$\boxed{LC}$ $\frac{Q}{C} + L\frac{dI}{dt} = \frac{1}{C}Q + L\ddot{Q} = 0$

Compare : $U(q)$, $K(\dot{q})$ is usual. Suggests $m \to L$, $k \to \frac{1}{C}$

if $T = \frac{1}{2}LI^2 = \frac{1}{2}L\dot{Q}^2$ energy in L! $\left\}\begin{array}{l}\dot{Q} \sim \text{velocity} \\ Q \sim \text{posn}\end{array}\right.$

and $U = \frac{1}{2}\frac{Q^2}{C}$ energy in C. $\left\}\begin{array}{l}Q \sim \text{posn} \\ 1/C \sim k\end{array}\right.$

looks like mass spring w/ $L = m$!, $k = \frac{1}{C}$

$$\mathcal{L} = \frac{1}{2}L\dot{Q}^2 - \frac{Q^2}{2C}$$

$$\frac{\partial \mathcal{L}}{\partial Q} = -\frac{Q}{C} \qquad \frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{Q}} = \frac{d}{dt}(L\dot{Q}) = L\ddot{Q}$$

$\Rightarrow L\ddot{Q} + \frac{1}{C}Q = 0$ SHM!

$\ddot{Q} = -\frac{1}{LC}Q$, $\omega_0 = \frac{1}{\sqrt{LC}}$, $Q = Q_0\cos(\omega_0 t)$

HW5 #2

O to P(1,1) making $\int_O^P (y'^2 + yy' + y^2)\,dx$ stationary

$f = y'^2 + yy' + y^2$, apply Euler-Lagrange, note $y = y(x)$

$$\frac{\partial f}{\partial y} = y' + 2y \qquad \frac{d}{dx}\frac{\partial f}{\partial y'} = \frac{d}{dx}(2y' + y) = 2y'' + y'$$

$$\Rightarrow 0 = y' + 2y - (2y'' + y') = 2y - 2y'' = 0$$

$$\Rightarrow y'' = y$$

Soln known: $y(x) = Ae^{x} + Be^{-x}$

know $y(0)=0$, $y(1)=1$ from given coords

$\Rightarrow$ $0 = A + B \Rightarrow A = -B$

$1 = Ae + \frac{B}{e} = Ae - A/e$

$\Rightarrow$ $A = \frac{1}{e - 1/e} = -B$

$$y(x) = \frac{e^x}{e - 1/e} - \frac{e^{-x}}{e - 1/e} = \frac{e^x - e^{-x}}{e - e^{-1}} = \frac{2\sinh(x)}{2\sinh(1)}$$

$$\boxed{y(x) = \frac{\sinh(x)}{\sinh(1)}}$$

3. $f = f(y', x)$ indep y?

$\frac{\partial f}{\partial y} - \frac{d}{dt}\frac{\partial f}{\partial y'} = 0$ but $\frac{\partial f}{\partial y} = 0$

$\Rightarrow$ $\frac{d}{dt}\frac{\partial f}{\partial y'} = 0$ $\Rightarrow$ $\frac{\partial f}{\partial y'} = \text{const}$

Lagrangian?

$f = T(\dot{q}) - U(q) \Rightarrow \frac{\partial T}{\partial \dot{q}} = \text{const}$

particle: $T = \frac{1}{2}m\dot{x}^2 \Rightarrow \frac{\partial T}{\partial \dot{x}} = m\dot{x} = \text{const}$

cons p!