University of Alabama
Department of Physics and Astronomy

PH 301 / LeClair Fall 2018

Exam 1

Instructions

1. **Solve any 4 problems below.** All problems have equal weight.
2. Do not hesitate to ask if you are unsure what a problem is asking for.
3. **Show your work** for full credit. Significant partial credit will be given.

1. A mass m is constrained to move along the x axis subject to a force $F(v) = -F_o e^{-v/u}$, where F_o and u are constants. **(a)** Find $v(t)$ if the initial velocity is $v_o > 0$ at $t = 0$. **(b)** At what time does it come instantaneously to rest?

2. To perform a rescue, a lunar landing craft needs to hover just above the surface of the moon, which has a gravitational acceleration of $g/6 \approx 1.62\,\mathrm{m/s^2}$. The exhaust velocity is $2000\,\mathrm{m/s}$, but fuel amounting to only 20 percent of the total mass may be used. How long can the landing craft hover?

3. Consider a gun of mass M (when unloaded) that fires a shell of mass m with muzzle speed v. (That is, the shell's speed *relative to the gun* is v.) Assuming the gun is completely free to recoil (no external forces on the gun or shell), use conservation of momentum, find the shell's speed relative to the ground in terms of v, m, and M.

4. Near the surface of planet X, the gravitational force on a particle of mass m is vertically downward (along $-\hat{\mathbf{y}}$) but has magnitude $m\gamma y^2$, where γ is a constant and y is the mass' height above horizontal ground. **(a)** Find the work done by gravity on a mass m moving from $\mathbf{r}_1$ to $\mathbf{r}_2$. **(b)** Use this result to show that the force, though unusual, is conservative (hint: what is the work done in going from y_i to y_i?). **(c)** If I release a mass from height h above the planet's surface, how fast will it be going just before it reaches the ground?

5. An undamped oscillator has period $\tau_o = 1$ second. When weak damping is added, it is found that the amplitude of the oscillator drops by 50% in one period $\tau_1 = 2\pi/\omega_1$. **(a)** How big is β compared to ω_o? **(b)** What is τ_1? *Hint: recall* $x(t) = Ae^{-\beta t}\cos(\omega_1 t - \delta)$ *and* $\omega_1^2 = \omega_o^2 - \beta^2$ *for weak damping.*

6. An approximation for the potential energy of two ions as a function of their separation is

$$U(r) = -\frac{ke^2}{r} + \frac{b}{r^9} \tag{1}$$

The first term is the usual Coulomb interaction, while the second term is introduced to account for the repulsive effect of the two ions at small distances. **(a)** Find b as a function of the equilibrium spacing r_o (recall that the equilibrium spacing is when U is minimum). **(b)** Find the frequency of small oscillations about $r = r_o$, assuming the molecule oscillates according to its effective mass m. *Hint: look at the definition of a Taylor series. Use your result from the first part in the second part to simplify.*