

Exam 1 redux

Instructions

1. **Solve any 2 problems below.** All problems have equal weight.
2. Do not hesitate to ask if you are unsure what a problem is asking for.
3. **Show your work** for full credit. Significant partial credit will be given.

1. A particle of mass m traveling along a horizontal line with constant velocity v_o suddenly experiences a resistive force $f = -bv^3$. **(a)** Find an expression for $v(t)$. *Hint: it is always the same method.* **(b)** Does the object eventually halt, either at finite time or as $t \rightarrow \infty$? If so, under what condition, and if not, why?

bonus +1: What is the time constant of the $v(t)$ decay? That is, your result should have a term that looks like t/τ , what is τ ?

2. A rectangular block of height h and area A floats in water. If the density of the block is ρ_b and the density of the water is ρ_w , find the frequency of small oscillations when the block bobs up and down on the surface of the water. *Hints: The buoyant force is the weight of the displaced fluid. If the object is floating in equilibrium, what is the net force if it is displaced by some δx ?*

bonus, +2: show that the frequency of oscillation is equivalent to that of a pendulum of length d_o , where d_o is the submerged depth of the block in equilibrium.

3. **(a)** Write down the total energy E for a mass on a spring in 1D in terms of \dot{x} and x . **(b)** Show, as is the case for any conservative 1D system, that you can obtain the equation of motion for the coordinate x by differentiating the equation $E = \text{const}$. It should be the familiar Newton's 2nd law result.

bonus +1: Suppose the energy were not constant, but you had it on good authority that energy was lost to the environment at a rate proportional to velocity, i.e., $\dot{E} \propto -\dot{x}$. With the result of (b), show that this is in fact just a damped harmonic oscillator. *Hint: consider a constant of proportionality 2β to produce a familiar-looking result.*