UNIVERSITY OF ALABAMA
Department of Physics and Astronomy

PH 301 / LeClair Fall 2018

Problem Set 8 Answers
due 9 November 2018

1. A massless spring (force constant k_1) is suspended from the ceiling, with a mass m_1 hanging from its lower end. A second massless spring (force constant k_2) is suspended from m_1, and a second mass m_2 is suspended from the spring's lower end. Assuming that the masses move only in a vertical direction and using coordinates y_1 and y_2 measured from the masses' equilibrium positions, show that the equations of motion can be written in the matrix form $\underline{\underline{\boldsymbol{M}}}\ddot{\mathbf{y}} = -\underline{\underline{\boldsymbol{K}}}\mathbf{y}$, where $\mathbf{y}$ is the 2x1 column made up of y_1 and y_2. Find the 2x2 matrices $\underline{\underline{\boldsymbol{M}}}$ and $\underline{\underline{\boldsymbol{K}}}$.

Answer:

$$M = \begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix} \tag{1}$$

$$K = \begin{bmatrix} k_1 + k_2 & -k_2 \\ -k_2 & k_2 \end{bmatrix} \tag{2}$$

2. **(a)** Find the normal frequencies for the system of two carts and three springs shown in Fig. 11.1 in your text, for the case that $m_1 = m_2$ and $k_1 = k_3$ (but k_2 can be different). Check that your answer is correct for the case that $k_1 = k_2$ as well. **(b)** Find and describe the motion in each of the two normal modes in turn. Compare with the motion found for the case that $k_1 = k_2$ in section 11.2 of your textbook. Explain any similarities.

Answer:

$$\omega^2 = \left\{\frac{k_1}{m}, \frac{k_1 + 2k_2}{m}\right\} \tag{3}$$

The motion of the two modes is the same as for equal springs.

3. **(a)** Find the normal frequencies for small oscillations of the double pendulum of Fig. 11.9 in your text for arbitrary values of the masses and lengths. **(b)** Check that your answers are correct for the special case that $m_1 = m_2$ and $L_1 = L_2$. **(c)** Discuss the limit that $m_2 \to 0$.

Answer:

$$\omega^2 = \frac{(m_1+m_2)g(L_1+L_2) \pm \sqrt{(m_1+m_2)^2(L_1+L_2)^2g^2 - 4m_1L_1L_2g^2(m_1+m_2)}}{2m_1L_1L_2} \tag{4}$$

For equal masses and lengths, reduces to $\omega^2 = (g/L)(2 \pm \sqrt{2})$. For $m_2 \to 0$, $\omega^2 = \{g/L_1, g/L_2\}$.

4. **(a)** Referring again to Fig. 11.9 in your textbook, find the normal frequencies and modes of the double pendulum given that $m_1 = 8m$, $m_2 = m$, and $L_1 = L_2 = L$. **(b)** Find the actual motion $[\varphi_1(t), \varphi_2(t)]$ if the pendulum is released from rest with $\varphi_1 = 0$ and $\varphi_2 = \alpha$. Is the motion periodic? (*Hint:* recall a condition for periodicity from a previous homework.)

Answer: **(a)**

$$\omega = \{\frac{3g}{4L}, \frac{3g}{2L}\} = \{\frac{3}{4}\omega_o, \frac{3}{2}\omega_o\} \tag{5}$$

$$\mathbf{a}_1 = A_1 \begin{bmatrix} 1 \\ 3 \end{bmatrix} \qquad \text{in phase, lower has 3x ampl.} \tag{6}$$

$$\mathbf{a}_2 = A_2 \begin{bmatrix} 1 \\ -3 \end{bmatrix} \qquad \text{out of phase, lower has 3x ampl.} \tag{7}$$

(b) $A_1 = -A_2 = \alpha/6$; not periodic.

5. We desire to superpose the oscillations of several simple harmonic oscillators having the same frequency ω and amplitude A, but differing from one another by constant phase increments α; that is,

$$x(t) = A\cos\omega t + A\cos(\omega t + \alpha) + A\cos(\omega t + 2\alpha) + A\cos(\omega t + 3\alpha) + \cdots \tag{8}$$

You should be able to show that the sum on the right-hand side above is equivalent to a single oscillator with frequency ω, amplitude A_o, and phase φ. Specifically, show that for N oscillators

$$x(t) = (NA)\frac{\sin(N\alpha/2)}{N\sin(\alpha/2)}\cos\left[\omega t + \left(\frac{N-1}{2}\right)\alpha\right] \tag{9}$$

[**Bonus, +2:** Explain why an array of oscillators with a constant phase offset might be useful.]

Answer: "Sum of sines and cosines with arguments in arithmetic progression."

6. Two resistors are connected in parallel, with values R_1 and R_2. A total current I_o divides somehow between them. Show that the condition $I_1 + I_2 = I_o$, together with the requirement of minimum power dissipation, leads to the same current values that we would calculate with normal circuit formulas. This illustrates a general variational principle that holds for direct current networks: the distribution of currents within the networks, for a given input current I_o, is always that which gives the least total power dissipation.

Answer: Again, you know the answer. If the current is I_1 in one resistor, it is $I - I_1$ in the other. Write down the total power and minimize with respect to I_1.