University of Alabama
Department of Physics and Astronomy

PH 301 / LeClair

Fall 2018

Problem Set 1

due 31 Aug 2018

Instructions:

1. Answer all questions below (bonus questions optional).
2. Show your work for full credit.
3. All problems are due by 11:59pm on 31 Aug 2018.
4. You may collaborate, but everyone must turn in their own work.

1. If $\mathbf{a}(t)$, $\mathbf{b}(t)$, and $\mathbf{c}(t)$ are functions of t, verify the following results:

$$\frac{d}{dt}\left[\mathbf{a}\cdot(\mathbf{b}\times\mathbf{c})\right] = \mathbf{a}\cdot\left(\mathbf{b}\times\frac{d\mathbf{c}}{dt}\right) + \mathbf{a}\cdot\left(\frac{d\mathbf{b}}{dt}\times c\right) + \frac{d\mathbf{a}}{dt}\cdot(\mathbf{b}\times\mathbf{c}) \tag{1}$$

$$\frac{d}{dt}\left[\mathbf{a}\times(\mathbf{b}\times\mathbf{c})\right] = \mathbf{a}\times\left(\mathbf{b}\times\frac{d\mathbf{c}}{dt}\right) + \mathbf{a}\times\left(\frac{d\mathbf{b}}{dt}\times c\right) + \frac{d\mathbf{a}}{dt}\times(\mathbf{b}\times\mathbf{c}) \tag{2}$$

2. Find the angle between a body diagonal of a cube and any one of its face diagonals. [*Hint:* Choose a unit cube with one corner at the origin and the opposite corner at point (1,1,1). Write down the vectors for the two diagonals and use the scalar product.]

3. **(a)** Prove that if $\mathbf{v}(t)$ is any vector that depends on time but which has constant *magnitude*, then $\dot{\mathbf{v}}(t)$ is orthogonal to $\mathbf{v}(t)$. **(b)** Prove the converse that if $\dot{\mathbf{v}}(t)$ is orthogonal to $\mathbf{v}(t)$, then $|\mathbf{v}(t)|$ is constant. [*Hint:* Consider the derivative of $\mathbf{v}^2$.] This is a very handy result. It explains why, in two-dimensional polars, $d\mathbf{r}/dt$ has to be in the direction of $\hat{\boldsymbol{\varphi}}$ and vice versa. It also shows that the speed of a charged particle in a magnetic field is constant, since the acceleration is perpendicular to the velocity.

4. When a baseball flies through the air, the ratio $f_{\text{quad}}/f_{\text{lin}}$ of the quadratic to the linear drag force is given by

$$\frac{f_{\text{quad}}}{f_{\text{lin}}} = \frac{\gamma D}{\beta}v = \left(1.6\times10^3\,\text{s/m}^2\right)Dv \tag{3}$$

Given that a baseball has a diameter of about 7 cm, find the approximate speed v at which the two drag forces are equally important. For what range of speeds is it safe to treat the drag force as purely quadratic? Under normal conditions is it a good approximation to ignore the linear term? Answer the same questions for a golf ball of diameter 4.3 cm.

5. A projectile is launched with initial velocity $\mathbf{v}_i$ from the start of a ramp, with the ramp making an angle φ with respect to the horizontal. The projectile is launched with an angle $\theta > \varphi$ with respect to the horizontal. **(a)** At what position along the ramp does the projectile land? **(b)** What angle θ maximizes the distance the particle makes it along the ramp (your answer will be in terms of the angle φ? Note no numeric solution is required.

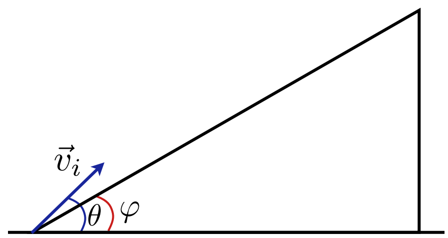

Figure 1: *A projectile is launched onto a ramp.*

6. The origin of the quadratic drag force on any projectile in a fluid is the inertia of the fluid that the projectile sweeps up. **(a)** Assuming the projectile has cross-sectional area A (normal to its velocity) and speed v, and that the density of the fluid is ρ, show that the rate at which the projectile encounters fluid (mass/time) is $\rho A v$. **(b)** Making the simplifying assumption that all of this fluid is accelerated to the speed v of the projectile, show that the net drag force on the projectile is $\rho A v^2$. (It is certainly not true that all the fluid the projectile encounters is accelerated to the full speed v, and one might guess the actual force has a correction factor $\kappa < 1$, so that $f_{\text{quad}} = \kappa \rho A v^2$, with κ depending on the shape of the body, smaller for more streamlined objects).

7. Problem 2.7 from your textbook is about a class of 1-D problems that can always be reduced to doing an integral. Specifically, if F is a function of v alone ($F = F(v)$), then you can show

$$t = m \int_{v_o}^{v} \frac{dv'}{F(v')} \tag{4}$$

Here is another class of problems. Show that if the net force on a 1-D particle depends only on position, $F = F(x)$, then Newton's second law can be solved to find v as a function of x given by

$$v^2 = v_o^2 + \frac{2}{m} \int_{x_o}^{x} F(x')\, dx' \tag{5}$$

[*Hint:* Use the chain rule to prove the following handy relationship (the "vdv/dx rule"): if you regard v as a function of x, then $\dot{v} = vdv/dx = (1/2)dv^2/dx$. Use this to rewrite Newton's second law in the separated form $md(v^2) = 2F(x)\, dx$ and then integrate from x_o to x.] Comment on the result for the case that $F(x)$ is actually a constant. (You may also recognize your solution as a statement about kinetic energy and work.)

8. Using the result of the previous problem, consider a mass m constrained to move on the x axis and subject to a force $F = -kx$, where k is a positive constant. The mass is released from rest at $x = x_o$ at time $t = 0$. First find the speed using equation 5, and then using $v = dx/dt$, separate the equation and integrate. You should recognize this as one way – not the easiest – to solve the simple harmonic oscillator.

9. A mass m has speed v_o at the origin and coasts along the x axis in a medium where the drag force is $F(v) = -cv^{3/2}$. Use the "vdv/dx rule" above to write the equation of motion in separated form $md(v^2) = 2F(x)\,dx$, and then integrate both sides to give x in terms of v (or vice versa). Show that it will eventually travel a distance $2m\sqrt{v_o}/c$.

10. A basketball has mass $m = 0.6\,\mathrm{kg}$ and diameter $D = 0.24\,\mathrm{m}$. **(a)** What is its terminal speed? (See example 2.5 ...) **(b)** If it is dropped from a 30-m tower, how long does it take to hit the ground and how fast is it going when it does so? [*Hint:* review the "vertical motion with quadratic drag" section.] Compare with the corresponding numbers in a vacuum

[Bonus Question; Computer] The differential equation for the skateboard in example 2, $\ddot{\varphi} = -\frac{g}{R}\sin\varphi$ cannot be solved in terms of elementary functions, but is easily solved numerically. **(a)** Do this for the case $\varphi_o = 20°$, with $R = 5\,\mathrm{m}$ and $g = 9.8\,\mathrm{m/s^2}$. Make a plot of $\varphi(t)$ for two or three periods. **(b)** On the same plot, show the approximate solution given by (1.57) with the same $\varphi_o = 20°$. Comment on the two graphs. **(c)** Repeat for $\varphi_o = \pi/2$.