University of Alabama
Department of Physics and Astronomy

PH 301 / LeClair — Fall 2018

# Problem Set 9

## due 21 November 2018

**Instructions:**

1. Answer all questions below (bonus questions optional).
2. Show your work for full credit.
3. All problems are due by 11:59pm on 21 November 2018.
4. You may collaborate, but everyone must turn in their own work.

**1.** A donut-shaped space station (outer radius $R$) arranges for artificial gravity by spinning on the axis of the donut with angular velocity $\omega$. Sketch the forces on, and accelerations of, an astronaut standing in the station **(a)** as seen from an inertial frame outside the station and **(b)** as seen from the astronaut's personal rest frame (which has an acceleration $A = \omega^2 R$ as seen from the inertial frame). What angular velocity is needed if $R = 40\,\mathrm{m}$ and the apparent gravity is equal to the usual value of about $10\,\mathrm{m/s^2}$? **(c)** What is the percentage difference between the perceived $g$ at a six-foot astronaut's feet ($R = 40\,\mathrm{m}$) and at her head ($R = 38\,\mathrm{m}$)?

**2.** **(a)** Consider the tidal force (given by equation 9.12 in your text) on a mass $m$ at the position $P$ in figure below. Write $d$ as $(d_o - R_e) = d_o(1 - R_e/d_o)$ and use the binomial approximation $(1-\epsilon)^{-2} \approx 1 + 2\epsilon$ to show that $F_{\text{tid}} \approx -(2GM_m m R_e/d_o^3)\,\hat{\mathbf{x}}$. Confirm the direction of the force as shown below and make a numerical comparison of the tidal force with the gravitational force $m\mathbf{g}$ of the earth. **(b)** Do the corresponding calculations for point $R$ in the figure below. Compare this force you found in part (a) in magnitude and direction.

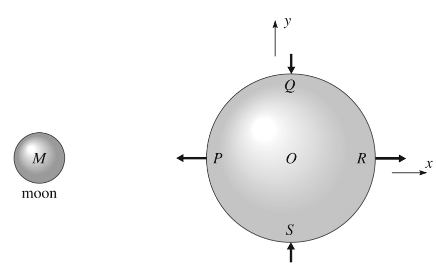



Figure 9.4
*Taylor* CLASSICAL MECHANICS


**3.** The equation of motion for a rotating frame with constant angular velocity $\boldsymbol{\Omega}$ was (equation 9.34 in your text)

$$m\ddot{\mathbf{r}} = \mathbf{F} = 2m\dot{\mathbf{r}} \times \boldsymbol{\Omega} + m\,(\boldsymbol{\Omega} \times \mathbf{r}) \times \boldsymbol{\Omega} \tag{1}$$

If we do *not* make the assumption that $\boldsymbol{\Omega}$ is constant, show that for $\dot{\boldsymbol{\Omega}} \neq 0$ there is a third "fictitious force," sometimes called the *azimuthal force*, on the right-hand side of the equation above equal to $m\mathbf{r} \times \dot{\boldsymbol{\Omega}}$

**4.** I am spinning a bucket of water about its vertical axis with angular velocity $\Omega$. Show that, once the water has settled in equilibrium relative to the bucket, its surface will be a parabola. (Use cylindrical polar coordinates and remember that the surface is an equipotential under the combined effect of the gravitational and centrifugal forces.)

**5.** The center of a long frictionless rod is pivoted at the origin and the rod is forced to rotate at a constant angular velocity $\Omega$ in a horizontal plane. Write down the equation of motion for a bead that is threaded on the rod, using the coordinates $x$ and $y$ of a frame that rotates with the rod (with $x$ along the rod and $y$ perpendicular to it). Solve for $x(t)$. What is the role of the centrifugal force? What of the Coriolis force?

**6.** In lecture we used a method of successive approximations to find the orbit of an object that is dropped from rest, correct to first order in the earth's angular velocity $\Omega$. Show that in the same way that if an object is thrown with initial velocity $\mathbf{v}_o$ from a point $O$ on the earth's surface at colatitude $\theta$, then to first order in $\Omega$ its orbit is

$$x = v_{xo}t + \Omega\left(v_{yo}\cos\theta - v_{zo}\sin\theta\right)t^2 + \frac{1}{3}\Omega g t^3 \sin\theta \tag{2}$$

$$y = v_{yo}t - \Omega\left(v_{xo}\cos\theta\right)t^2 \tag{3}$$

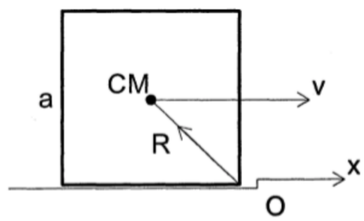
$$z = v_{zo}t - \frac{1}{2}gt^2 + \Omega\left(v_{xo}\sin\theta\right)t^2 \tag{4}$$

[First solve the equations of motion, equations 9.53 in your text, in zeroth order ignoring $\Omega$ entirely. Use this $\Omega = 0$ solution for $\dot{x}$, $\dot{y}$, and $\dot{z}$ in the right-hand side of (9.53) and integrate to give the next approximation. Assume that $v_o$ is small enough that air resistance is negligible and that $\mathbf{g}$ is a constant throughout the flight.]

**7.** Use the result of the previous problem. A naval gun shoots a shell at colatitude $\theta$ in a direction that is $\alpha$ above the horizontal and due east, with muzzle speed $v_o$. **(a)** Ignoring the earth's rotation (and air resistance), find how long ($t$) the shell would be in the air and how far away ($R$) it would land. If $v_o = 500\,\mathrm{m/s}$ and $\alpha = 20°$, what are $t$ and $R$? **(b)** A naval gunner spots an enemy ship due east at the range $R$ of part (a) and, forgetting about the Coriolis effect, aims his gun exactly as in part (a). Find out by how far north or south, and in what direction, the shell will miss the target, in terms of $\Omega$, $v_o$, $\alpha$, $\theta$, and $g$. (It will also miss in the east-west direction, but this is perhaps less critical.) If the incident occurs at latitude 50° north (so $\theta = 40°$), what is this distance?

[This problem is an issue in long-range gunnery, though the story about the British navy in your textbook is probably apocryphal.]

**8.** Find the moment of inertia for a uniform cube of mass $M$ and edge $a$, rotating about an edge. (See example 10.2 in your textbook.). Now do the following. The cube is sliding with velocity $\mathbf{v}$ across a flat horizontal frictionless table when it hits a straight very low step perpendicular to $\mathbf{v}$, and the leading lower edge comes abruptly to rest. **(a)** By considering what quantities are conserved before, during, and after the brief collision, find the cube's angular velocity just after the collision. **(b)** Find the minimum speed $v$ for which the cube rolls over after hitting the step.




**9.** The definition of the inertia tensor was

$$I_{xx} = \sum m_\alpha \left(y_\alpha^2 + z_\alpha^2\right) \tag{5}$$

$$I_{xy} = -\sum m_\alpha x_\alpha y_\alpha \tag{6}$$

and so on, with corresponding expressions for all 9 components. This has the rather ugly feature that the diagonal and off-diagonal elements are defined by completely different equations. Show that the two definitions can be combined into a single equation (which is slightly less messy in integral form)

$$I_{ij} = \int \varrho \left(r^2 \delta_{ij} - r_i r_j\right) \, dV \tag{7}$$

where $\delta_{ij}$ is the Kronecker delta symbol,

$$\delta_{ij} = \begin{cases} 1 & i = j \\ 0 & i \neq j \end{cases} \tag{8}$$