UNIVERSITY OF ALABAMA
Department of Physics and Astronomy

PH 301 / LeClair Fall 2018

Problem Set 7

due 29 October 2018

Instructions:

1. Answer all questions below (bonus questions optional).
2. Show your work for full credit.
3. All problems are due by 11:59pm on 29 October 2018.
4. You may collaborate, but everyone must turn in their own work.

1. A spacecraft is being designed to dispose of nuclear waste either by carrying it out of the solar system or crashing into the sun. Assume that no planetary flybys are permitted and that thrusts occur only in the orbital plane. Which mission requires the least energy? *Explain.*

2. Assume earth's orbit to be circular and that Thanos has snapped his fingers and decreased the sun's mass by half for inadequately explained reasons. What orbit does the earth have then? Will the earth escape the solar system?

3. **(a)** Using elementary Newtonian mechanics find the period of a mass m_1 in a circular orbit of radius r around a *fixed* mass m_2. **(b)** Using the separation into CM and relative motions, find the corresponding period for the case that m_2 is not fixed and the masses circle each other at a constant distance r apart. Discuss the limit of this result if $m_2 \to \infty$. **(c)** What would be the orbital period if the earth were replaced by a star of mass equal to the solar mass, in a circular orbit, with the distance between the sun and star equal to the present earth-sun distance?

4. Two masses m_1 and m_2 move in a plane and interact by a potential energy $U(r) = \frac{1}{2}kr^2$. Write down their Lagrangian in terms of the CM and relative positions $\mathbf{R}$ and $\mathbf{r}$, and find the equations of motion for the coordinates X, Y and x, y. Describe the motion and find the frequency of the relative motion.

5. By examining the effective potential energy

$$U_{\text{eff}}(r) = -\frac{Gm_1m_2}{r} + \frac{l^2}{2\mu r^2} \tag{1}$$

find the radius at which a planet (or comet, etc.) with angular momentum l can orbit the sun in a circular orbit with fixed radius. [Look at dU_{eff}/dr.] **(b)** Show that this circular orbit is stable, in the sense that a small radial nudge will cause only small radial oscillations. [Look at d^2U_{eff}/dr^2.] **(c)** Show that the period of these oscillations is equal to the planet's orbital period.

6. In deriving Kepler's third law, we made an approximation based on the fact that the sun's mass M_s is much greater than that of the planet m. **(a)** Show that the law should actually read

$$\tau^2 = \left[\frac{4\pi^2}{G(M_s + m)}\right] a^3 \tag{2}$$

and hence the "constant" of proportionality is actually a little different for different planets. **(b)** By what percentage would you expect the "constant" in Kepler's third law to vary among the planets? [You may consider Pluto to be a planet if it makes you feel better, but note this promotion in your answer.]

7. An earth satellite is observed at perigee to be 250 km above the earth's surface and traveling at about 8500 m/s. Find the eccentricity of its orbit and its height above the earth at apogee. [Hint: you will need a bunch of numbers and constants and definitions that google knows about.]

8. For a given earth satellite with given angular momentum l, show that the distance of closest approach r_{min} on a parabolic orbit is half the radius of the circular orbit.