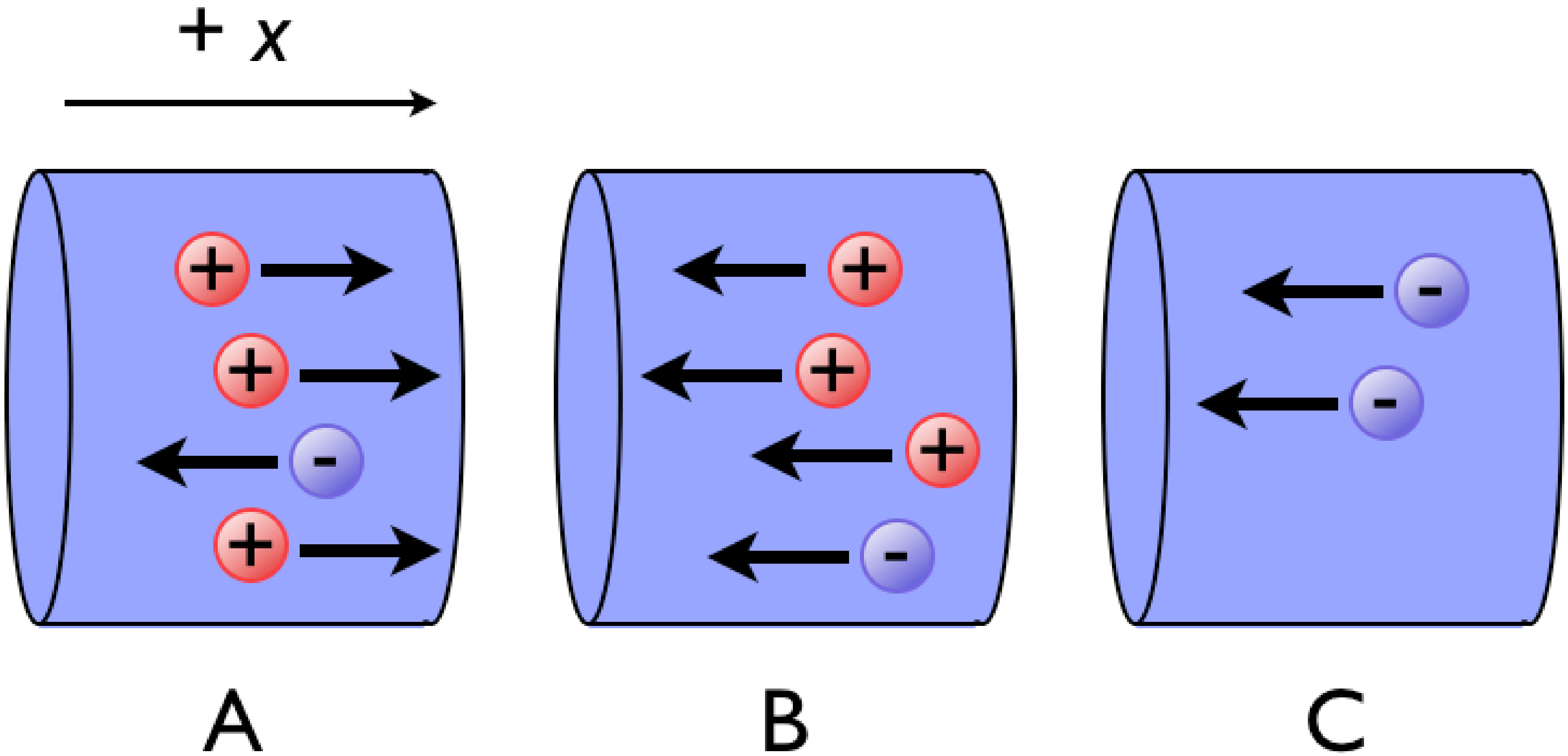
today: dc circuits

mostly current & resistance

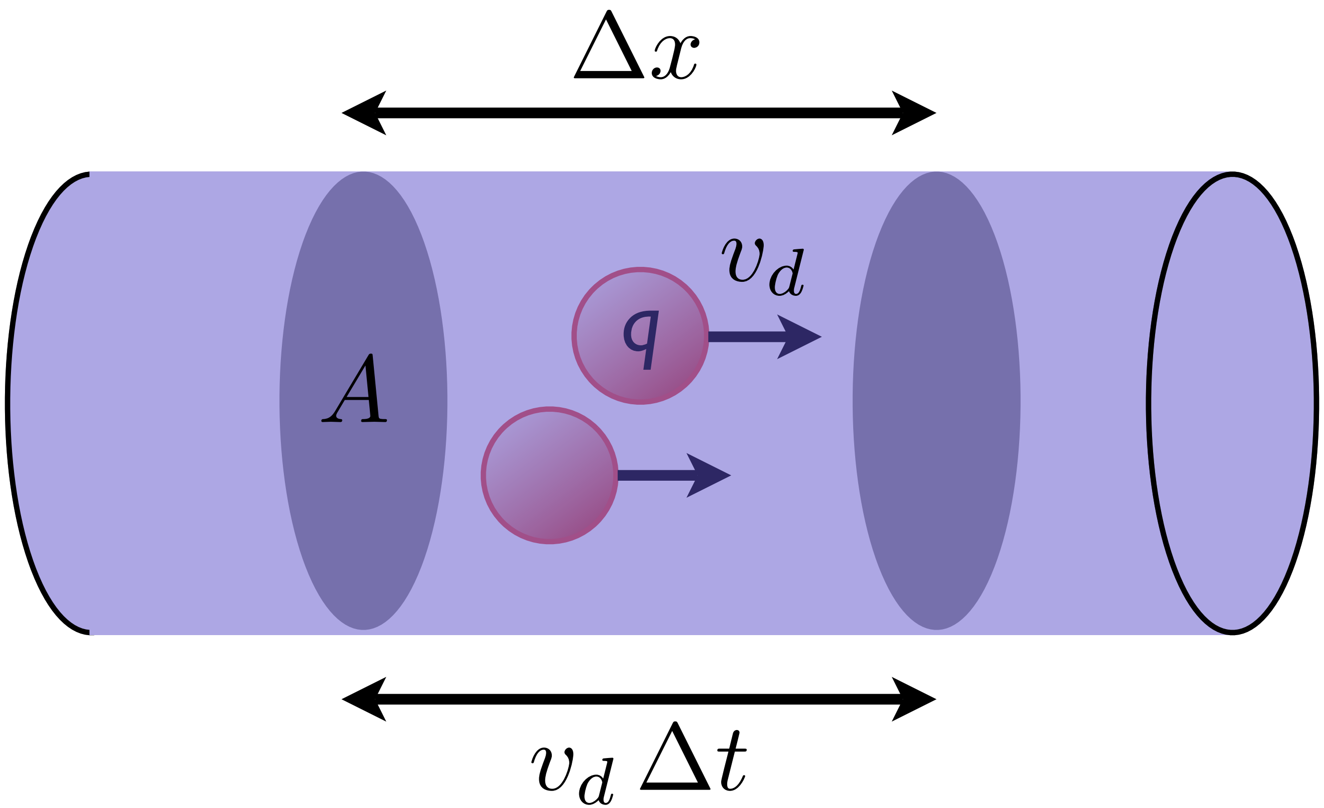


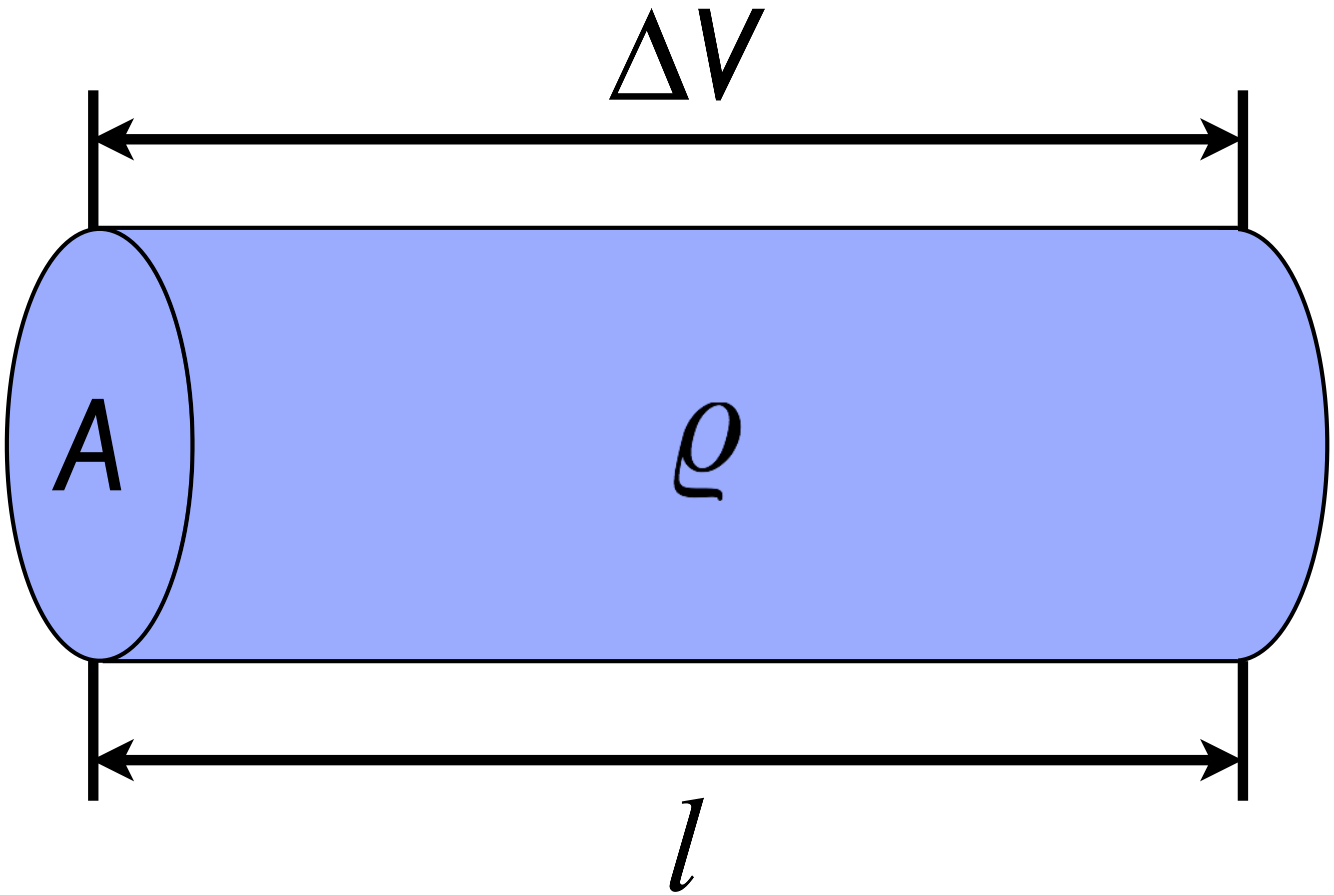
$I = \frac{\Delta Q}{\Delta t}$
A
ΔQ
Δt


+ x
A
B
C

not so funny now.

just wait ...


Δx
v_d
A
q
$v_d \, \Delta t$

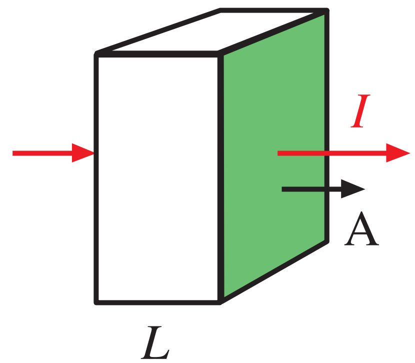


$$R = \frac{\rho l}{A} = \frac{\Delta V}{I}$$

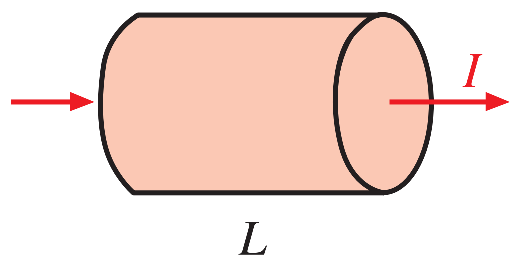
I = Cause/Resistance

I is the current, or flow rate,
describes different scenes:

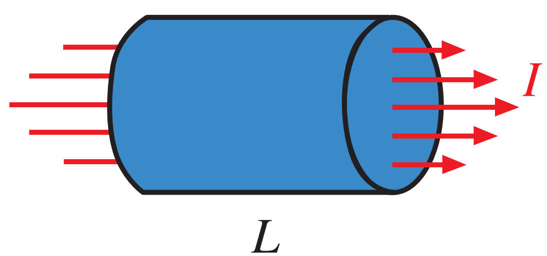
(a) Heat flow through a wall



(b) Charge flow through a wire



(c) Fluid flow through a pipe

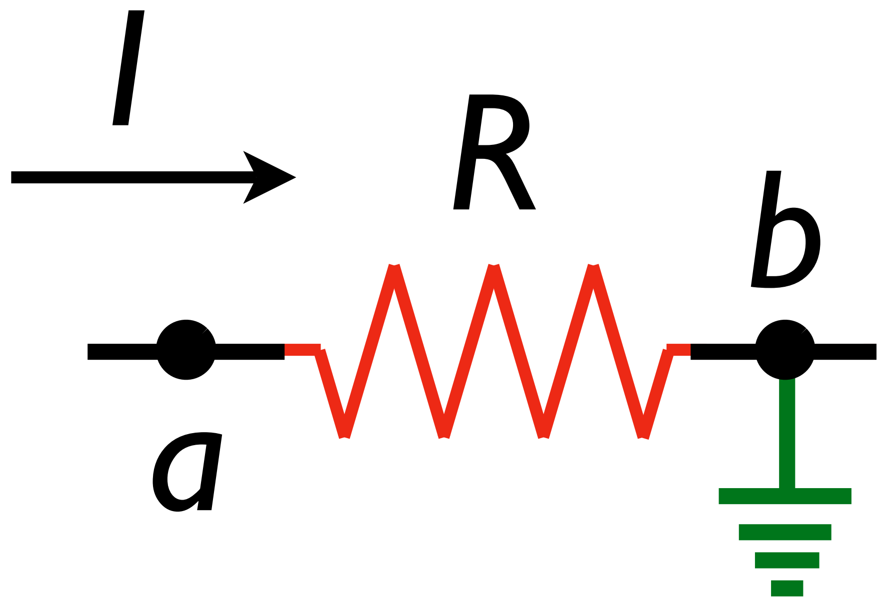


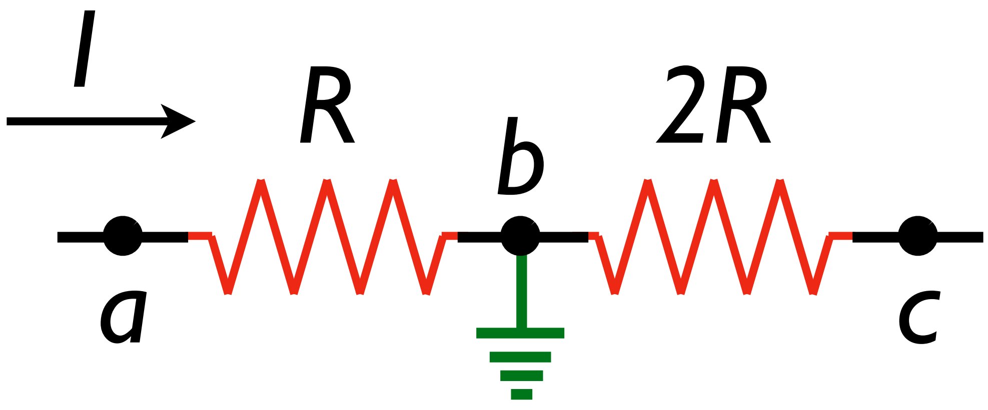
Resistance R
has the same form in most cases,

$$R = \rho L/A$$

Transport what?	Heat	Electric charges	Displacement of a molecule in a fluid	Volume of fluid
Current form (items/second)	$I = -\Delta T/R$	$I = -\Delta V/R$	$v_{av} \equiv \mathrm{I} = -\Delta P/R$	$I = -\Delta P/R$
Current units	J/s or W	C/s or amperes	m/s	m^3/s
Resistance form	$R = \rho L/A$	$R = \rho L/A$	$R = \rho L/A$	$R = \rho L/A^2$
Detail of ρ (resistivity)	ρ = 1/heat conductivity	ρ = electrical resistivity	$\rho = 6\eta\pi$	$\rho = 8\eta\pi$

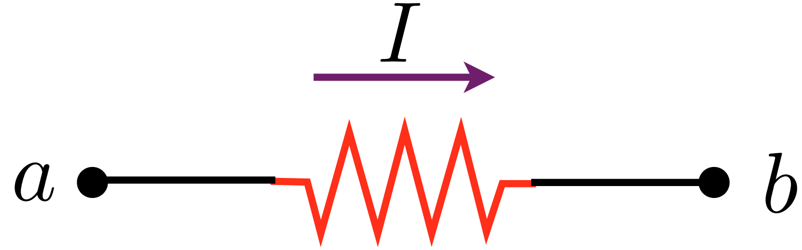
battery = pump
voltage = pressure
current = flow
resistor = constriction
capacitor = diaphragm / flexible reservoir
diode = check valve
inductor = paddle wheel


I
R
b
a



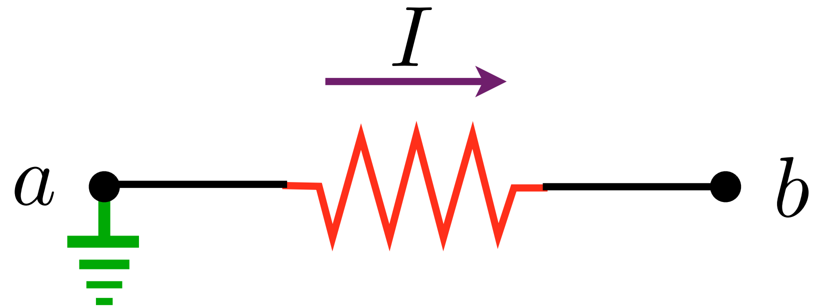
I
R
2R
b
a
c

(a)



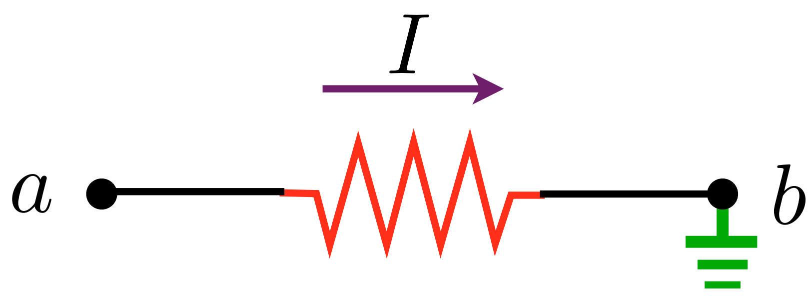
$$\Delta V = V_b - V_a = -IR$$

(b)



$$V_a = 0 \quad V_b = -IR$$

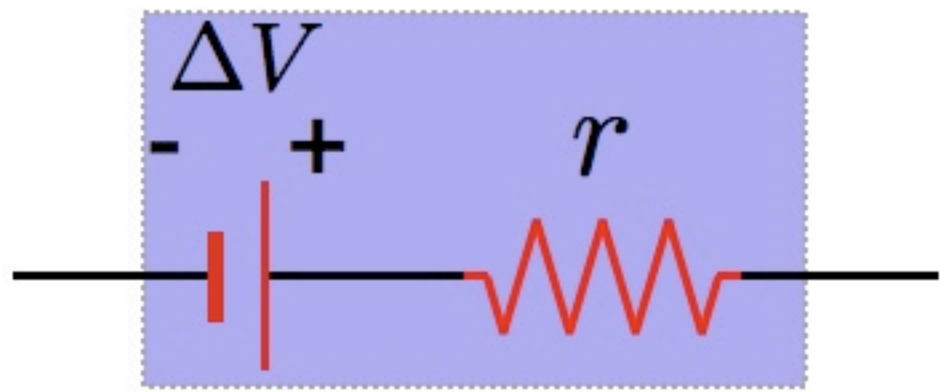
(c)



$$V_a = +IR \quad V_b = 0$$

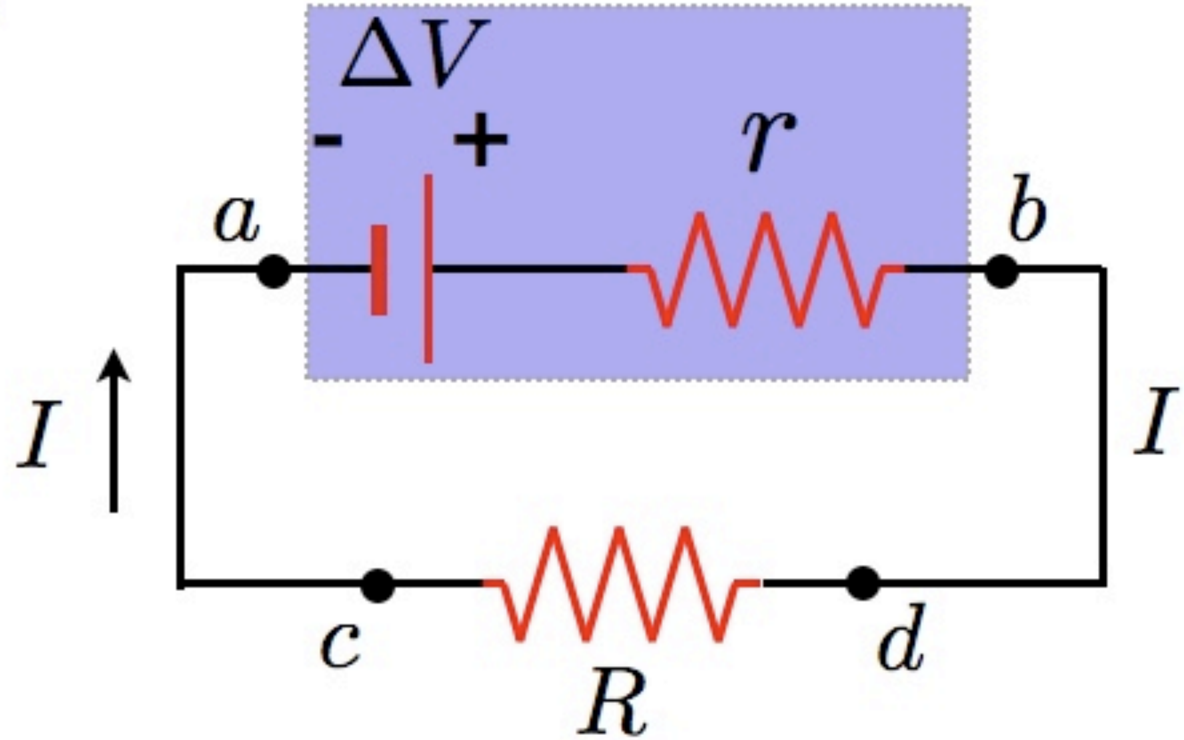
real V source = ideal V source + R

=



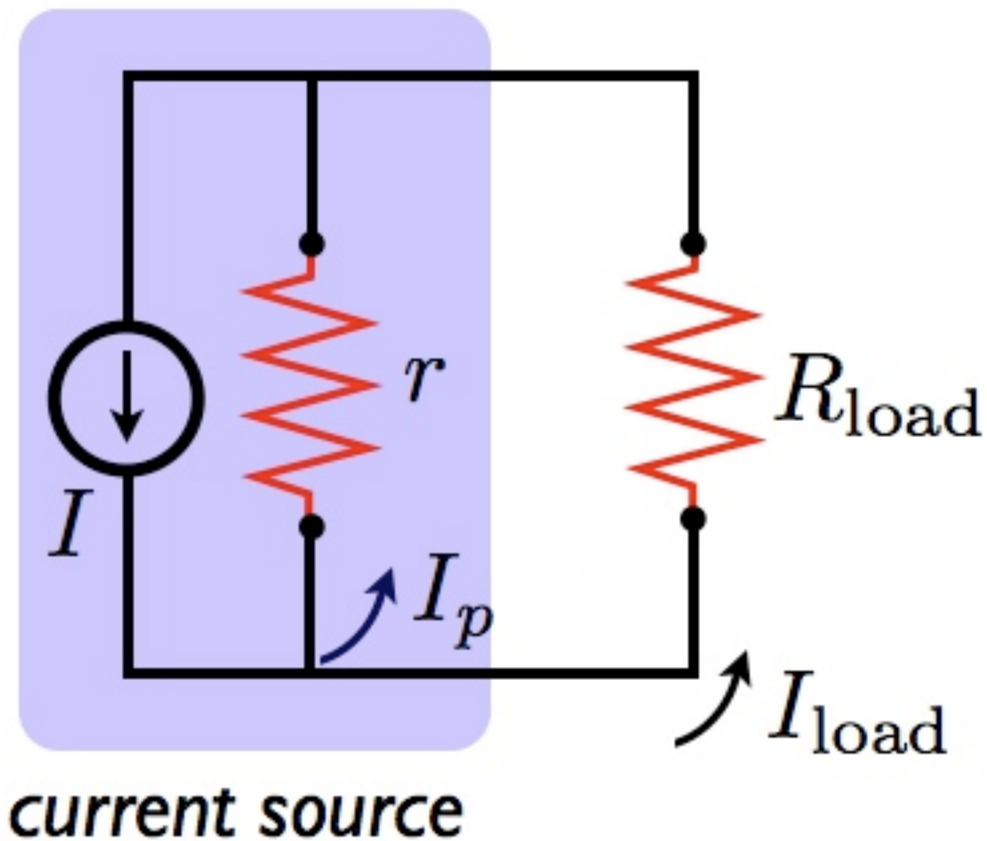
actual circuit has a parasitic r

b)



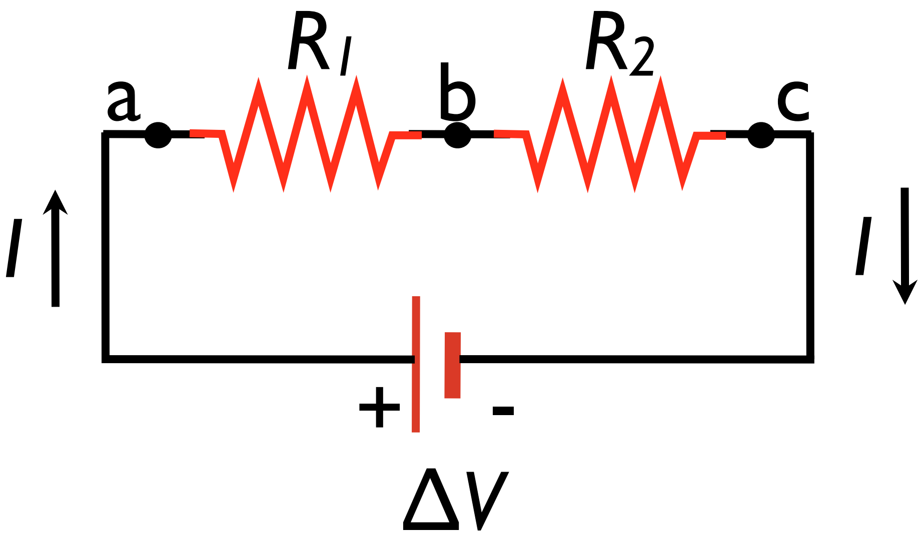
R in series with output
("steals" V)

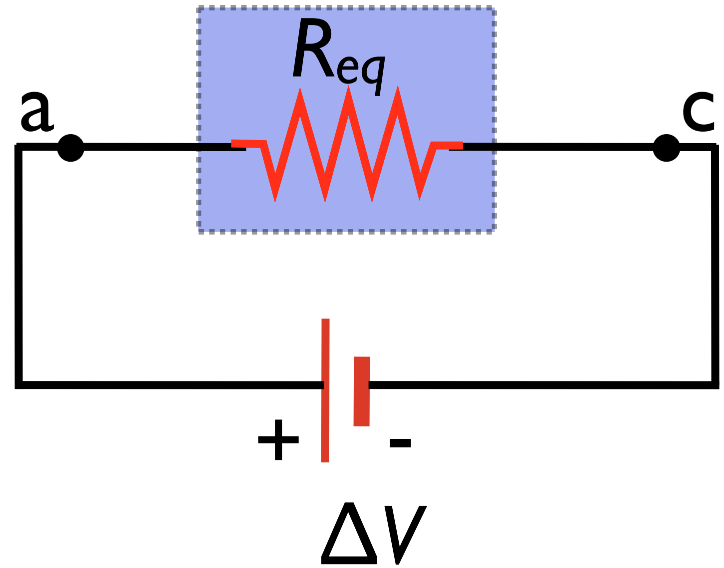
real current sources



R in parallel with output
("steals" I)

series resistors: conservation of energy





Two Resistors in Series:

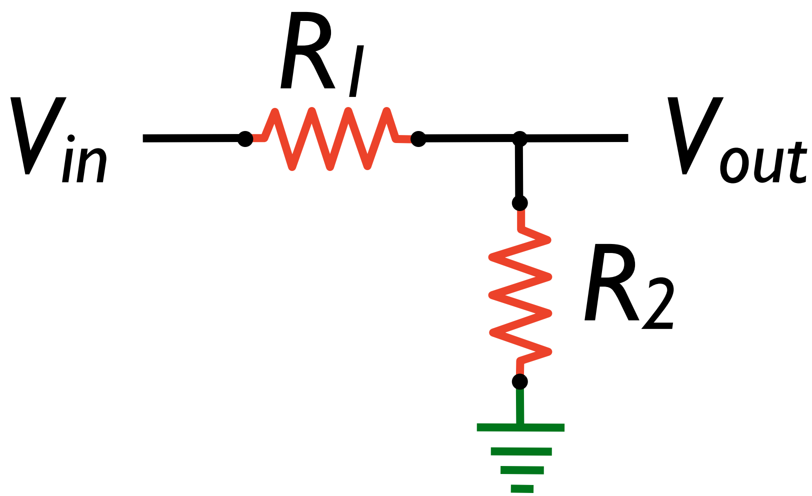
$$R_{eq} = R_1 + R_2$$

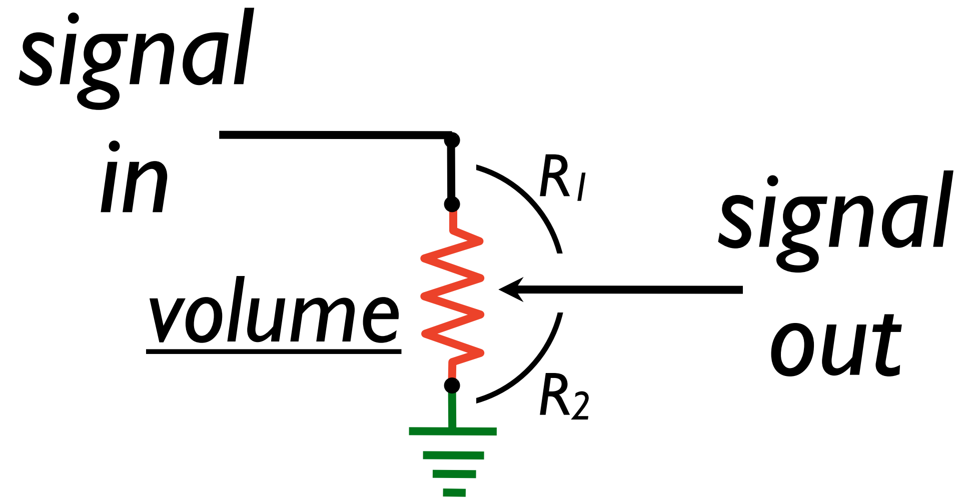
Three or More Resistors in Series:

$$R_{eq} = R_1 + R_2 + R_3 + \dots$$

The current through resistors in series is the same.

voltage divider

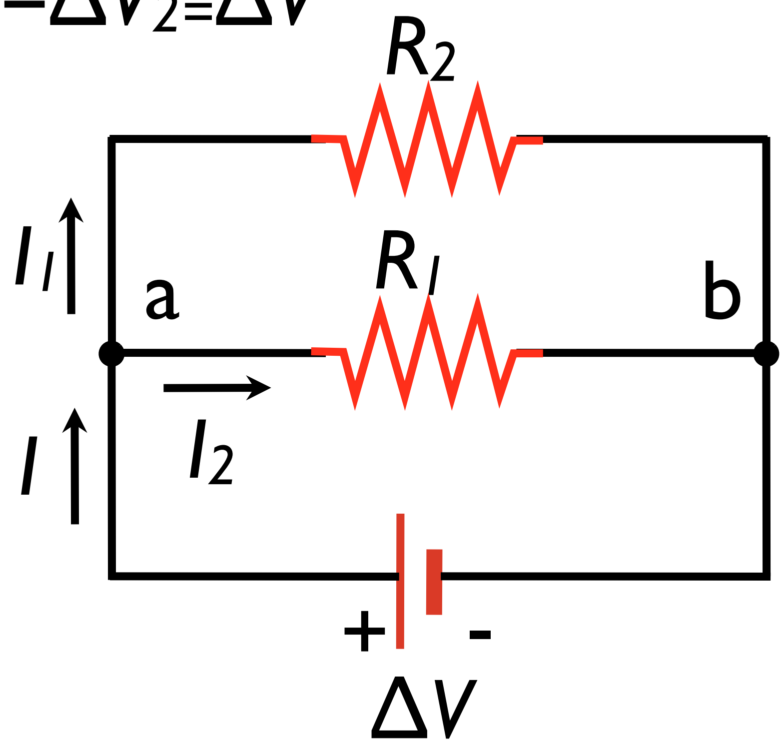




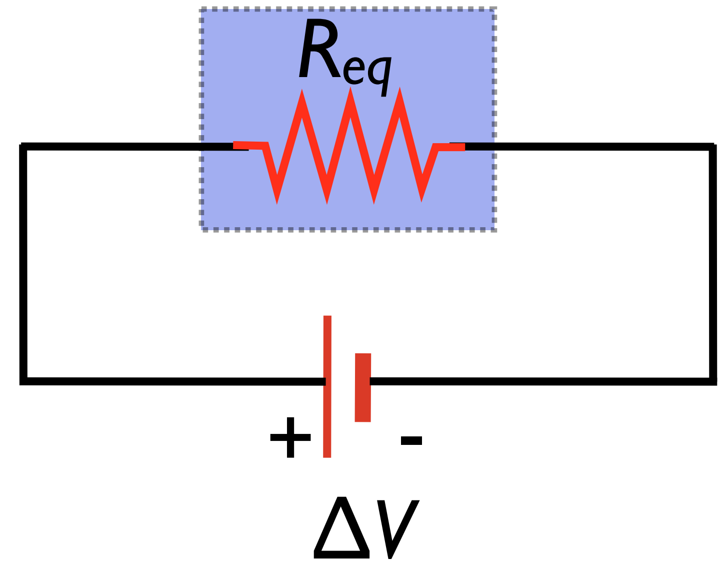
$$V_{out} = \frac{R_2}{R_1+R_2} V_{in}$$

parallel resistors: conservation of charge

$\Delta V_1 = \Delta V_2 = \Delta V$



$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$$



Two Resistors in Parallel:

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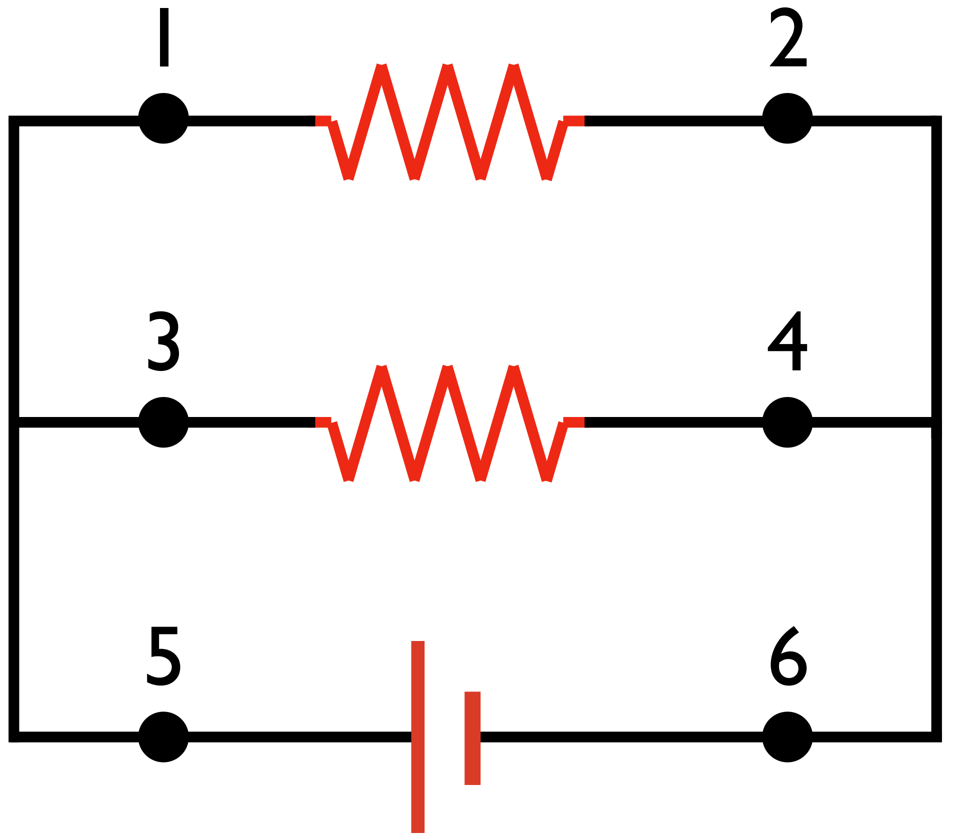
Three or More Resistors in Parallel:

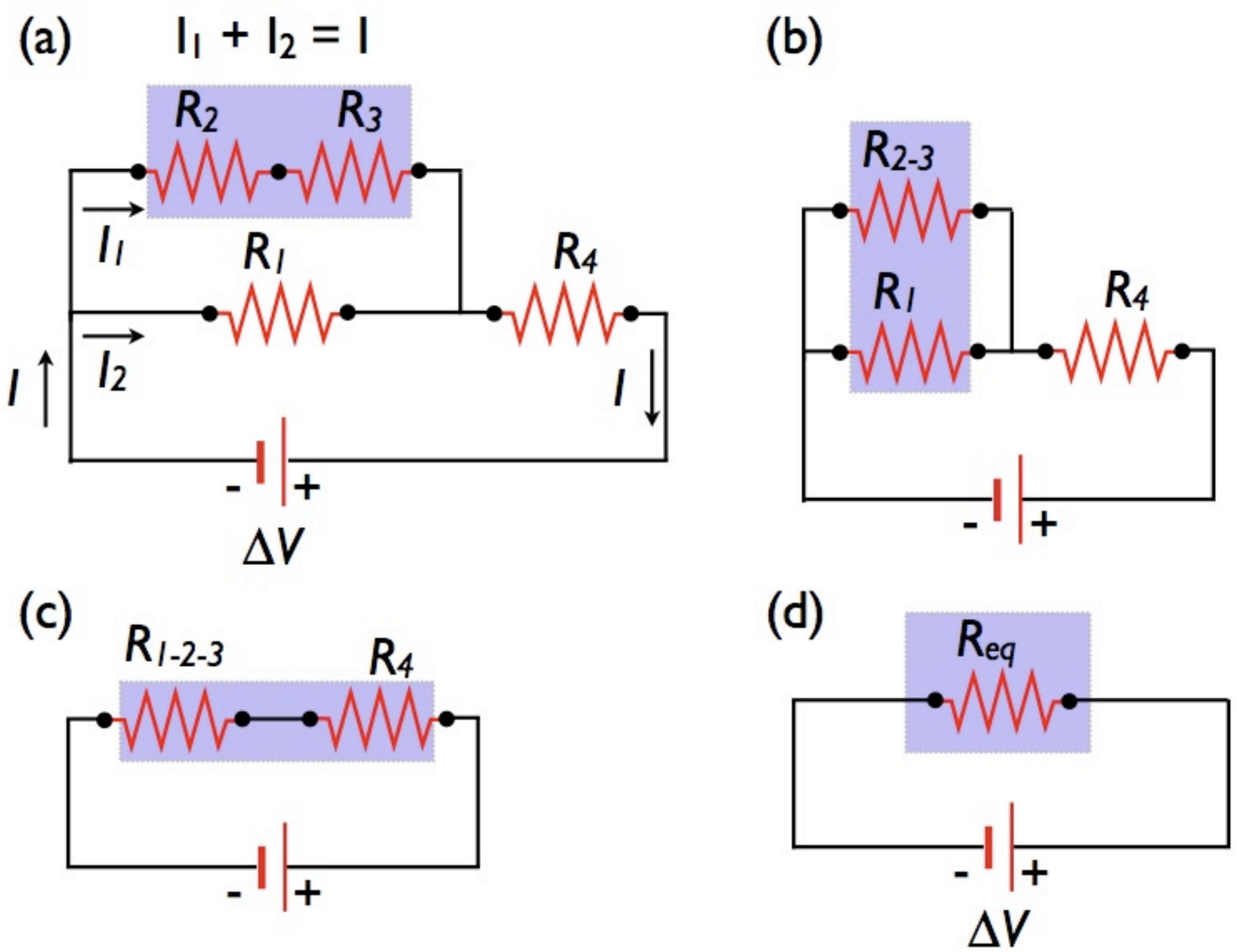
$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots$$

current divider

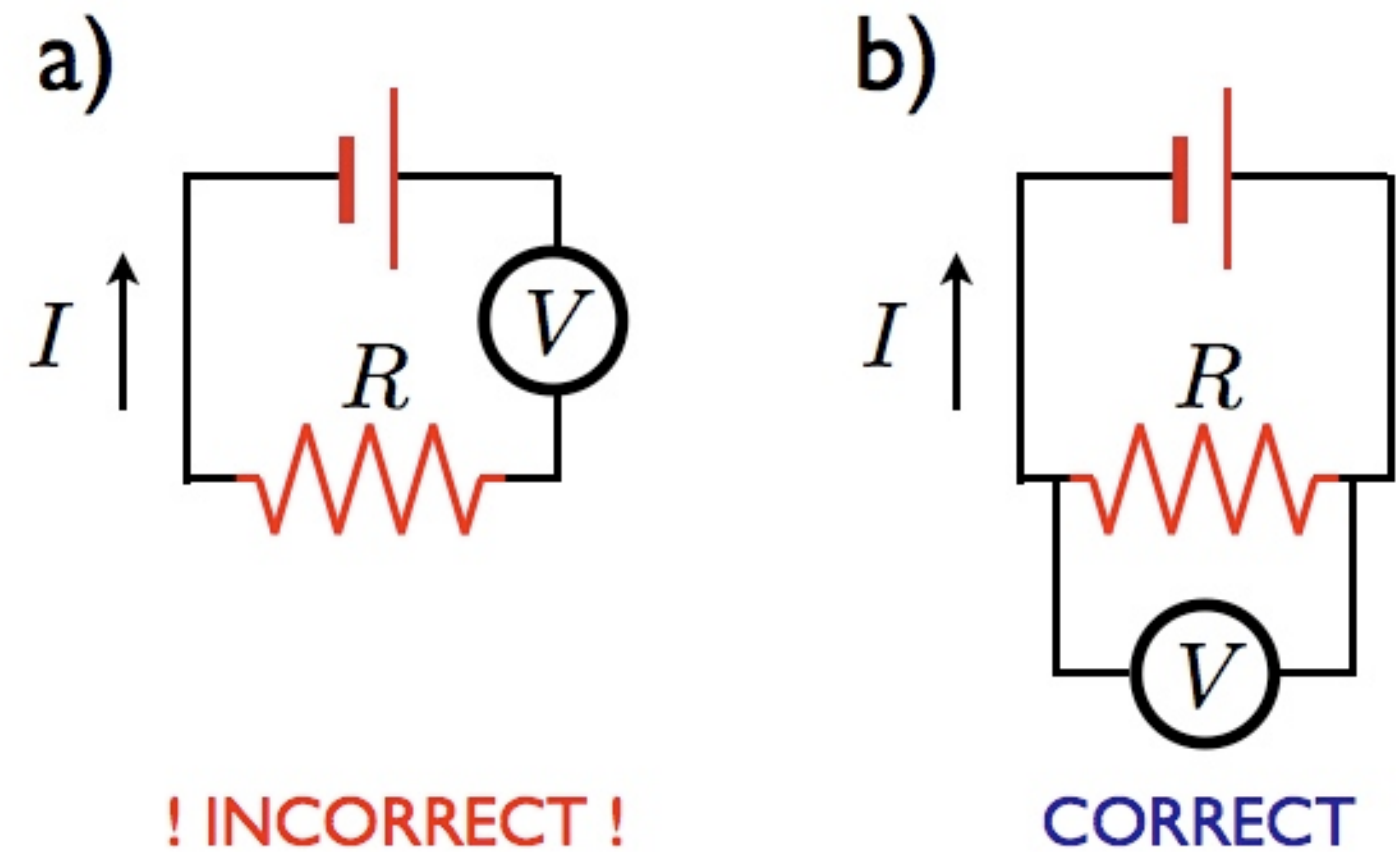
rank the currents



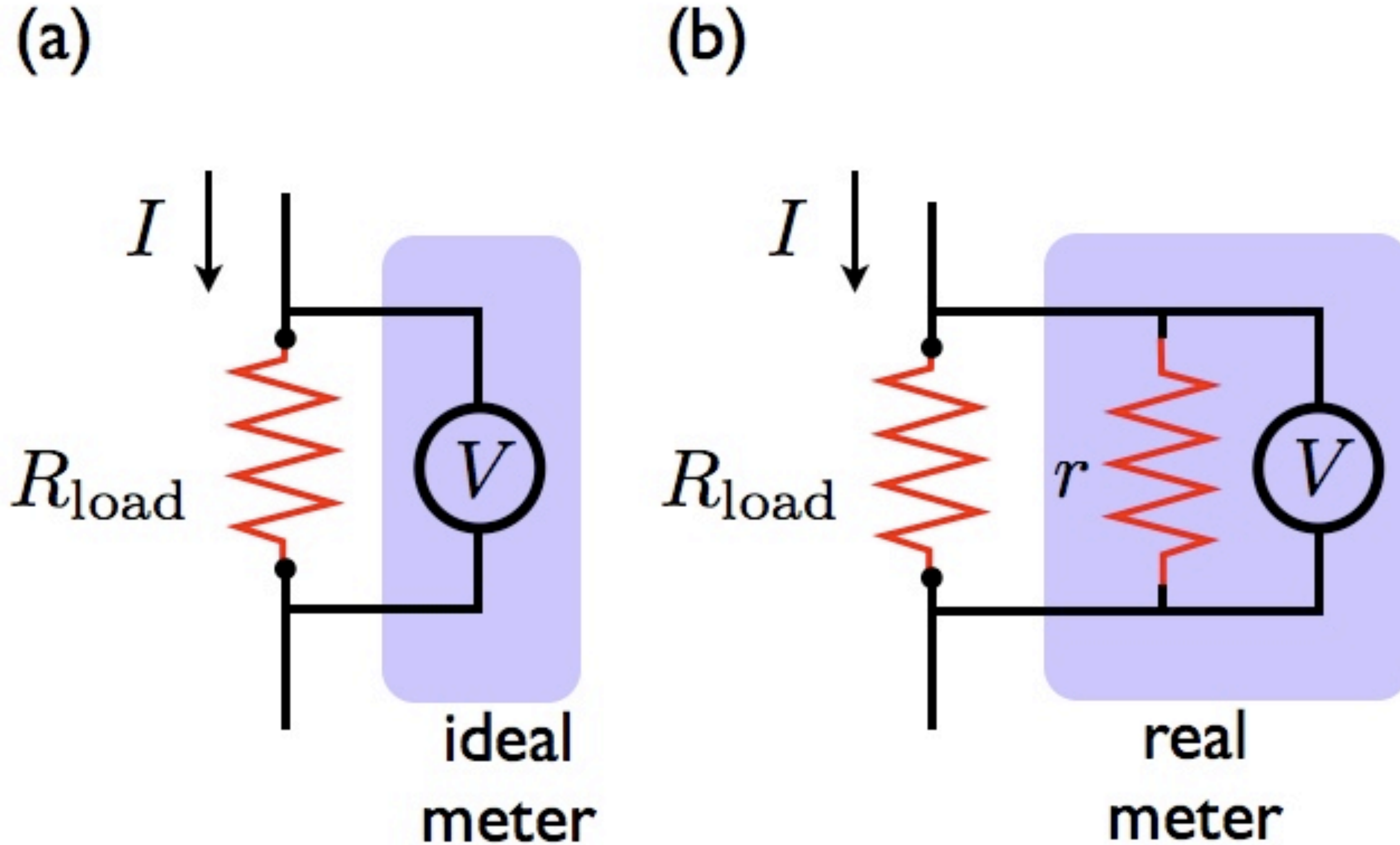
more complex arrangements



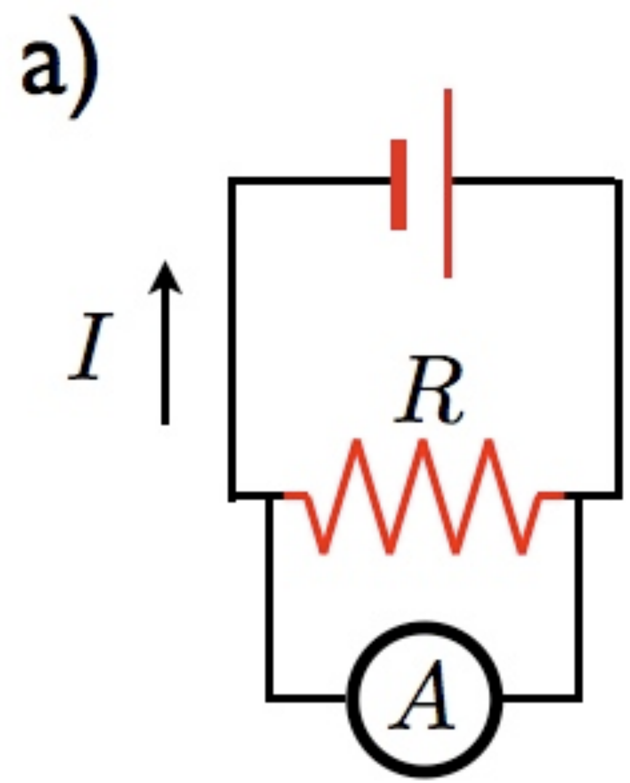
measuring voltage



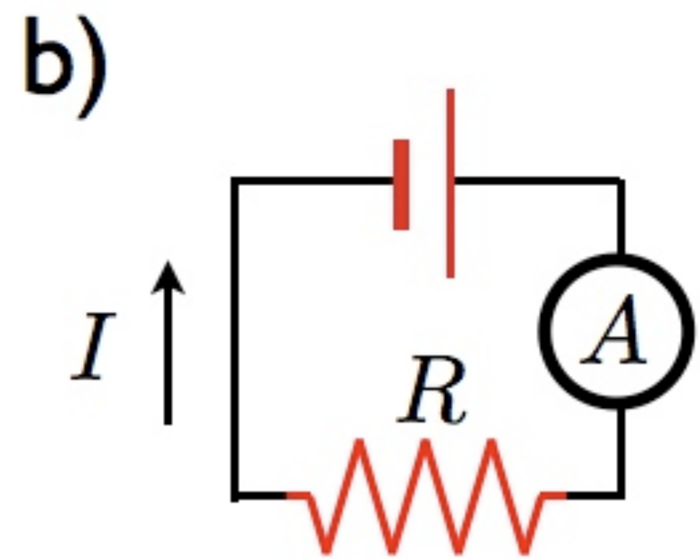
real voltmeters



measuring current

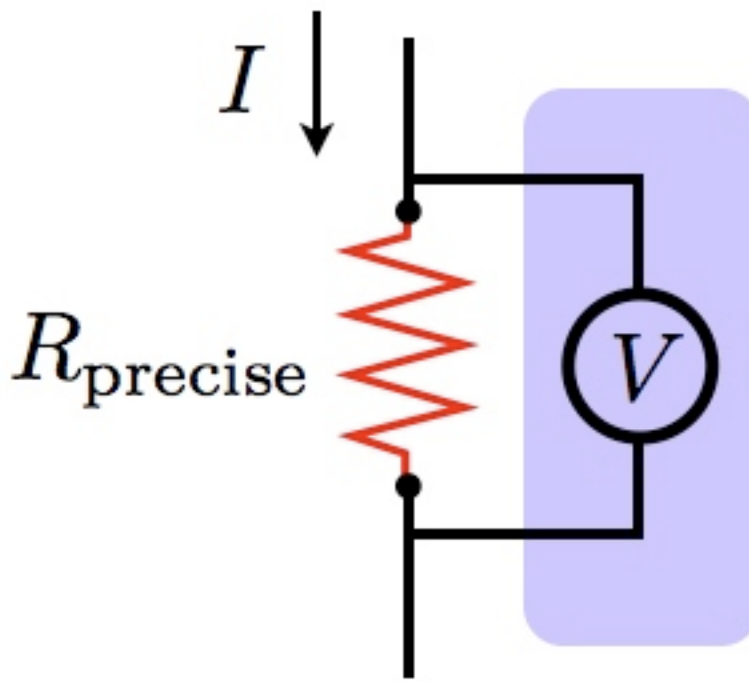


! INCORRECT !



CORRECT

a simple ammeter

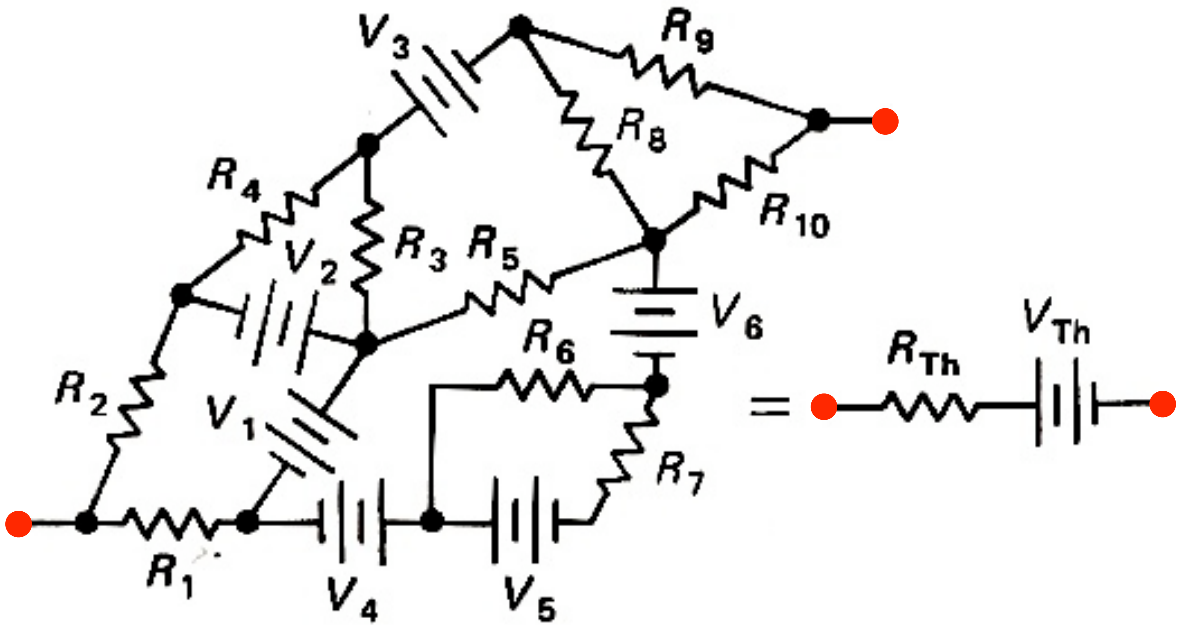


dc Circuits, part II

same thing, just more of it

Thévenin equivalents

This image: Horowitz & Hill, *The art of electronics*



$$V_{th} = V\ (\text{open circuit})$$

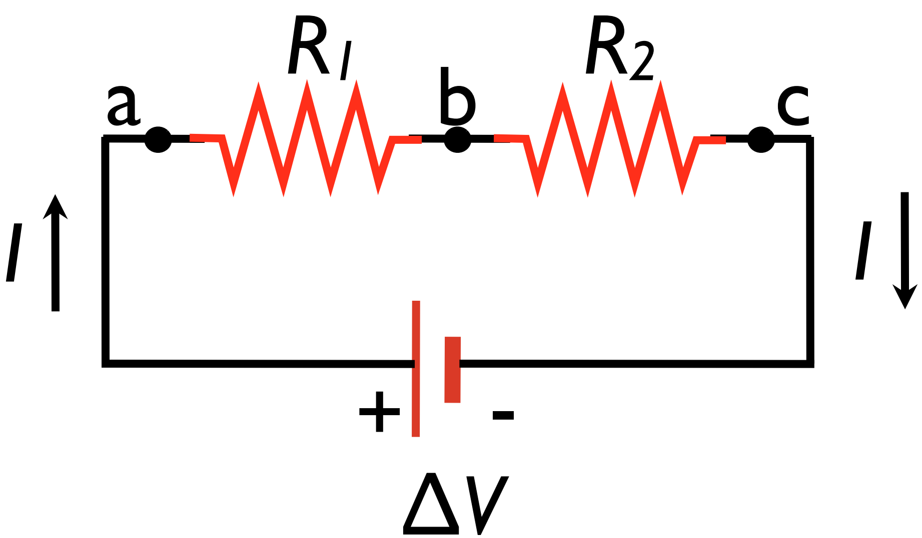
$$R_{th} = \frac{V\ (\text{open circuit})}{I\ (\text{closed circuit})}$$

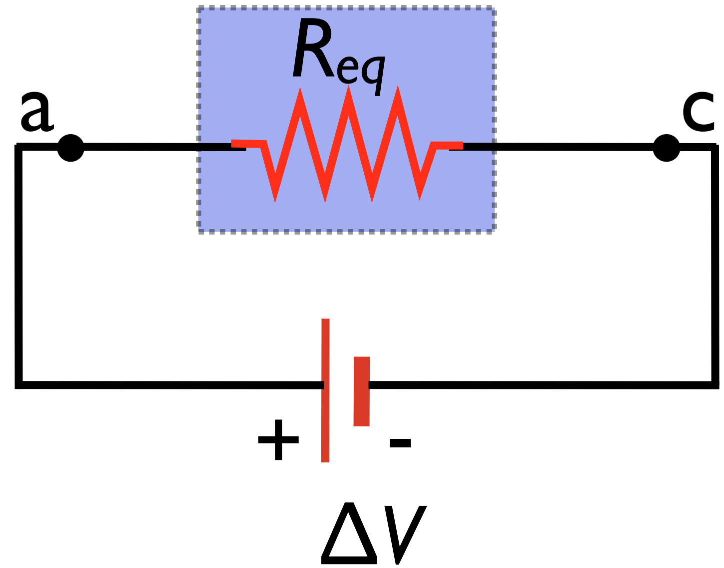
any combination of R's and V's
is equivalent to a SINGLE R and V

disconnect from red dots = open circuit voltage
short red dots, current *there* is closed-circuit current.

(Norton equivalent: a single I source in parallel with R)

series resistors: conservation of energy





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Three or More Resistors in Series:

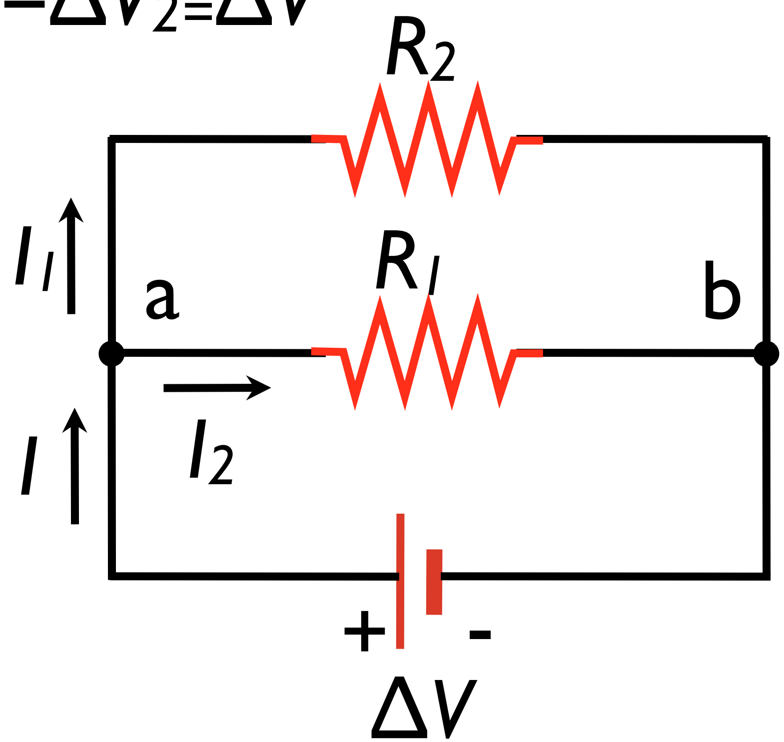
$$R_{eq} = R_1 + R_2 + R_3 + \ldots$$

The current through resistors in series is the same.

source voltage =
sum of voltages
on resistors

parallel resistors: conservation of charge

$\Delta V_1 = \Delta V_2 = \Delta V$



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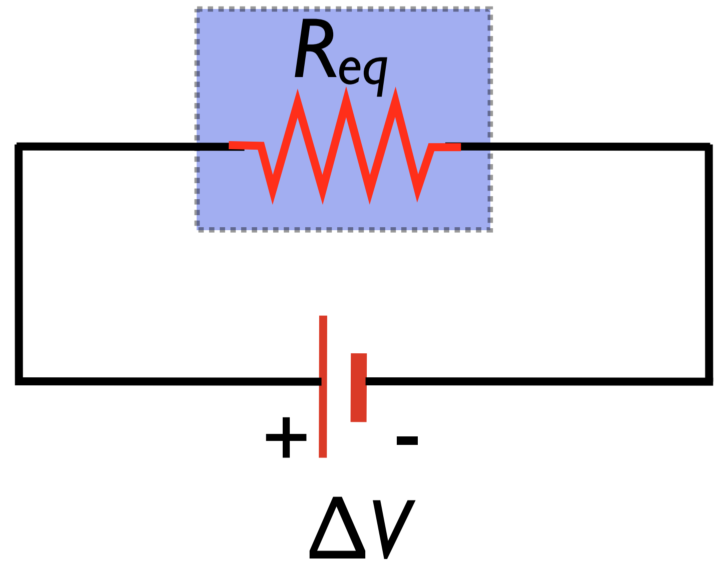


Two Resistors in Parallel:

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$$

Three or More Resistors in Parallel:

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots$$

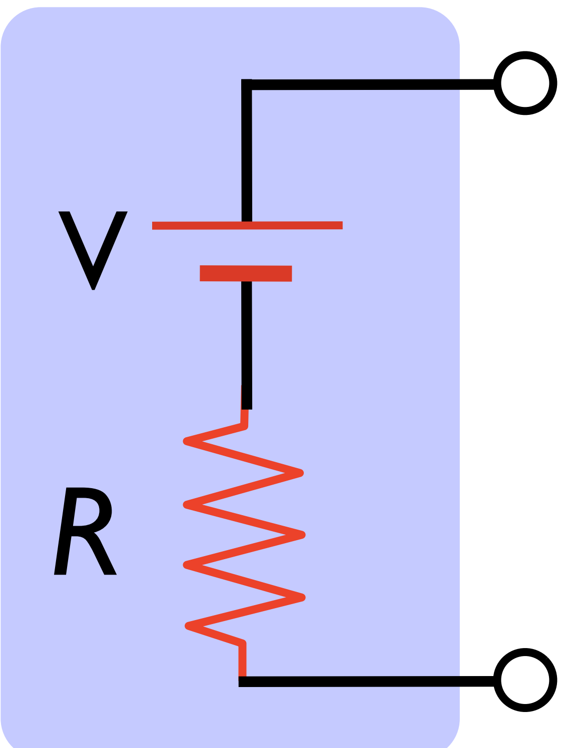
source current =
sum of currents
in resistors

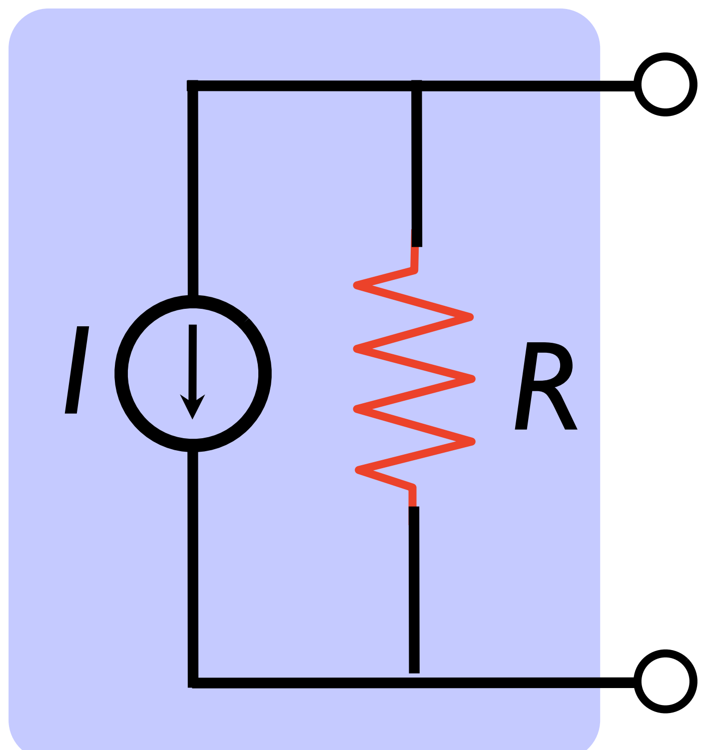
current divider

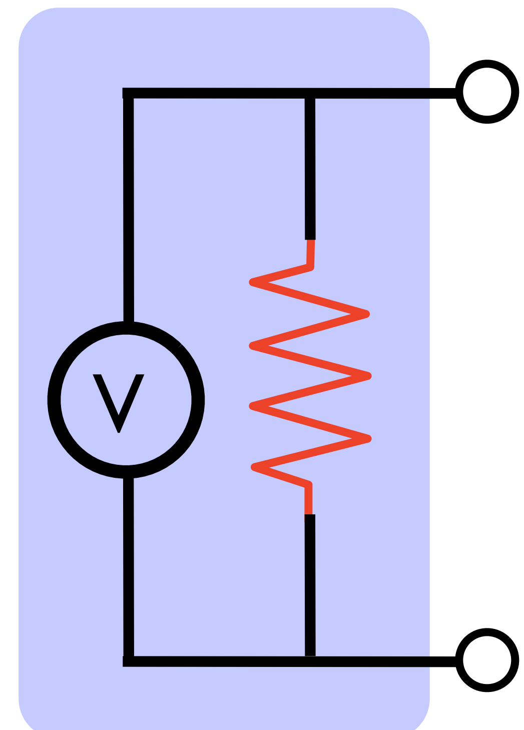
so what?

real sources =
ideal sources + R

real meter =
ideal meter with R

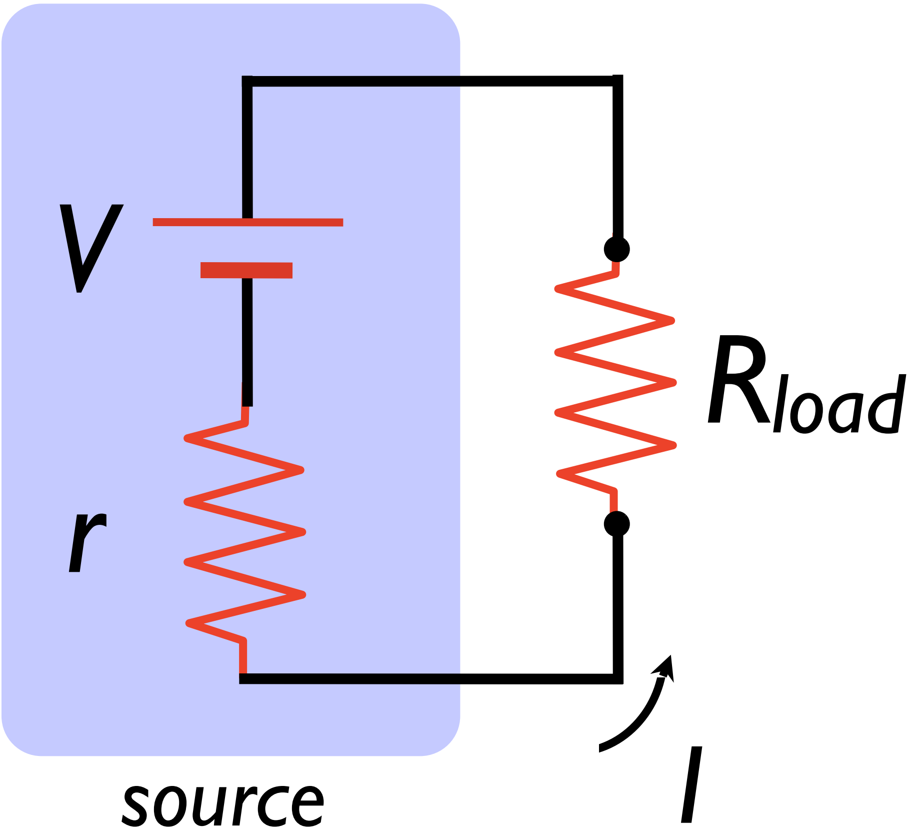






what about
ammeter?

V source loading



extra series resistance

$$\Delta V_{load} = V - Ir$$

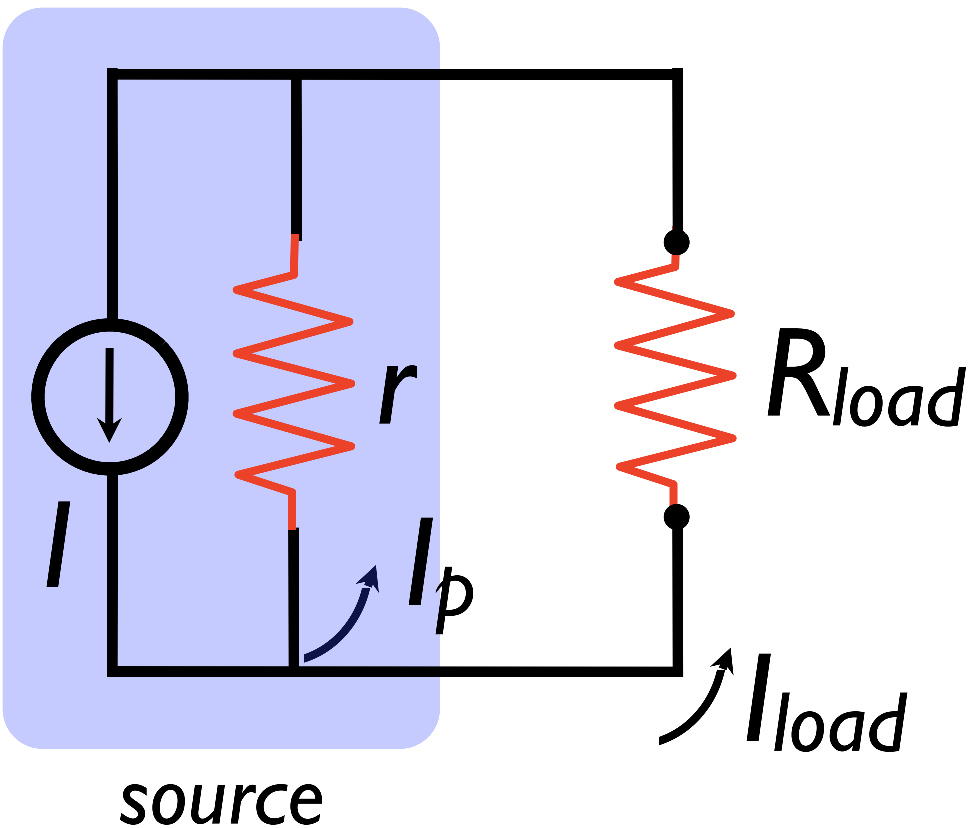
$$\text{for } r \ll R_{load},$$

$$\Delta V_{load} \approx V$$

V source wants R **high**

one solution:
large resistor in parallel with load

I source loading



$$I_{load} = I \frac{r}{r+R}$$

for $R_{load} \ll r$,

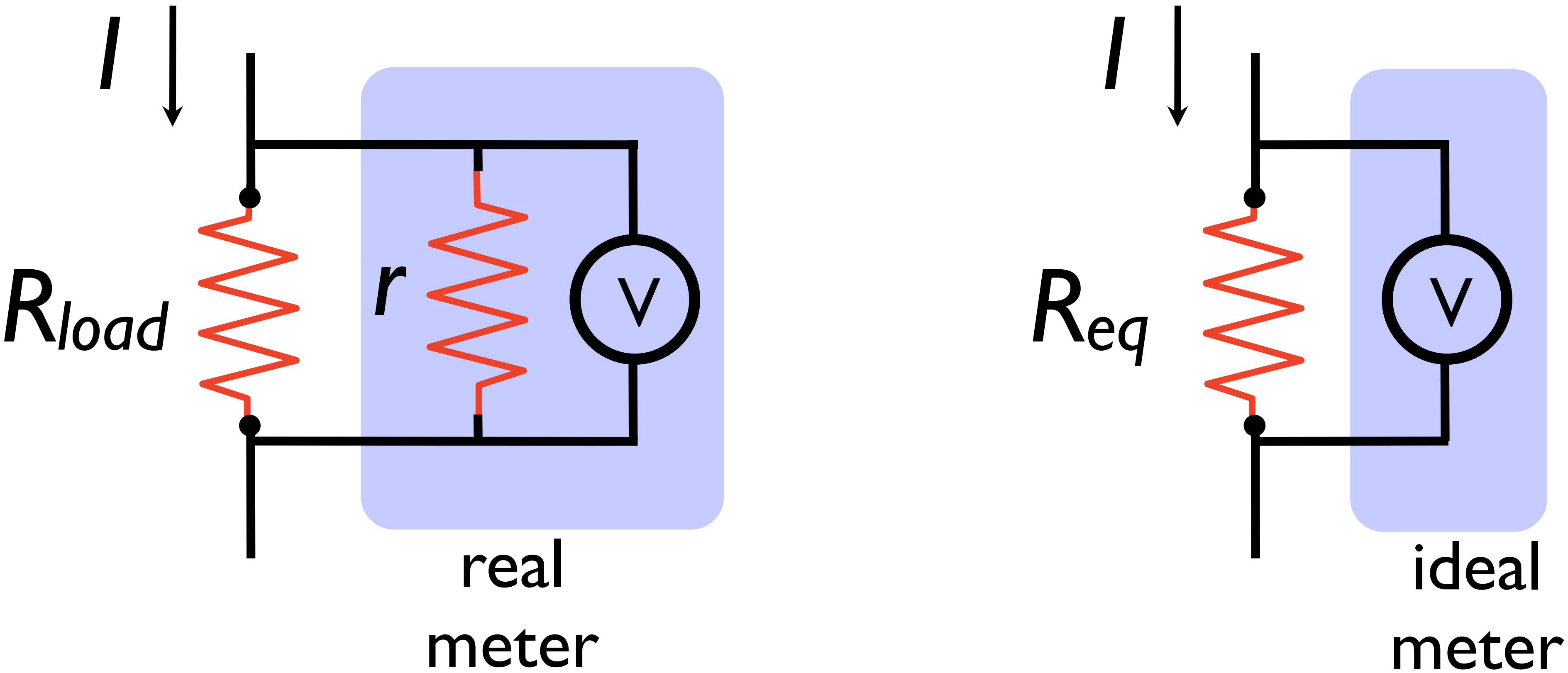
$$I_{load} \approx I$$

extra parallel resistance

I source wants R **low**

sourcing currents at high R_{load} is hard

measuring the meter



$$\Delta V_{load} = IR_{eq} = \frac{R}{1+R/r} I$$

$$R_{load} \ll r, \;\; \Delta V_{load} \approx IR$$

summary

voltmeter wants R **low**!

 can use a buffer/follower ... later

I source wants R **low**

 transformer pre-amp

 consider sourcing V

V source wants R **high**

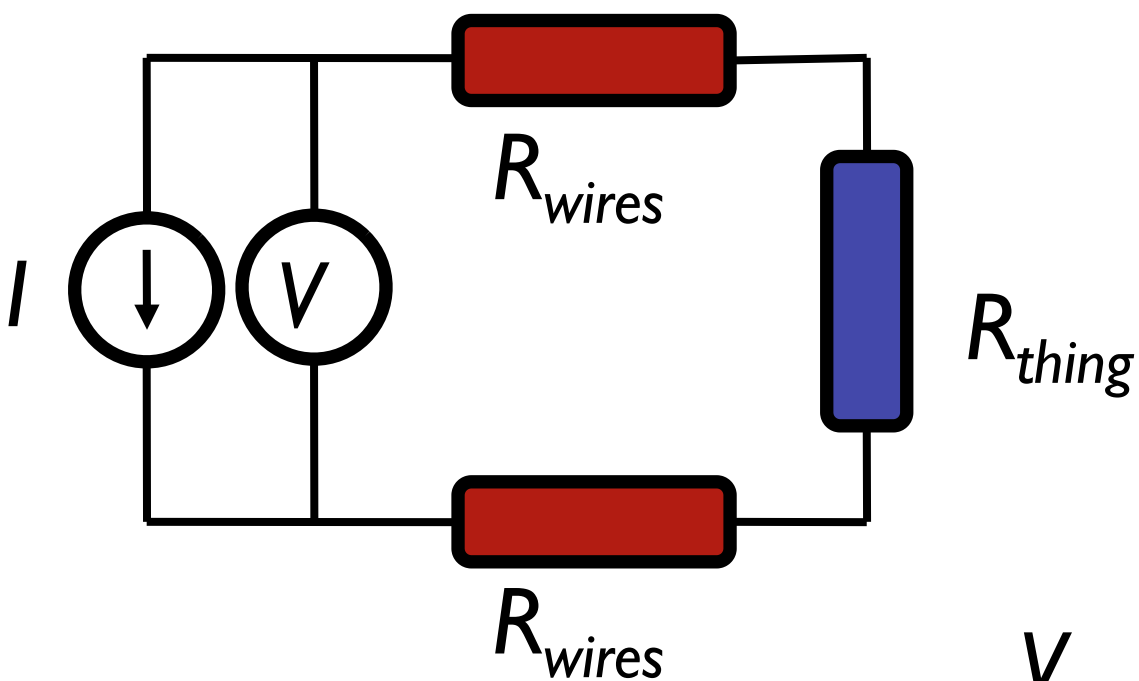
 large series + parallel resistors

 present large R

Sourcing current

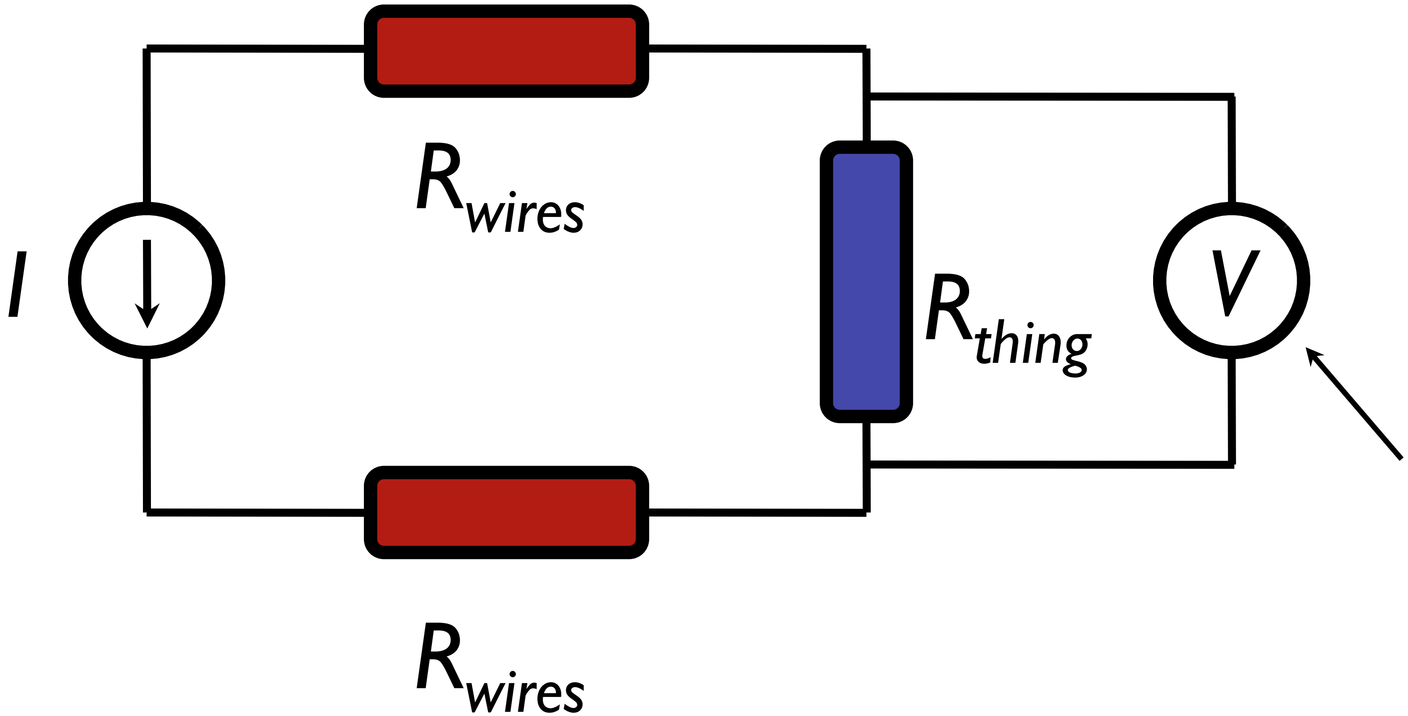
This is what a hand meter does.

Why is it no good?



$$V_{meter} = I(R_{thing} + 2R_{wires})$$

Sourcing current, properly

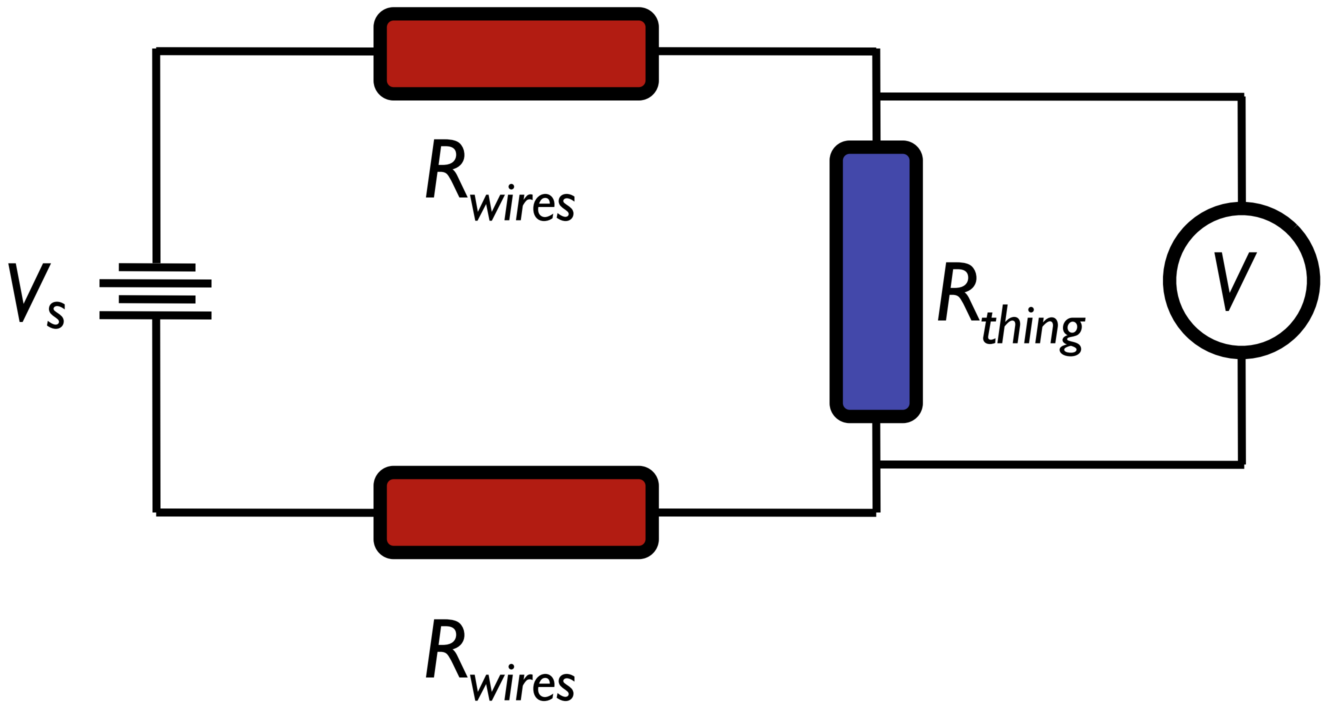


$R_{thing} \ll R_{internal}$

or add buffer

No problem.
You just need four wires.

Sourcing voltage

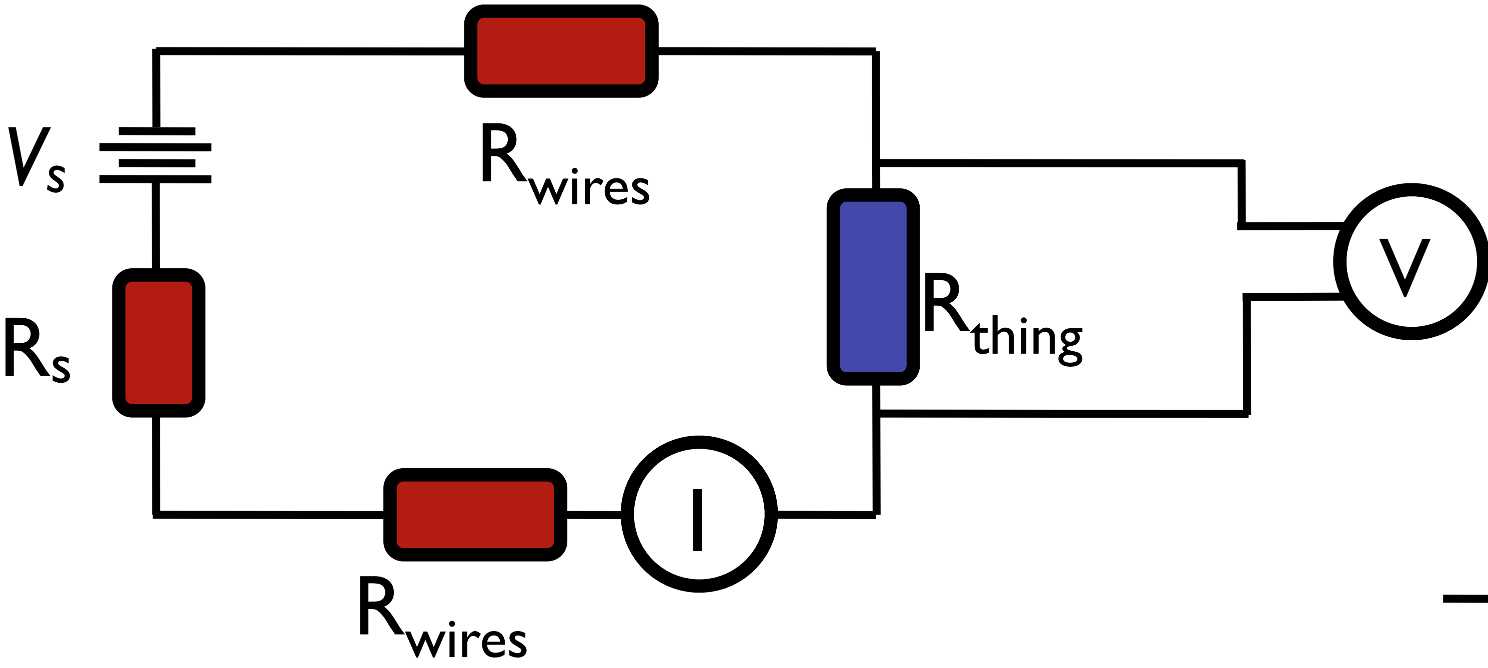


Still have to measure voltage on device

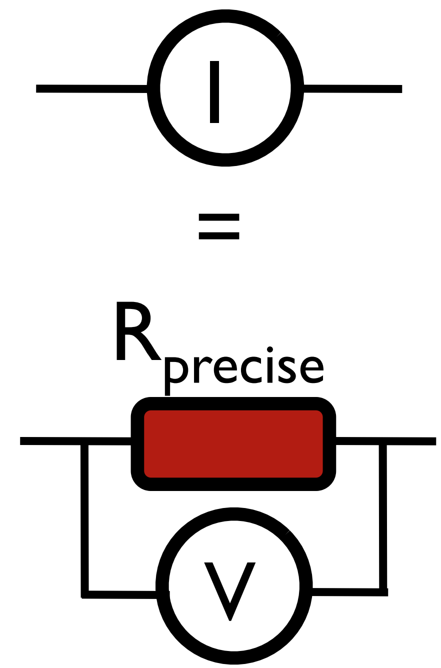
the wires still use up some of V

What about current?

Sourcing voltage better



$R = \Delta V / I$



Note we need 4 wires again
current meter - not hard
still problems?

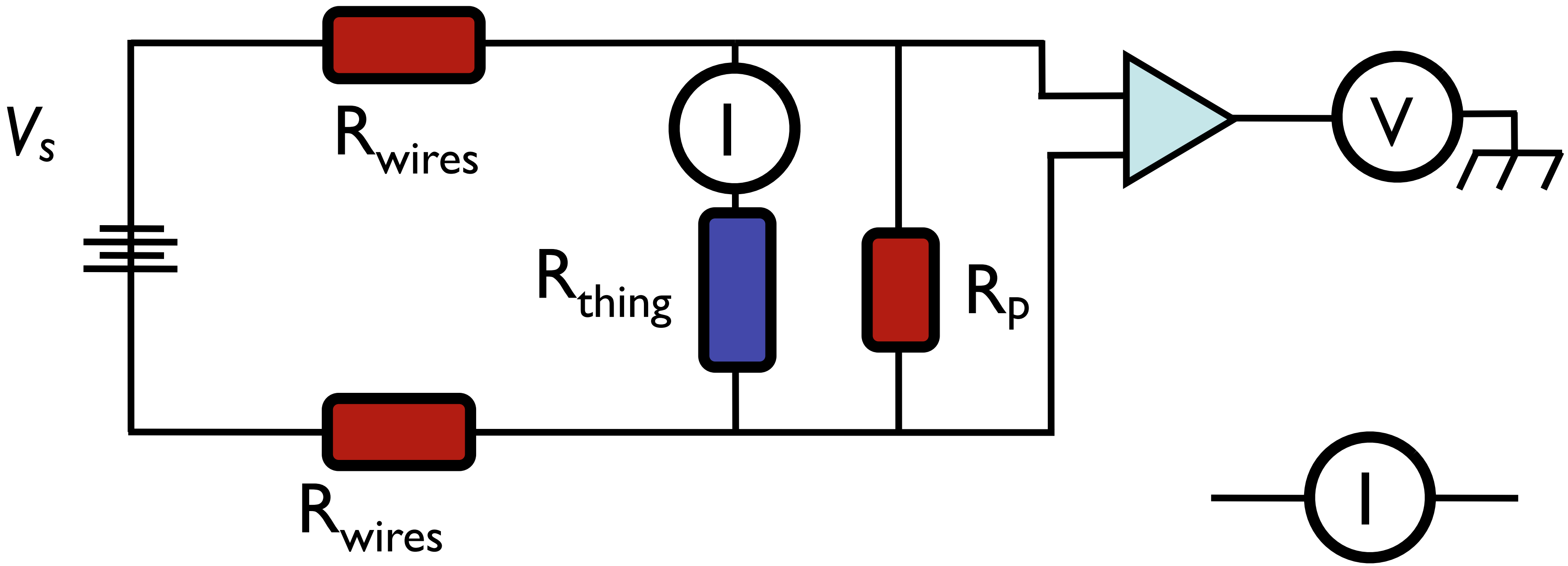
source/meter resistances

voltmeter wants R low
but V source wants R high

need buffer/amp on V meter
resistor in parallel with source

if V source is problem, R is too low
consider sourcing I

what if I want to measure a *really* high R?

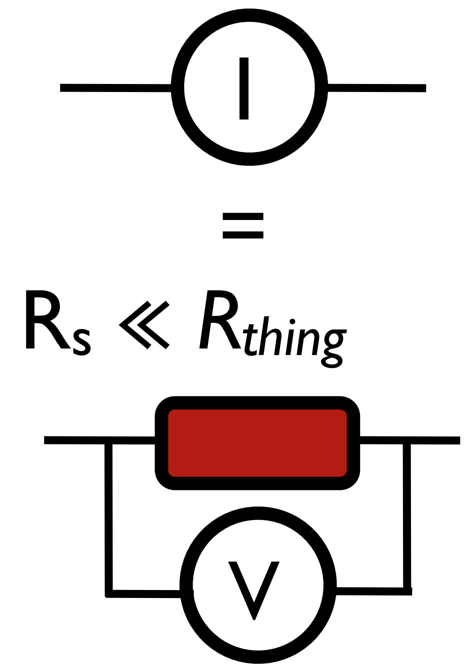


source *voltage*

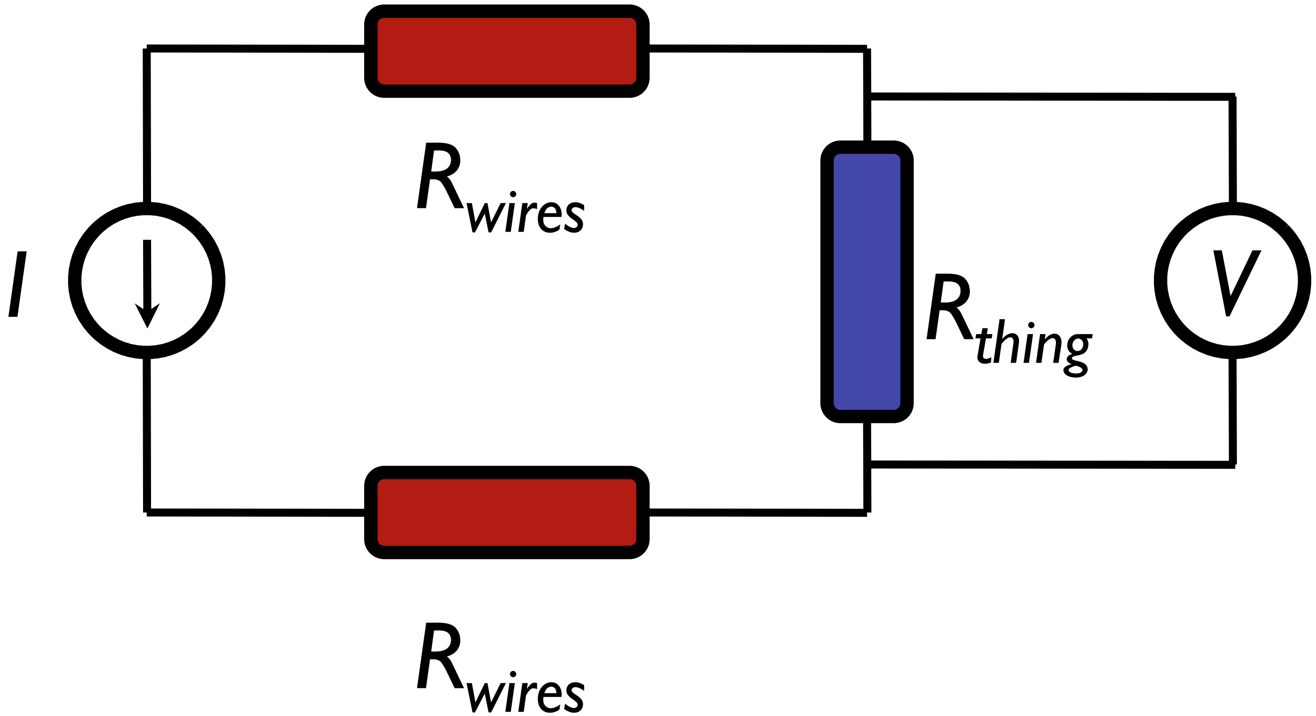
R_p has same voltage as R_{thing}

R_s has same current

have done $>10^{10}$ Ohm

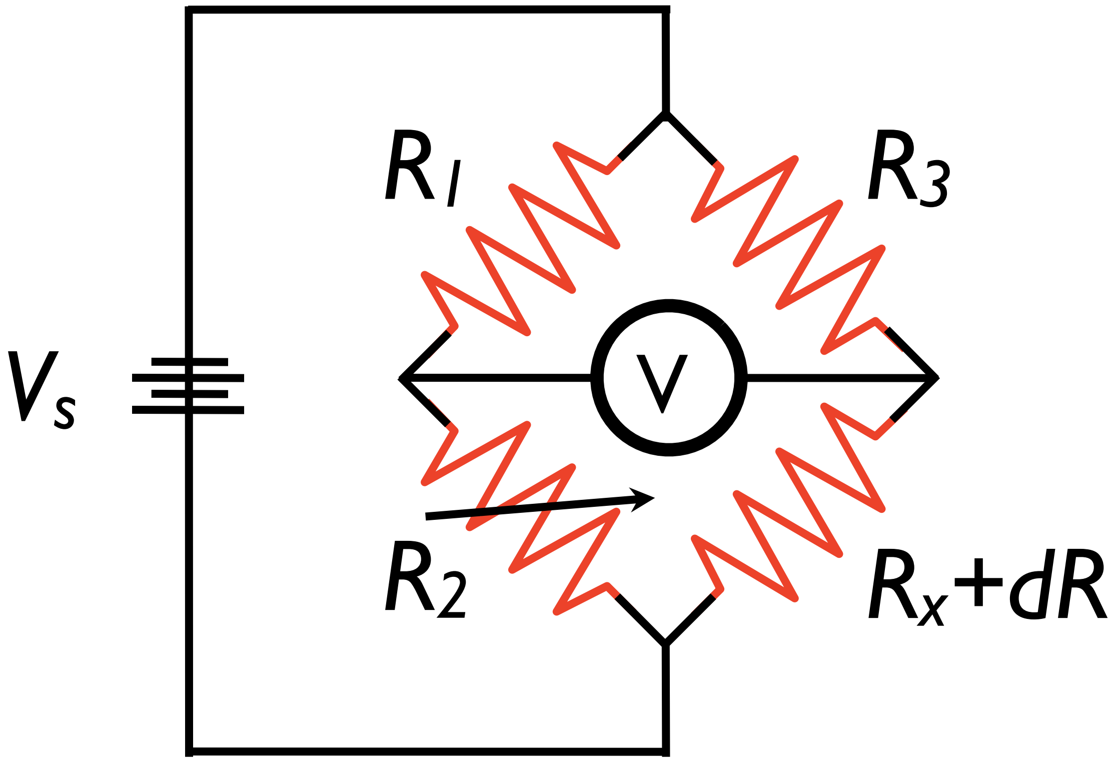


what if I want to measure a *really* *low* R?



this works just fine ...
so long as your V meter is good or you can tolerate large I
v. good amp / part of a bridge

what if I want to measure a small change in R?



balance bridge to $V=0$
detect small changes from null

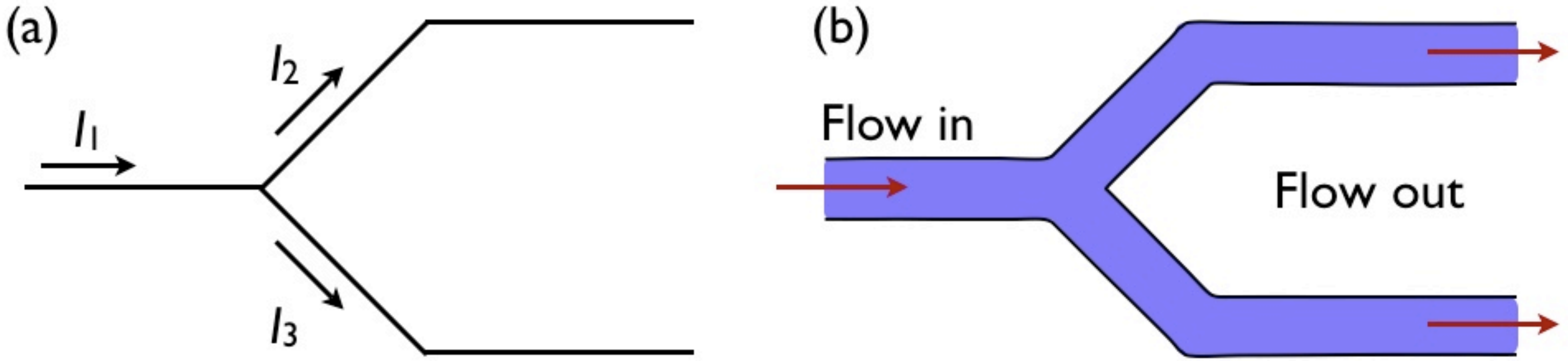
$R_2 =$  $\approx R_3$

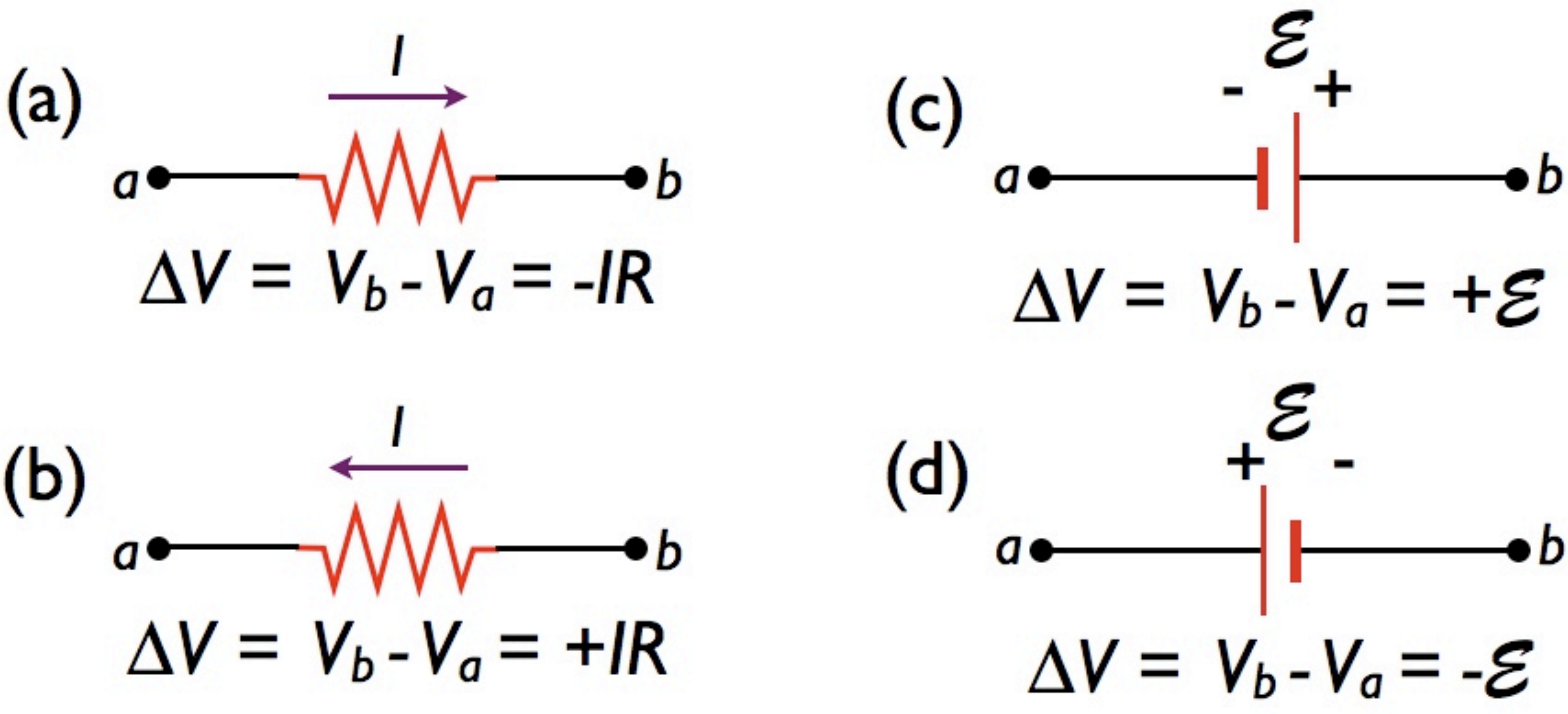
make R_1-R_3 *about the same*
trimming resistor on $R_2 = dR$

$$V = \left(\frac{R_x}{R_3+R_x} - \frac{R_2}{R_1+R_2}\right)V_s$$

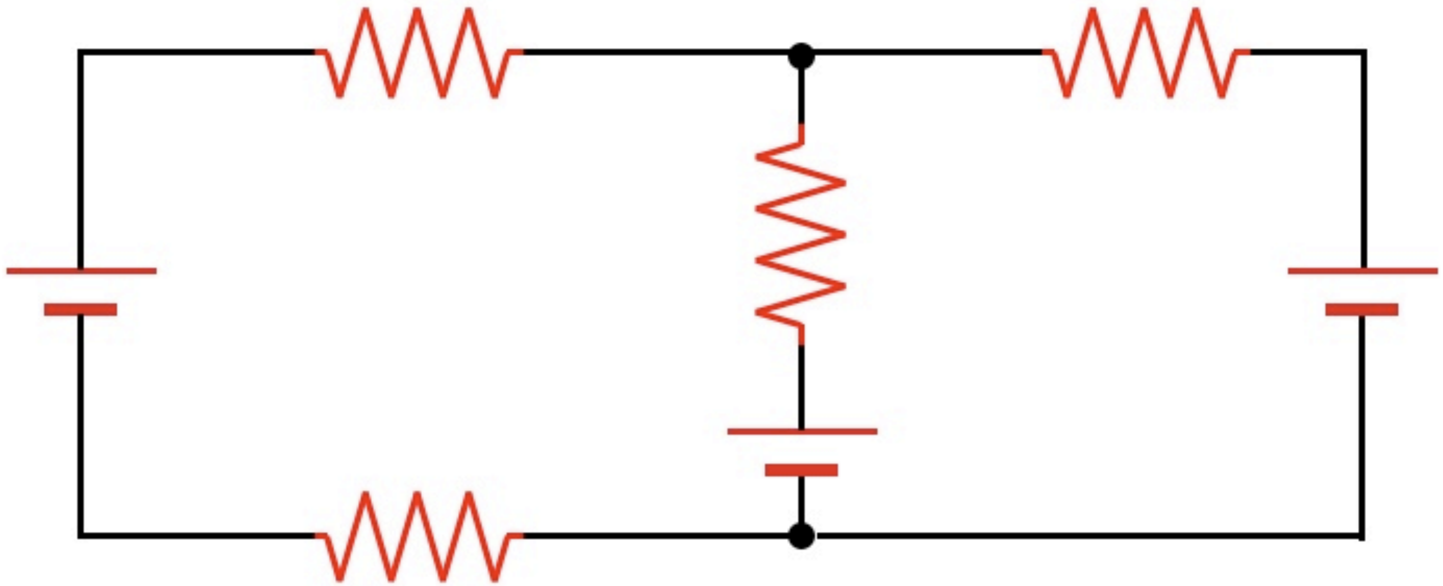
$$R_x = \frac{R_3 R_2}{R_1}$$

Rules for analyzing more complicated circuits

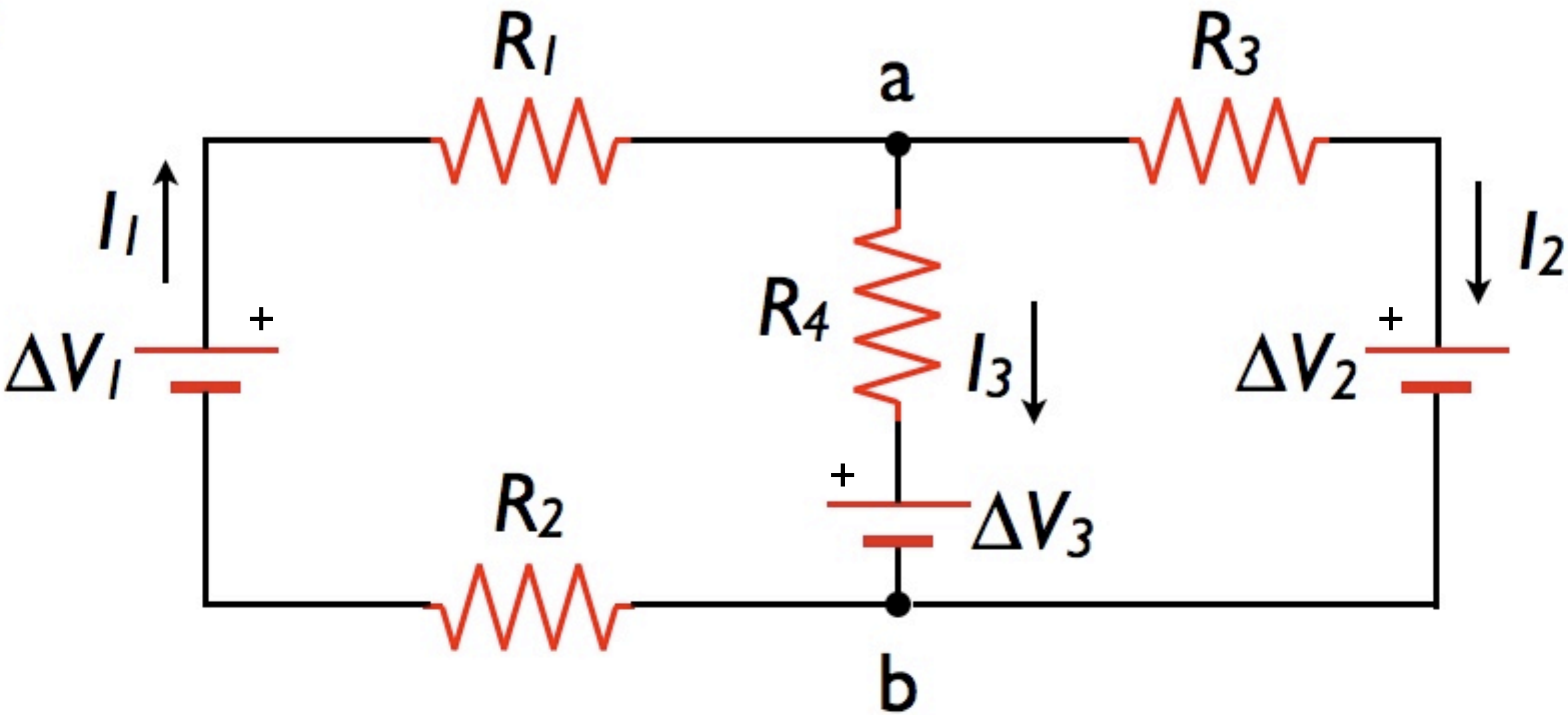

(a)
I_1
I_2
I_3
(b)
Flow in
Flow out


(a)
I
a
b
ΔV = V_b - V_a = -IR
(b)
I
a
b
ΔV = V_b - V_a = +IR
(c)
ℰ
- +
a
b
ΔV = V_b - V_a = +ℰ
(d)
ℰ
+ -
a
b
ΔV = V_b - V_a = -ℰ

(a)



(b)



capacitors

Definition of Capacitance: the capacitance C is the ratio of the charge stored on one conductor (or the other) to the potential difference between the conductors:

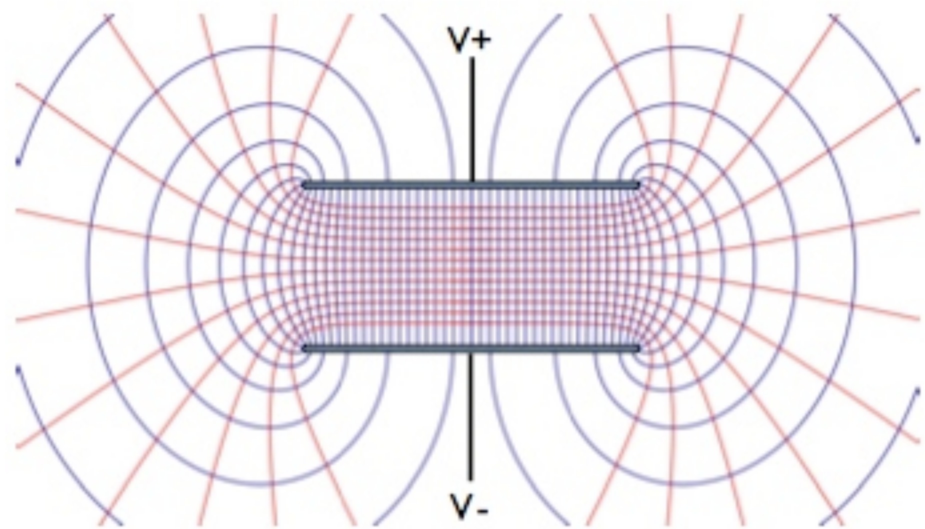
$$C \equiv \frac{|Q|}{|\Delta V|} \qquad (4.12)$$

frequency-dependent resistor

I and *V* are 90° out of phase

can't dissipate power, ideally

$$I = \frac{dQ}{dt} = \frac{d(CV)}{dt} \rightarrow C\frac{dV}{dt}$$



Capacitance of a parallel plate capacitor:

$$C = \epsilon_0 \frac{A}{d}$$

where d is the spacing between the plates, and A is the area of the plates.

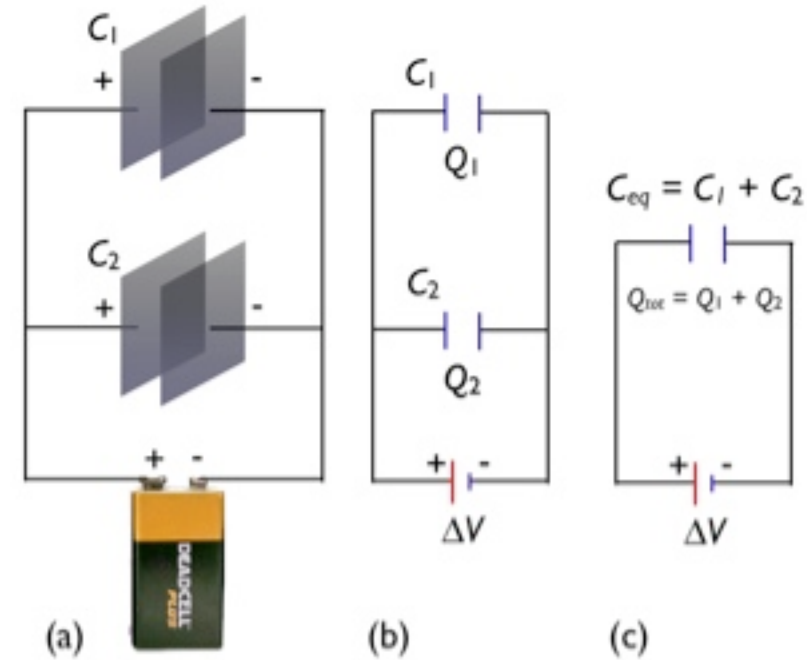
combinations of capacitors

Two Capacitors in Parallel:

$$C_{eq} = C_1 + C_2$$

Three or More Capacitors in Parallel:

$$C_{eq} = C_1 + C_2 + C_3 + \ldots$$



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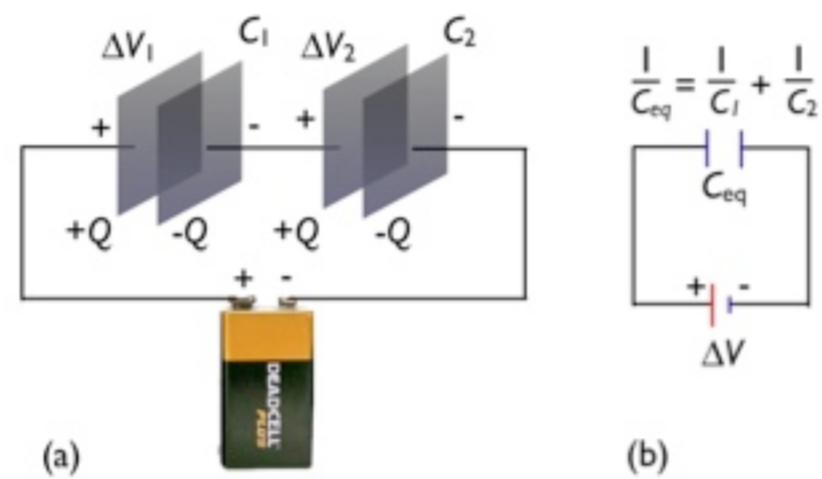
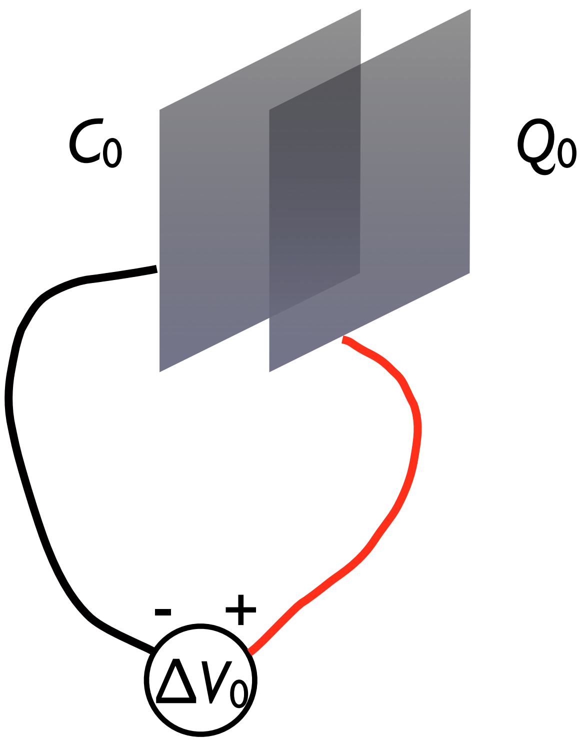
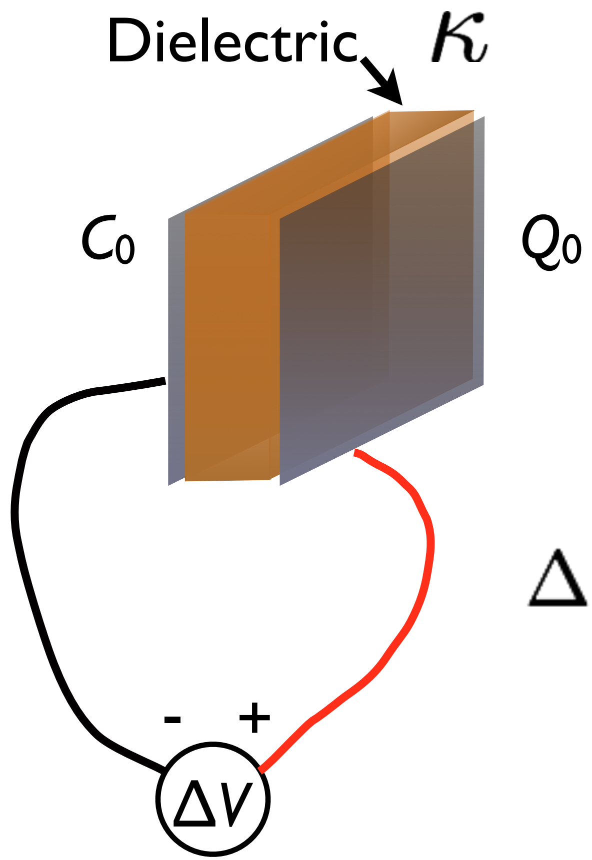
Three or More Capacitors in Series:

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} \ldots$$



capacitors with stuff inside





$$\Delta V = \frac{\Delta V_0}{\kappa} = \frac{\Delta V_0}{\epsilon_r}$$

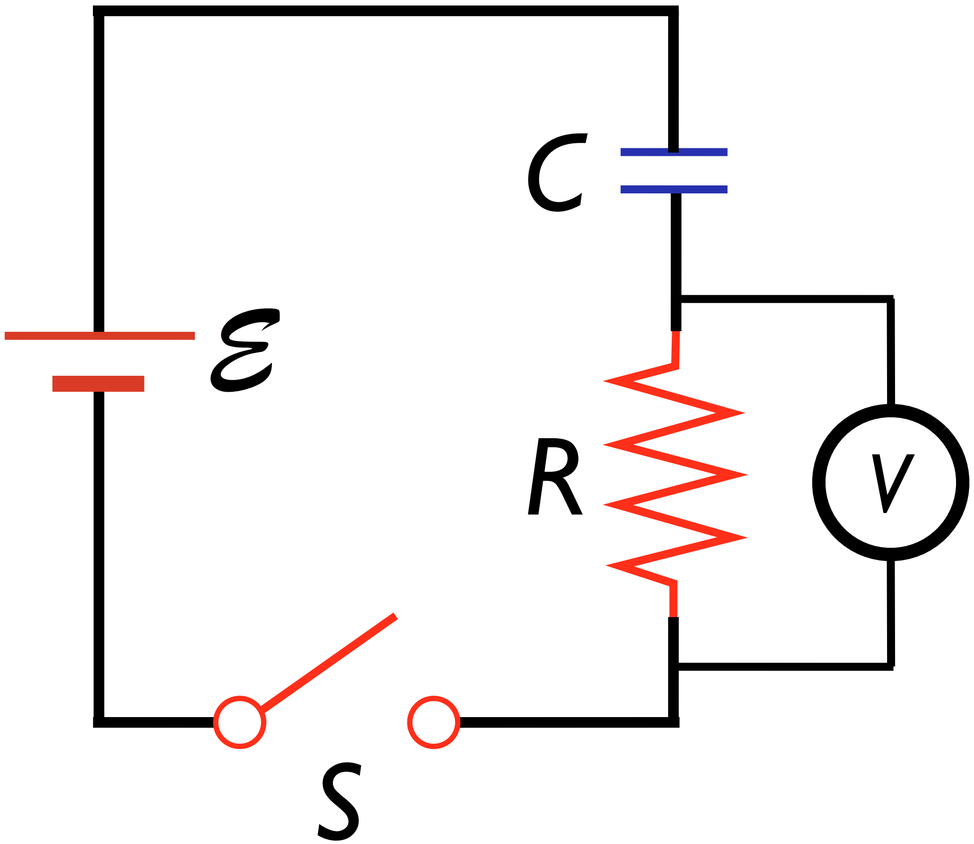
$$C = \frac{Q_0}{\Delta V} = \frac{\kappa Q_0}{\Delta V_0}$$

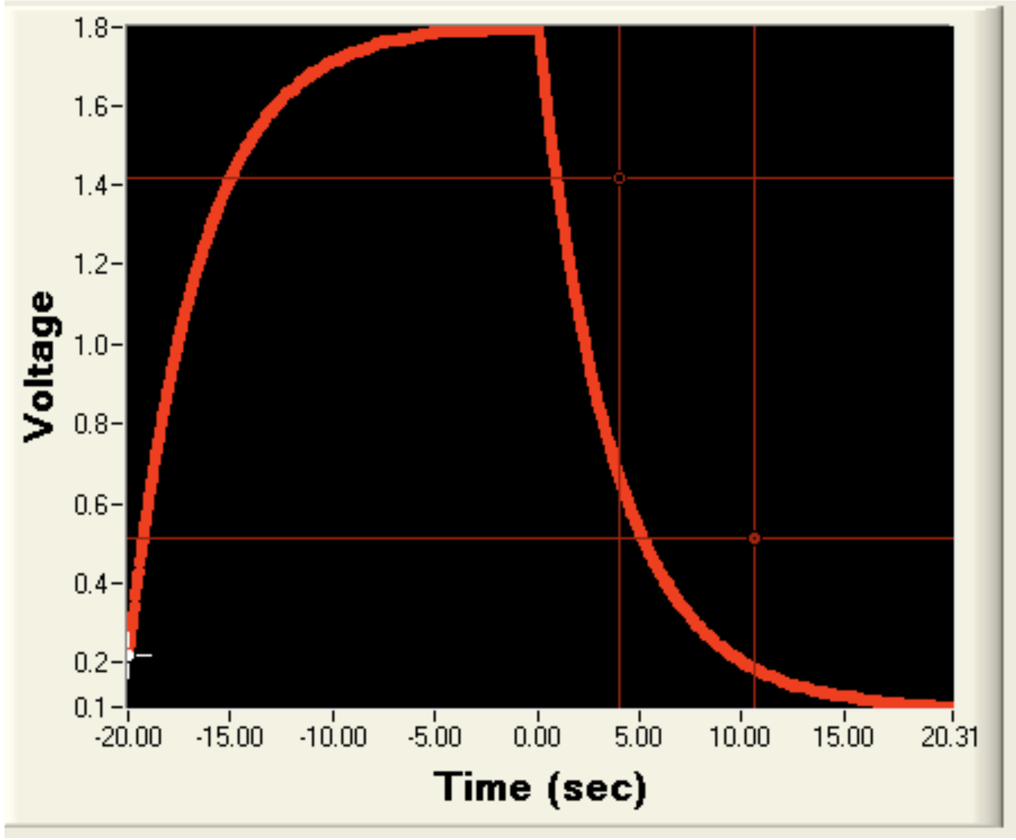
Parallel plate capacitor with a dielectric between the plates:

$$C = \kappa\epsilon_0\frac{A}{d} = \epsilon_r\epsilon_0\frac{A}{d} \qquad (4.29)$$

the dielectric *increases* the capacitance by a factor κ, the dielectric constant. The dielectric constant is also written ϵ_r sometimes.

rc circuits





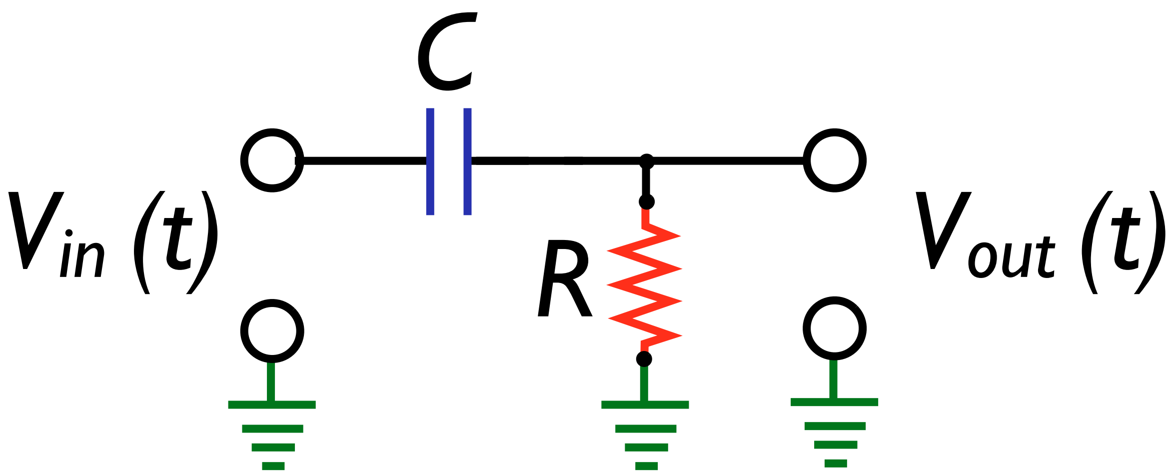
$$V \propto e^{-\tau/RC}$$

Time constant τ **of an** RC **circuit:**

$$\tau = RC \quad (6.27)$$

This gives τ in seconds [s] when R is in Ohms [Ω] and C is in farads [F].

RC differentiator



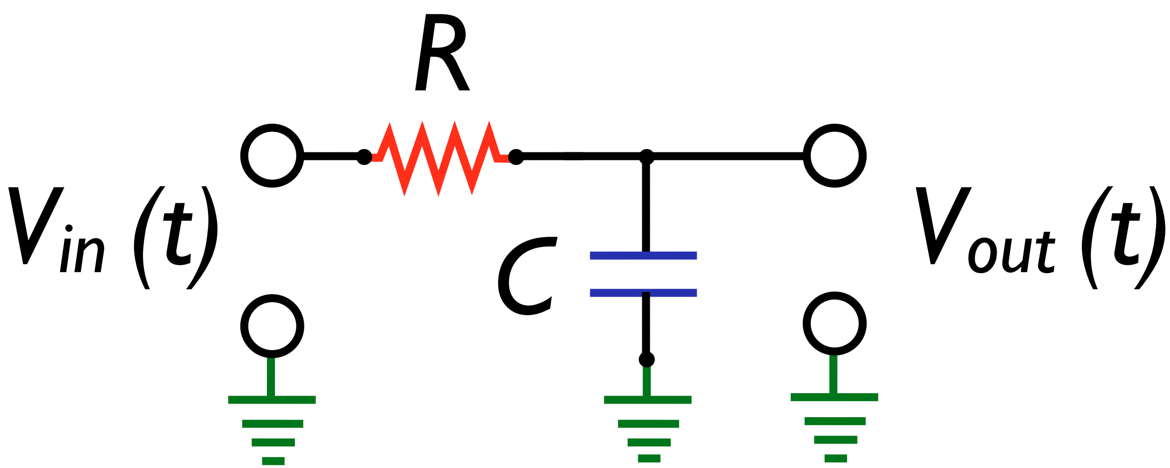
$$I = C\frac{d}{dt}\left(V_{in} - V\right) = \frac{V}{R}$$

for small RC,

$$C\frac{dV_{in}}{dt} \approx \frac{V}{R}$$

$$V(t) \approx RC\frac{d}{dt}V_{in}(t)$$

RC integrator



$$I = C\frac{dV}{dt} = \frac{V_{in} - V}{R}$$

for large RC (V ≪ V_{in})

$$C\frac{dV}{dt} \approx \frac{V_{in}}{R}$$

$$V(t) = \frac{1}{RC}\int^{t} V_{in}(t)\,dt + \text{const}$$

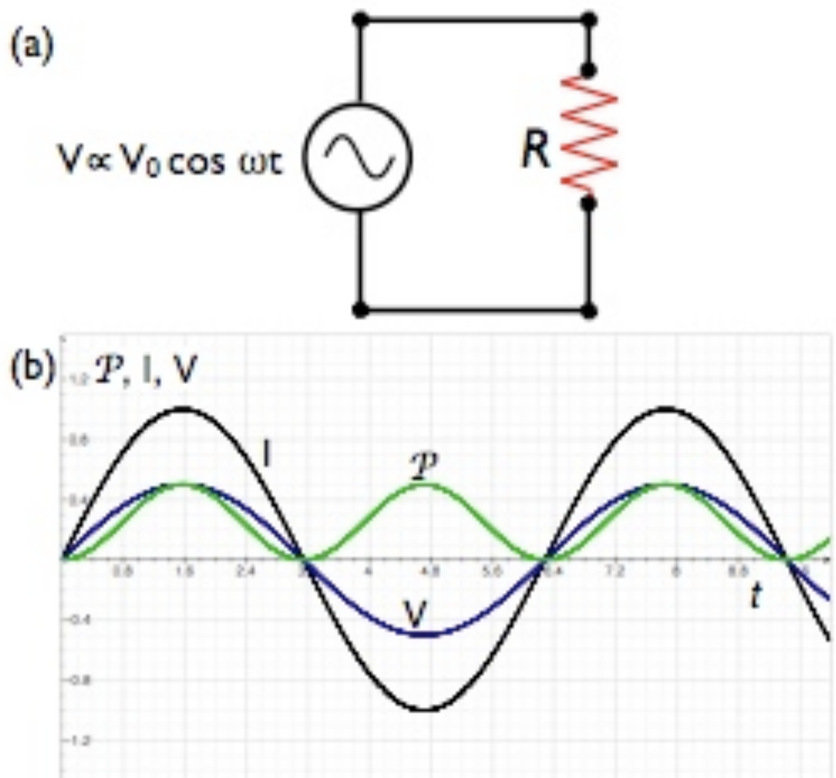
so what?

filtering of signals

unintentional capacitive coupling
see from waveform shape

more later

ac resistive circuits

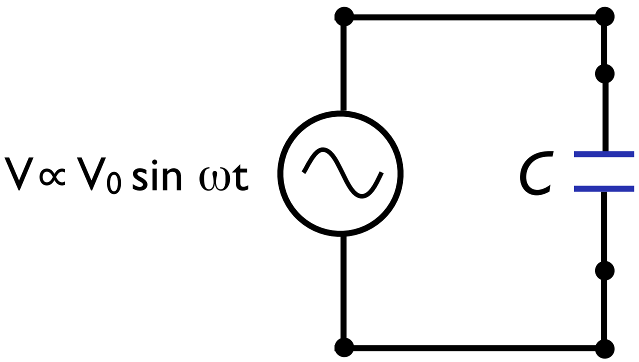


nothing earth-shattering happens

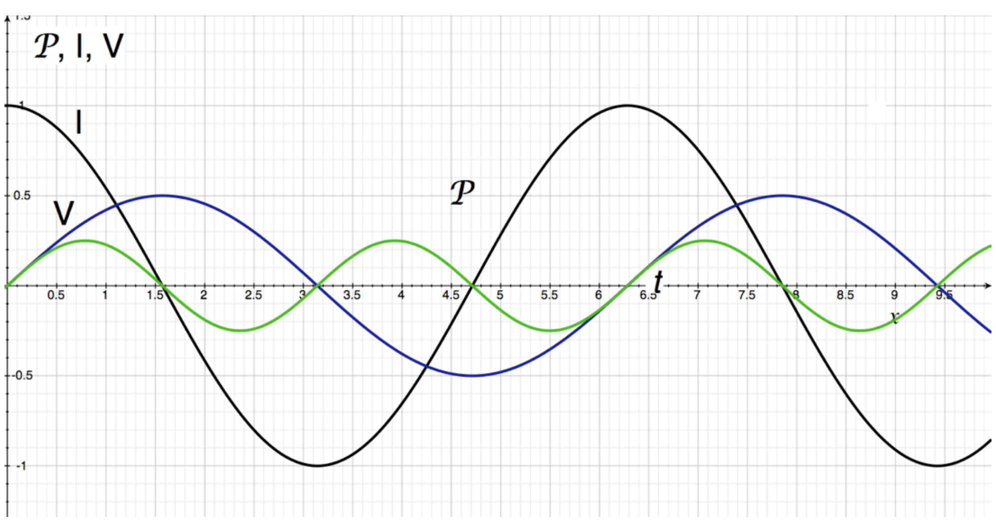
except P is lower than you expect

ac capacitive circuits

(a)



(b)



I and V 90° out of phase
average power is ZERO

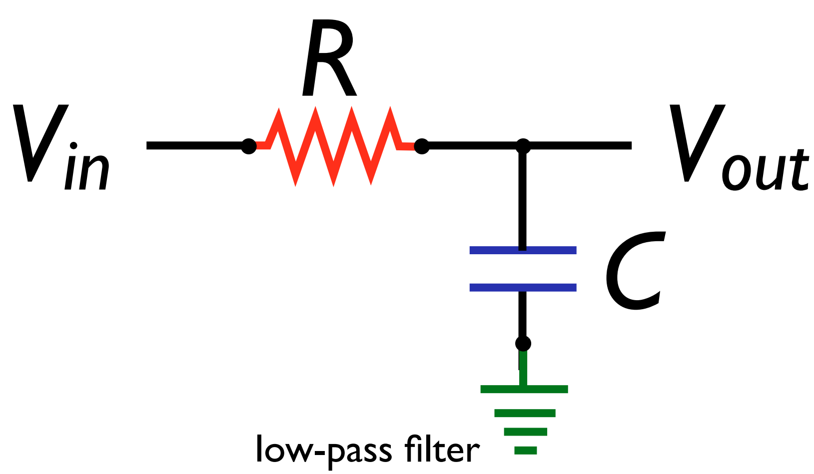
frequency response?
- insulating at dc
- conducting at high *f*

voltage “lags” current

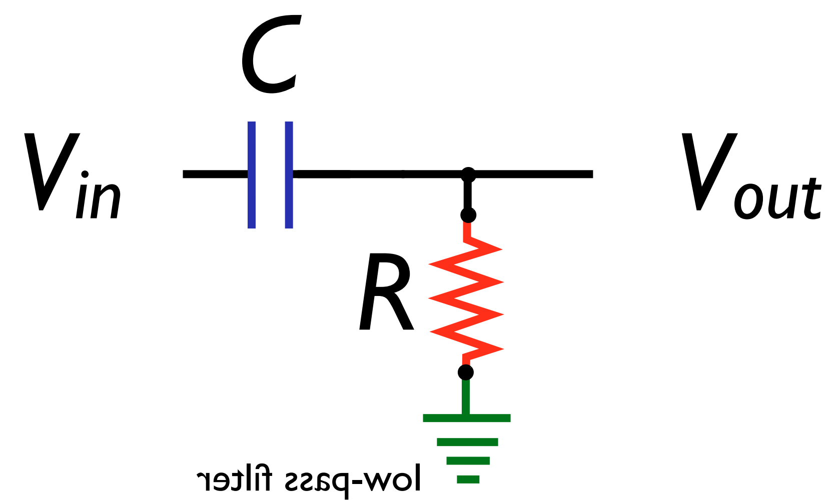
$$Z = \frac{1}{iwC} = \frac{-1}{2\pi i f C}$$

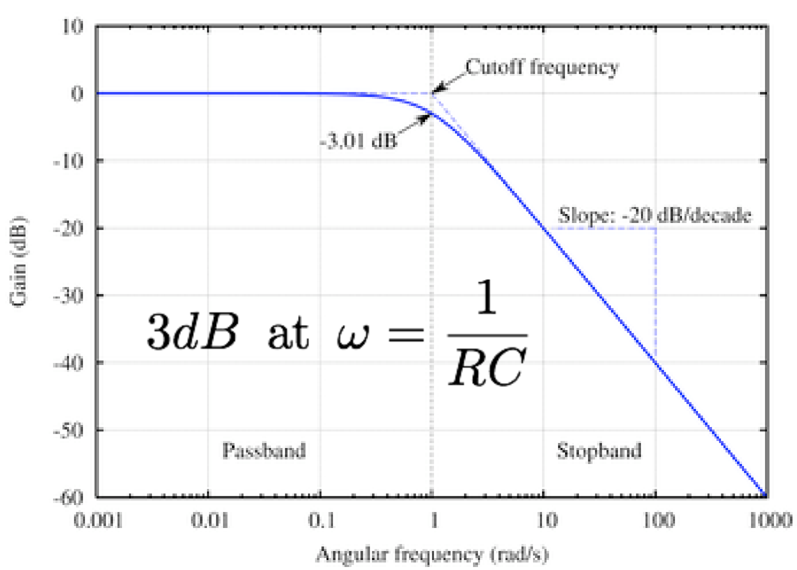
filters

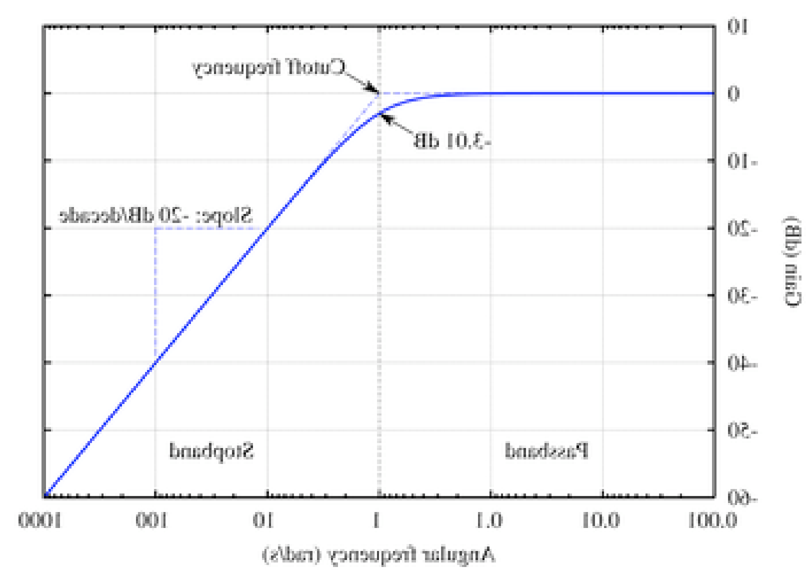
low-pass



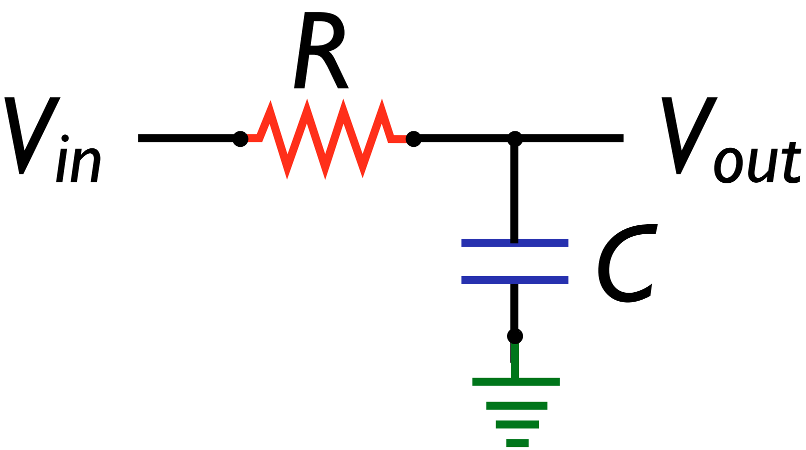
high-pass

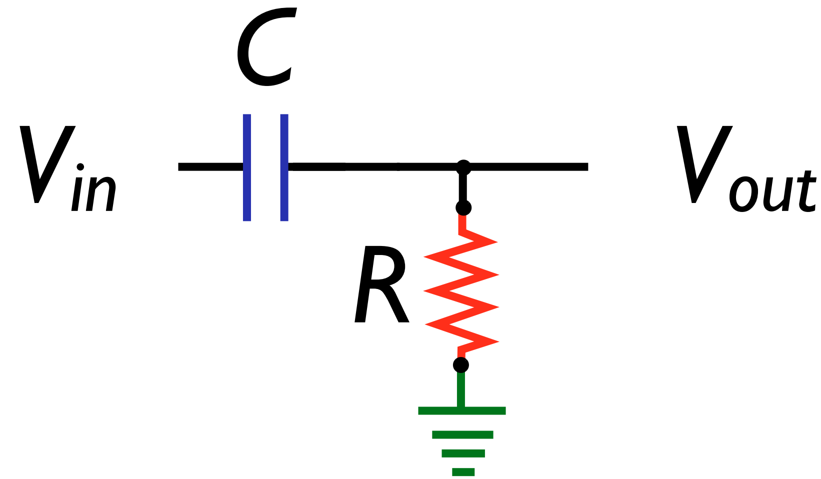






familiar?





low-pass filter

high-pass filter

frequency domain

⇕

time domain

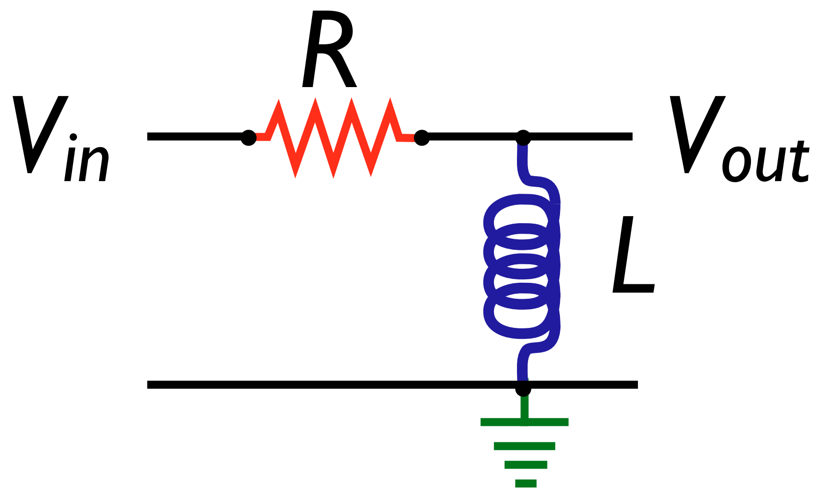
integrator

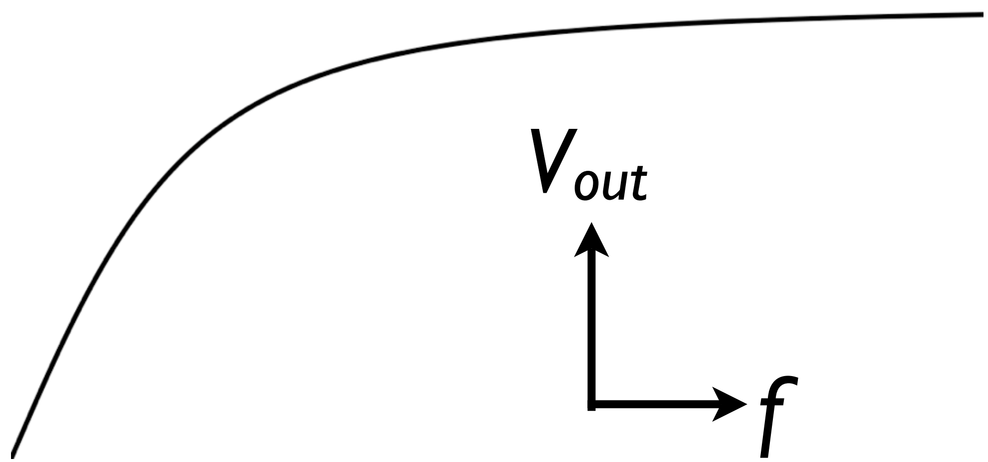
$$V(t) = \frac{1}{RC}\int^t V_{in}(t)\,dt + \text{const}$$

differentiator

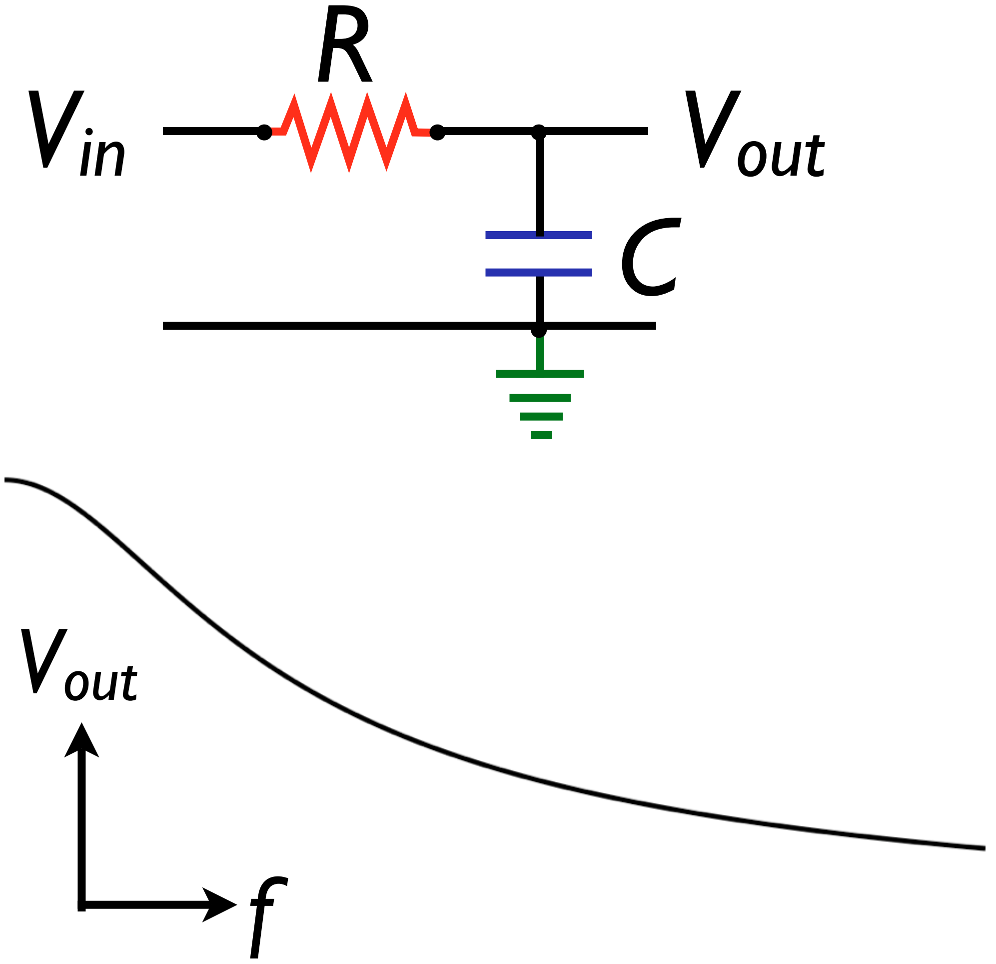
$$V(t) \approx RC\frac{d}{dt}V_{in}(t)$$

high-pass

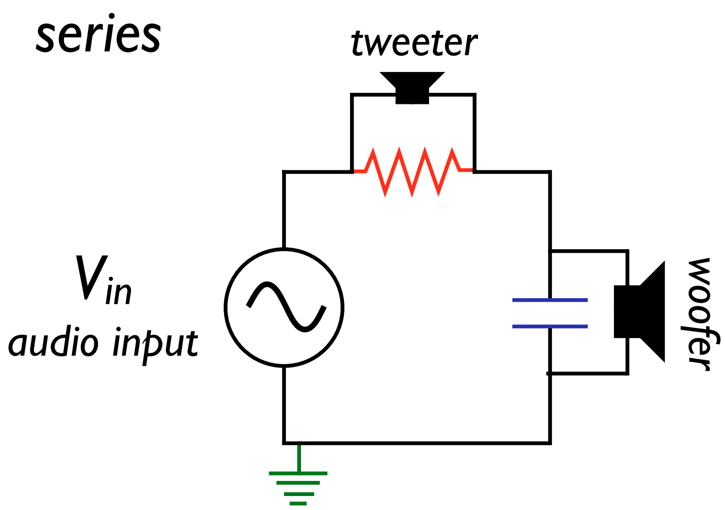


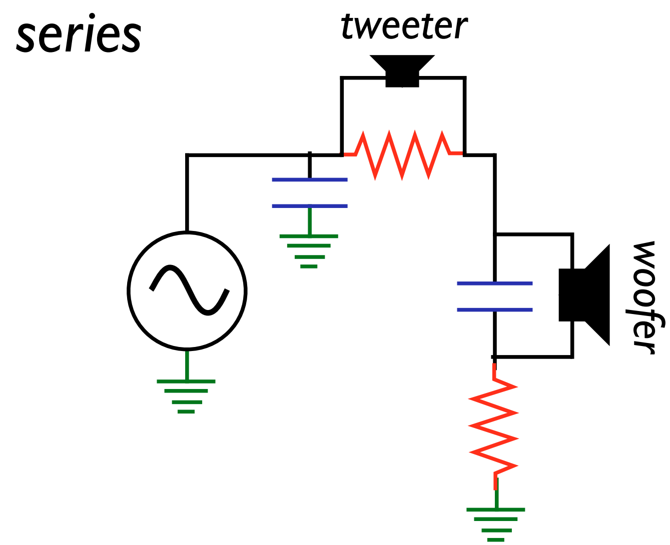


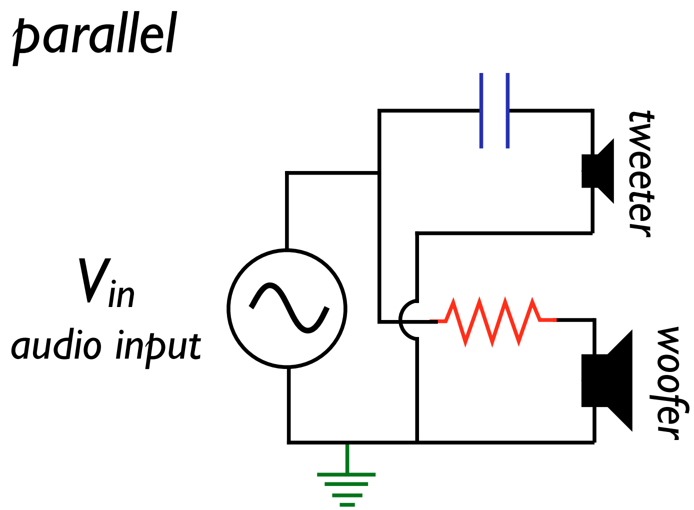
low-pass

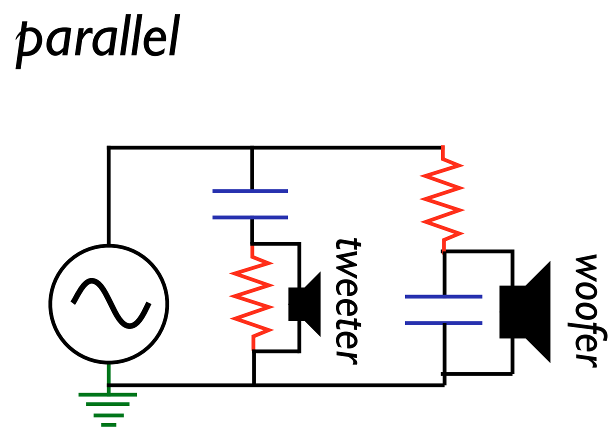


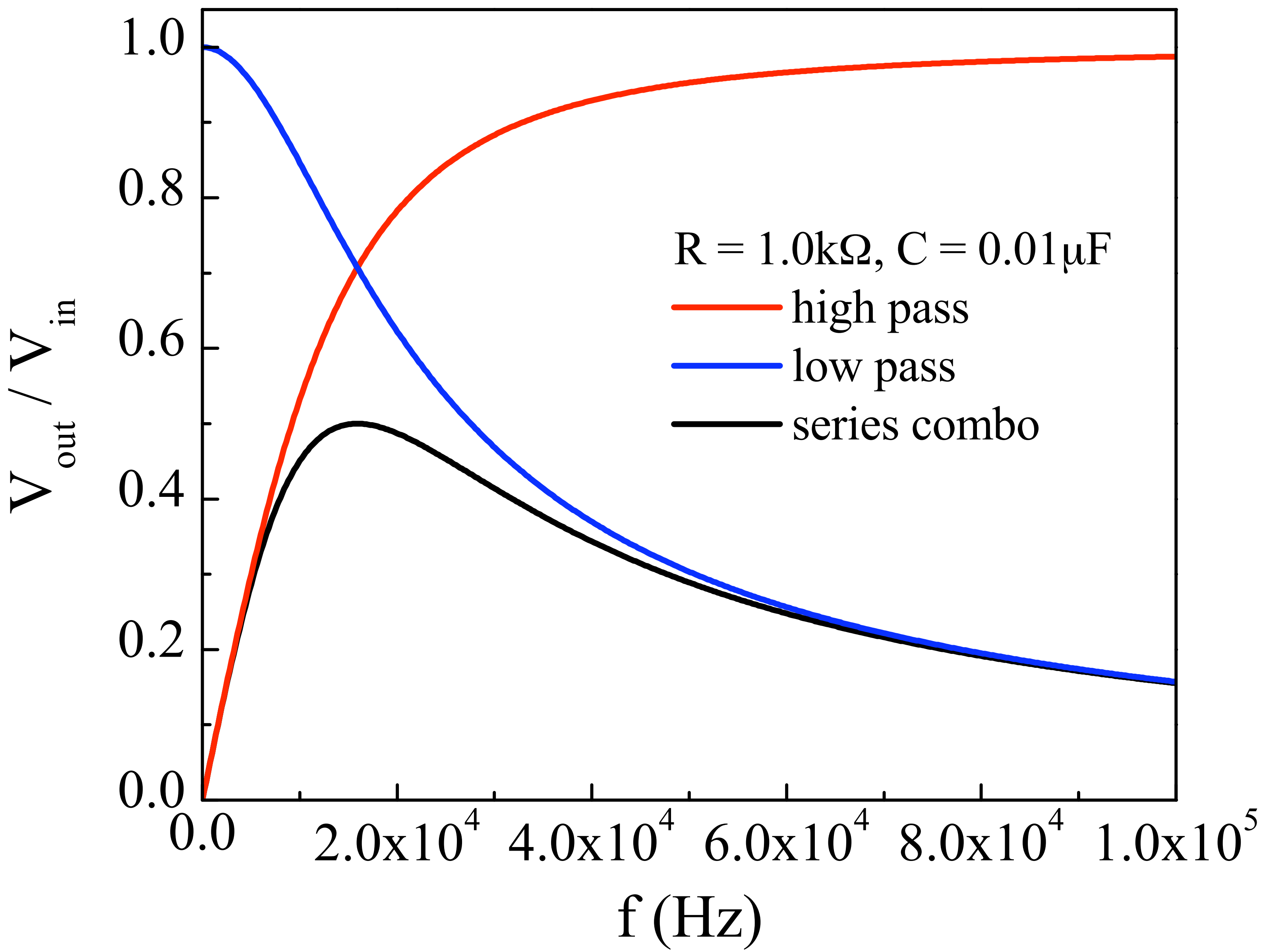
audio crossovers



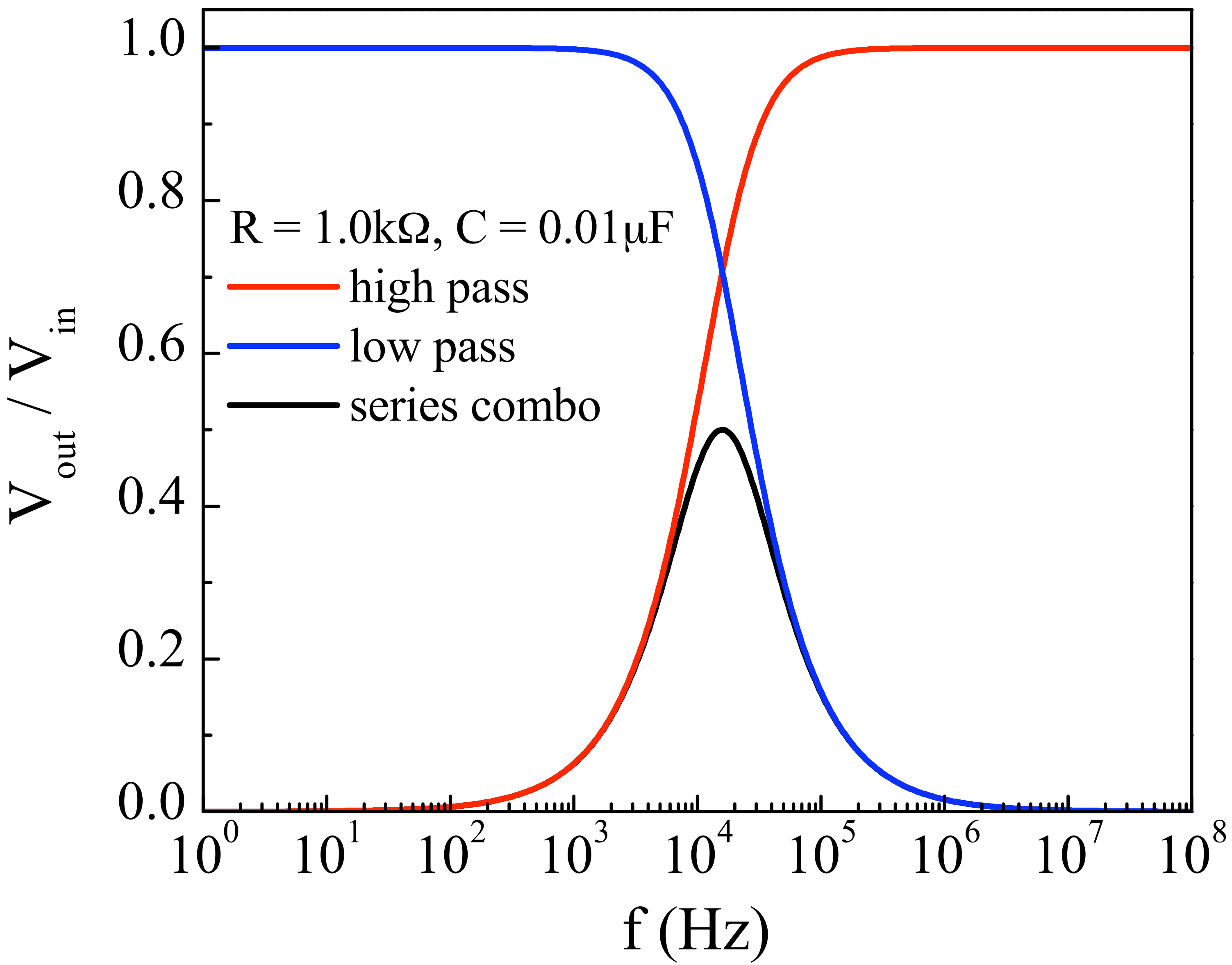




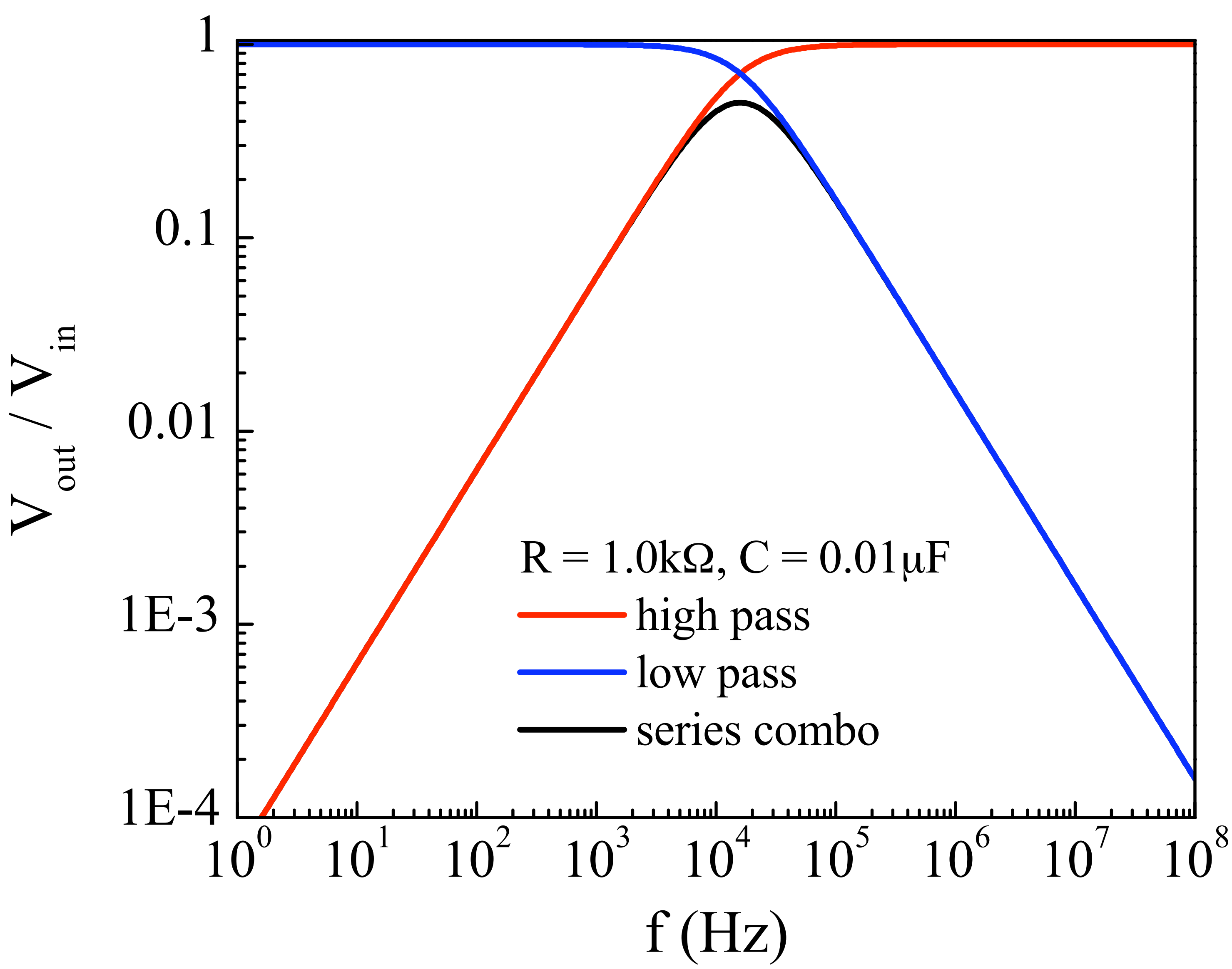




R = 1.0kΩ, C = 0.01μF
high pass
low pass
series combo
1.0
0.8
0.6
0.4
0.2
0.0
0.0
2.0x10^4
4.0x10^4
6.0x10^4
8.0x10^4
1.0x10^5
f (Hz)
V_out / V_in



1.0
0.8
0.6
0.4
0.2
0.0
V_{out} / V_{in}
R = 1.0kΩ, C = 0.01μF
high pass
low pass
series combo
10^0 10^1 10^2 10^3 10^4 10^5 10^6 10^7 10^8
f (Hz)

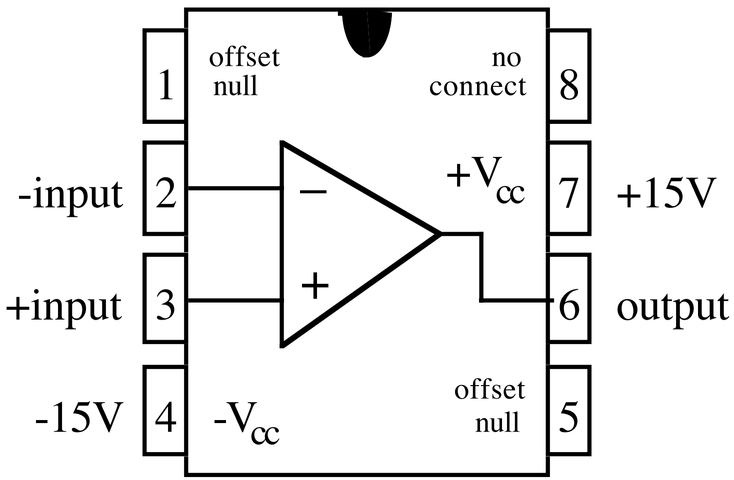


1
0.1
0.01
1E-3
1E-4
V_out / V_in
R = 1.0kΩ, C = 0.01μF
high pass
low pass
series combo
10^0 10^1 10^2 10^3 10^4 10^5 10^6 10^7 10^8
f (Hz)

dB/decade ...

Today:

amplify photodiode signal



741 pinout