UNIVERSITY OF ALABAMA
Department of Physics and Astronomy

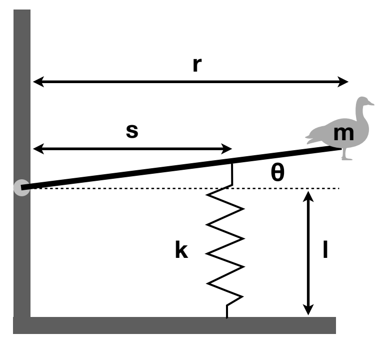
PH 301 / LeClair Fall 2018

Exam 2

Instructions

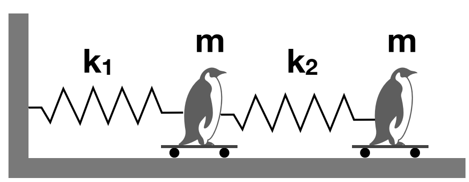
1. **Solve any 2 problems below.** All problems have equal weight.
2. **Show your work** for full credit. Significant partial credit will be given.

1. A duck of mass m sits on the end of a light rod pivoted at the wall. A spring of constant k supports the rod and duck a horizontal distance s from the pivot point, while the duck (which you may approximate as a point mass) sits a horizontal distance r from the pivot. Assume the system is in static equilibrium at $\theta = 0$ with the spring at its relaxed length for simplicity. **(a)** For small angular displacement θ, write down the Lagrangian for the system, making sure to include both elastic and gravitational potential energy. *Hint:* For small angles, arclengths are useful distances. **(b)** Use the Lagrange equations to find the equation of motion. You do not need to solve it. **(c)** Aside from any constant (θ-independent) terms, you should be able to find the frequency of small oscillations from the equation of motion. What is it?



2. A spacecraft releases a 470 kg satellite while in an orbit 280 km above the surface of the earth. A rocket engine on the satellite boosts it to a geosynchronous orbit. How much energy is required for the orbit boost? (Note: the earth's radius is 6378 km, its mass is 5.98×10^{24} kg, and $G = 6.67 \times 10^{-11} \mathrm{N \cdot m^2 kg^{-2}}$. Hint: "geosynchronous" means the satellite is always above the same point on the earth's surface, which implies something about the period of orbit.

3. Two skateboarding penguins of identical mass are connected together by a spring with force constant k_2, and the leftmost penguin is connected to the wall by a spring of force constant k_1. **(a)** Find the the normal frequencies ω_1 and ω_2. **(b)** Either show or comment on the motion of the normal modes (i.e., how are the penguins moving relative to each other in each mode?).



4. A mass m_1 rests on a frictionless horizontal table and is attached to a massless string. The string runs horizontally to the edge of the table, where it passes over a massless, frictionless pulley and then hangs vertically down. A second mass m_2 is now attached to the bottom end of the string. **(a)** Write down the Lagrangian for the system. **(b)** Find the Lagrange equation of motion, and solve it for the acceleration of the blocks. *Hint:* for your generalized coordinate, use the distance x of the second mass below the tabletop.