University of Alabama
Department of Physics and Astronomy

PH 301 / LeClair Fall 2018

Problem Set 6

due 15 October 2018

Instructions:

1. Answer all questions below (bonus questions optional).
2. Show your work for full credit.
3. All problems are due by 11:59pm on 15 October 2018.
4. You may collaborate, but everyone must turn in their own work.

1. **(a)** Write down the Lagrangian $\mathcal{L}(x_1, x_2, \dot{x}_1, \dot{x}_2)$ for two particles of equal mass $m_1 = m_2 = m$, confined to the x axis and connected by a spring with potential energy $U = \frac{1}{2}x^2$, where x is the extension of the spring, $x = (x_1 - x_2 - l)$ and l is the spring's unstretched length. You can assume mass 1 remains to the right of mass 2 at all times. **(b)** Rewrite $\mathcal{L}$ in terms of the new variables $X = \frac{1}{2}(x_1 + x_2)$ (the CM position) and x (the extension), and write down the two Lagrange equations for X and x. **(c)** Solve for $X(t)$ and $x(t)$ and describe the motion.

2. A mass m is suspended from a massless string, the other end of which is wrapped several times around a horizontal cylinder of radius R and moment of inertia I, which is free to rotate about a fixed horizontal axle. Using a suitable coordinate, set up the Lagrangian and the Lagrange equations of motion, and find the acceleration of the mass m. (The kinetic energy of a rotating cylinder is $\frac{1}{2}I\omega^2$.)

3. Using the usual angle φ as generalized coordinate, write down the Lagrangian for a simple pendulum of length l suspended from the ceiling of an elevator that is accelerating upward with constant acceleration a. (Be careful when writing T; it is probably safest to write the bob's velocity in component form.) Find the Lagrange equation of motion and show that it is the same as that for a normal, non-accelerating pendulum, except that g has been replaced by $g + a$. In particular, the angular frequency of small oscillations is $\sqrt{(g + a)/l}$.

4. We saw in example 7.3 in the text that the acceleration of an Atwood machine is $\ddot{x} = (m_1 - m_2)g/(m_1 + m_2)$. It is sometimes claimed that this result is "obvious" because, it is said, the effective force on the system is $(m_1 - m_2)g$ and the effective mass is $(m_1 + m_2)$. This is not, perhaps, all that obvious, but it does emerge very naturally int he Lagrangian approach. Recall that the Lagrange equation can be thought of as

$$\text{(generalized force)} = \text{(rate of change of generalized momentum)} \tag{1}$$

Show that for the Atwood machine the generalized force is $(m_1 - m_2)g$ and the generalized momentum is $(m_1 + m_2)\dot{x}$.

5. A pendulum consists of a mass m and a massless stick of length l. The pendulum support oscillates horizontally with a position given by $x(t) = A\cos\omega t$. What is the general solution for the angle of the pendulum as a function of time?

6. Consider a mass m connected to a spring of force constant k, confined to the x axis. We know the system executes simple harmonic motion with frequency $\omega = \sqrt{k/m}$ if the spring is massless. Using the Lagrangian approach, you can find the effect of the spring's mass M as follows. **(a)** Assuming the spring is uniform and stretches uniformly, show that its kinetic energy is $\frac{1}{6}M\dot{x}^2$. (As usual x is the extension of the spring from its equilibrium length.) Write down the Lagrangian for the system of cart plus spring (note $U = \frac{1}{2}kx^2$ still holds.) **(b)** Write down the Lagrange equation and show that the mass still executes SHM, but with angular frequency $\omega = \sqrt{k/(m + M/3)}$; that is, the effect of the spring's mass M is just to add $M/3$.