University of Alabama
Department of Physics and Astronomy

PH 301 / LeClair Fall 2018

Problem Set 0

due 24 Aug 2018

Instructions:

1. Answer all questions below
2. Show your work for full credit.
3. All problems are due by 11:59pm on 24 Aug 2018.
4. You may collaborate, but everyone must turn in their own work.

1. You throw a ball upward. After half of the time to the highest point, the ball has covered what fraction of its maximum height? Ignore air resistance.

Solution: This is a problem from "Problems and Solutions in Introductory Mechanics" by David Morin, a nice (and inexpensive) book you might find useful as a supplement if you need to review introductory mechanics.[i] Let the ground level be $x=0$, with $+x$ running upward. The ball's position at any time, assuming an initial velocity v_0, is then[ii]

$$x(t) = v_0 t - \frac{1}{2} g t^2 \tag{1}$$

The time to its highest point is found by maximizing $x(t)$, or equivalently, finding the time at which the velocity is zero.

$$v(t) = \frac{dx}{dt} = v_0 - gt = 0 \tag{2}$$

$$\implies \quad t_{\text{max}} = \frac{v_0}{g} \tag{3}$$

At this time, we can find the ball's height:

$$x\left(t_{\text{max}}\right) = v_0 \left(\frac{v_0}{g}\right) - \frac{1}{2} g \left(\frac{v_0}{g}\right)^2 = \frac{v_0^2}{2g} \tag{4}$$

At half this time, the ball's height is

$$x\left(\frac{1}{2} t_{\text{max}}\right) = v_0 \left(\frac{v_0}{2g}\right) - \frac{1}{2} g \left(\frac{v_0}{2g}\right)^2 = \frac{3v_0^2}{8g} \tag{5}$$

[i] See http://www.people.fas.harvard.edu/~djmorin/book.html.

[ii] We are not given the initial velocity, but we need it to work the problem. In most cases like this, quantities you had to introduce yourself will be part of the calculation and not the final answer. In this case, we introduced v_0 to solve the problem, but since it wasn't specified our final answer should be independent of v_0.

The fraction of maximum height is then

$$\text{fraction of max height} = \frac{x(\frac{1}{2}t_{\text{max}})}{x(t_{\text{max}})} = \frac{\frac{3v_0^2}{8g}}{\frac{v_0^2}{2g}} = \frac{3}{4} \tag{6}$$

Since the ball is going much faster during the first half of its motion, it covers more distance. The last half of the ball's flight only covers 1/4 of the net vertical distance.

2. A ball is dropped, and then another ball is dropped from the same spot one second later. As time goes on while the balls are falling, what is the distance between them at any given time? (Ignoring air resistance, as usual.)

Solution: Another problem from Morin's book (see previous problem). Let the starting point of the balls be $x=0$, with $+x$ upward. Let the time difference between dropping the two balls be $T=1\,\text{s}$ (so we can do this symbolically). The first ball falls a total of $t+T$ seconds after the second has fallen for t seconds, so the positions of the two balls can be written

$$x_1(t) = -\frac{1}{2}g\,(t+T)^2 \tag{7}$$

$$x_2(t) = -\frac{1}{2}gt^2 \tag{8}$$

Their difference is easily found:

$$\Delta x = x_2 - x_1 = -\frac{1}{2}gt^2 + \frac{1}{2}g\,(t+T)^2 = -\frac{1}{2}gt^2 + \frac{1}{2}gt^2 + gTt + \frac{1}{2}gT^2 = \frac{1}{2}gT^2 + gTt \tag{9}$$

The first term is constant, and represents how far the first ball falls before the second one is dropped. The second term increases linearly with time, representing the fact that while both balls have the same acceleration, the first ball has been accelerating a time T longer. While their relative velocity is constant (the two velocities will always differ by gT, the speed the first ball picks up before the second is dropped), the separation between the two balls increases linearly.