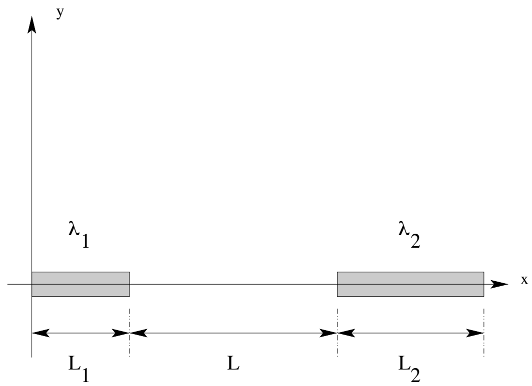
# Quiz 1: Solution

**1.** Two thin rigid rods lie along the $x$ axis, as shown below. Both rods are uniformly charged. Rod 1 has a length $L_1$ and a charge per unit length $\lambda_1$. Rod 2 has a length $L_2$ and a charge per unit length $\lambda_2$. The distance between the right end of rod 1 and the left end of rod 2 is $L$.




Which expression below could give the electric force between the two rods? Circle your answer.

$$\vec{\mathbf{F}}_{12,\text{tot}} = k_e\lambda_1\lambda_2 \left[\frac{(L_2+L)(L_1+L)}{L(L+L_1+L_2)}\right]\hat{\mathbf{x}} \tag{1}$$

$\longrightarrow$
$$\vec{\mathbf{F}}_{12,\text{tot}} = k_e\lambda_1\lambda_2 \ln\left[\frac{(L_2+L)(L_1+L)}{L(L+L_1+L_2)}\right]\hat{\mathbf{x}} \tag{2}$$

$$\vec{\mathbf{F}}_{12,\text{tot}} = k_e\lambda_1^2 \ln\left[\frac{L_1+L}{L+L_1+L_2}\right]\hat{\mathbf{y}} \tag{3}$$

$$\vec{\mathbf{F}}_{12,\text{tot}} = k_e\lambda_1\lambda_2 \frac{L_1+L_2}{\left(L_1^2+L_2^2\right)^{3/2}+L^2}\hat{\mathbf{x}} \tag{4}$$

See my PH106 problem sets from F08, HW1.1 for a full solution. We can figure out this one by eliminating the wrong answers though.

The third choice is right out, since the field is in the $\hat{\mathbf{y}}$ direction, violating the symmetry of the problem. Also, it does not depend on $\lambda_2$, so it is unrealistic.

The last choice is out because in the denominator we are add a distance cubed to a distance squared . . . it is not dimensionally correct, so it can't be right.

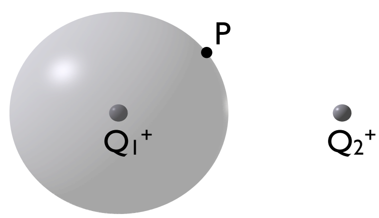
The first and second choices are plausible enough. I will probably give half credit for the wrong choice, since they are hard to distinguish. I would rule out the first by noting that the electric force goes as inverse distance squared, and this problem requires two integrations: one over the first rod, one over the second rod. Therefore, a log dependence seems in order, since we are working in 1D and have no other dimensions to muck things up.

**2.** Suppose three positively charged particles are constrained to move on a fixed circular track. If all the charges were equal, an equilibrium arrangement would obviously be a symmetrical one with the particles spaced 120° apart around the circle. Suppose two of the charges have equal charge $q$, and the equilibrium arrangement is such that these two charges are 140° apart rather than 120°. What is the *relative* magnitude and sign of the third charge?

- □ larger than either $q_1$ or $q_2$ and positive
- ■ smaller than either $q_1$ or $q_2$ and positive
- □ larger than either $q_1$ or $q_2$ and negative
- □ smaller than either $q_1$ or $q_2$ and negative

First, all the charges must be of the same sign. With only three charges on a circular track, there is no way to keep opposite charges separated. The charges must be separated due to mutual repulsion.

If the angle between the two charges is 140°, it means that their mutual repulsion is *greater* than the repulsive force each feels due to the third charge. Thus, the third charge must be of the same sign, but smaller than either $q_1$ or $q_2$. Remember this problem from your homework? In that case, the equilibrium angle was 90°, and the third charge was (approximately) three times larger than the other two.

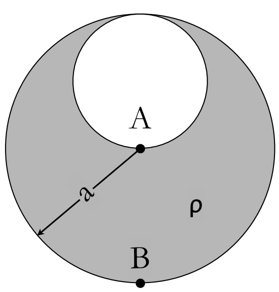



**3.** In the figure above, a point charge $1Q^+$ is at the center of an imaginary spherical Gaussian surface and another point charge $2Q^+$ is outside of the Gaussian surface. Point $P$ is on the surface of the sphere. Which one of the following statements is true?

- □ Both charges contribute to the net electric flux through the sphere but only $2Q^+$ contributes to the electric field at point $P$.
- ■ Only $1Q^+$ contributes to the net electric flux through the sphere but both charges contribute to the electric field at point $P$.

- ◻ Both contribute to the net electric flux through the sphere but only $1Q^+$ contributes to the electric field at point $P$.
- ◻ Only $2Q^+$ contributes to the net electric flux through the sphere but both charges contribute to the electric field at point $P$.
- ◻ Only $2Q^+$ contributes to the net electric flux through the sphere and to the electric field at point $P$ on the sphere.
- ◻ Only $1Q^+$ contributes to the net electric flux through the sphere and to the electric field at point $P$ on the sphere.
- ◻ I don't know (this answer is worth 1/5 of full credit)

We first note that only $Q_1^+$ can contribute to the flux through the Gaussian surface, since it is the only charge enclosed by the surface. However, *both* charges contribute to the electric field at the particular point $P$. Imagine that the spherical surface wasn't there at all - the electric field at $P$ would clearly have a contribution from both charges. The total flux through the *entire* sphere does depend on only $Q_1^+$, as Gauss' law says it must, but the field at any *particular* point on the sphere does depend on $Q_2^+$.




**4.** The sphere of radius $a$ was filled with positive charge at uniform density $\rho$. Then a smaller sphere of radius $a/2$ was carved out, as shown in the figure, and left empty. Which expression *could* give the expression for the electric field *anywhere* inside the cavity? The $\hat{\mathbf{y}}$ direction is vertical, and $r$ is measured from the center of the large sphere. *Hint: if it is true anywhere inside the cavity, pick an easy example point. What superposition of simple charge distributions could give the one shown?*

- ◻ $\vec{\mathbf{E}} = \frac{2k_e\pi\rho}{r}\,\hat{\mathbf{y}}$
- ◻ $\vec{\mathbf{E}} = \frac{2k_e\pi\rho a}{r^2}\,\hat{\mathbf{y}}$
- ■ $\vec{\mathbf{E}} = \frac{2k_e\pi\rho a}{3}\,\hat{\mathbf{y}}$
- ◻ $\vec{\mathbf{E}} = \frac{2k_e\pi\rho r}{a}\,\hat{\mathbf{y}}$

First, recognize that this is a superposition of a full sphere of positive charge of radius $a$ and a sphere of radius $a/2$ filled with *negative* charge. Then you can apply superposition and Gauss' law to actually solve the problem.

However, given choices, there are two very simple ways to eliminate answers here: first, not all of the choices have the proper units. Only the third choice gives the correct units for $E$. Second, if the expression is supposed to give the field anywhere inside the cavity, we can pick one easy example point where we know how to calculate the field. At point $A$, the total field would be equivalent to that of a negatively charged sphere of radius $a/2$ and charge density $\rho$ at a distance $a/2$. A simple application of Gauss' law gives the third answer as the correct field.

A full solution can be found in my PH106 problem sets, F08, HW2.