University of Alabama
Department of Physics and Astronomy

PH 301 / LeClair | Fall 2018

Problem Set 3
due 14 Sept 2018

Instructions:

1. Answer all questions below (bonus questions optional).
2. Show your work for full credit.
3. All problems are due by 11:59pm on 14 Sept 2018.
4. You may collaborate, but everyone must turn in their own work.

1. A projectile that is subject to quadratic air resistance is thrown vertically *up* with initial speed v_o. **(a)** Write down the equation of motion for the upward motion and solve it to give $v(t)$. **(b)** Show that the time to reach the top of the trajectory is

$$t_{\text{top}} = \frac{v_{\text{ter}}}{g} \arctan\left(\frac{v_o}{v_{\text{ter}}}\right) \tag{1}$$

(c) For the baseball of Example 2.5 in your text (with $v_{\text{ter}} = 35\,\text{m/s}$), find t_{top} for the cases that $v_o = 1, 10, 20, 30, 40\,\text{m/s}$ and compare with the corresponding values in a vacuum.

2. Two people, each of mass m_h, are standing at one end of a stationary railroad flatcar with frictionless wheels and mass m_{fc}. Either person can run to the other end of the flatcar and jump off with the same speed u (relative to the car). **(a)** Use conservation of momentum to find the speed of the recoiling car if the two people run and jump simultaneously. **(b)** What is it if the second person starts running only after the first has jumped? Which procedure gives the greater speed to the car? [*Hint:* The speed u is the speed of either person, *relative to the car* just after they have jumped; it has the same value for either person and is the same in parts (a) and (b).]

3. Many applications of conservation of momentum involve conservation of energy as well, and we haven't yet begun our discussion of energy. Nevertheless, you know enough about energy from your introductory physics course to handle some problems of this type. Here is one elegant example: an *elastic* collision between two bodies is defined as a collision in which the total kinetic energy of the two bodies is the same before and after the collision (for example, the collision of two billiard balls, which generally lose extremely little of their kinetic energy.) Consider an elastic collision between two equal mass bodies, one of which is initially at rest. Let their velocities be $\mathbf{v}_1$ and $\mathbf{v}_2 = 0$ before the collision, and $\mathbf{v}_1'$ and $\mathbf{v}_2'$ after. Write down the vector equation representing the conservation of momentum and the scalar equation which expresses that the collision is elastic. Use these to prove that the angle between $\mathbf{v}_1'$ and $\mathbf{v}_2'$ is 90°. This result was important in the history of atomic and nuclear physics: that two bodies emerged from a collision traveling on perpendicular paths was strongly suggestive that they had equal mass and had undergone an elastic collision.

4. A rocket (initial mass m_o) needs to use its engines to hover stationary, just above the ground. **(a)** If it an afford to burn no more than a mass λm_o of its fuel, how long can it hover? [*Hint:* Write down the condition that the thrust just balances the force of gravity. You can integrate the resulting equation by separating the variables t and m. Take v_{ex} to be constant.] **(b)** If $v_{ex} \approx 3000\,\mathrm{m/s}$ and $\lambda \approx 10\%$, for how long could the rocket hover just above the earth's surface?

5. Consider a rocket (initial mass m_o) accelerating from rest in free space. At first, as it speeds up, its momentum p in creases, but as its mass m decreases p eventually begins to decrease. For what value of m (in terms of m_o) is p maximum?

6. **(a)** Consider a rocket traveling in a straight line subject to an external force F^{ext} acting along the same line. Show that the equation of motion is

$$m\dot{v} = -\dot{m}v_{\text{ex}} + F^{\text{ext}} \tag{2}$$

[Review the derivation of equation (3.6) in the book but keep the external force term.] **(b)** Specialize to the case of a rocket taking off vertically (from rest) in a (constant) gravitational field g, so the equation of motion becomes

$$m\dot{v} = -\dot{m}v_{\text{ex}} - mg \tag{3}$$

Assume that the rocket ejects mass at a constant rate, $\dot{m} = -k$ (where k is a positive constant), so that $m = m_o - kt$. Solve the equation for v as a function of t. **(c)** Using the rough data from problem 3.7 in your textbook, find the space shuttle's speed two minutes into flight, assuming (what is nearly true) that is travels vertically up during this period and that g does not change appreciably. Compare with the corresponding result if there were no gravity. **(d)** Describe what would happen to a rocket that was designed so that the first term on the right of equation 3 was smaller than the initial value of the second.

7. Use the results of the previous problem giving $v(t)$ for a rocket accelerating vertically from rest in a gravitational field g. Now integrate $v(t)$ to show the rocket's height as a function of t is

$$y(t) = v_{\text{ex}} - \frac{1}{2}gt^2 - \frac{mv_{\text{ex}}}{k}\ln\left(\frac{m_o}{m}\right) \tag{4}$$

Using the numbers given in problem 3.7 in your textbook, estimate the space shuttle's height after 2 minutes.

8. **(a)** We know that the path of a projectile thrown from the ground is a parabola if we ignore air resistance. In the light of equation (3.12) in your textbook ($\mathbf{F}^{\text{ext}} = M\ddot{\mathbf{R}}$), what would be the subsequent path of the CM of the pieces if the projectile exploded in midair? **(b)** A shell is fired from ground level so as to hit a target 100 m away. Unluckily, the shell explodes prematurely and breaks into two equal pieces. The two pieces land at the same time, and one lands 100 m beyond the target. Where does the other piece land? **(c)** Is the same result true if they land at different times (with one piece still landing 100 m beyond the target)?

9. Use spherical polar coordinates (r, θ, φ) to find the CM of a uniform solid hemisphere of radius R whose flat face lies in the xy plane with its center at the origin. You will need the element of volume in spherical polar coordinates.

10. I have a hemispherical bowl of radius R, and I wish to fill it to a height h such that half the volume is filled. To what height h (in terms of R) do I need to fill it? (Imagine you have a hemispherical 1 tsp measuring spoon and need to fill it to 1/2 tsp.)