University of Alabama
Department of Physics and Astronomy

PH 301 / LeClair Fall 2018

Lecture 1
22 Aug 2018

1 Classical Physics

We know all of classical physics in principle.

$$\textbf{Maxwell} \tag{1}$$
$$\nabla \cdot \mathbf{E} = \rho/\epsilon_o \qquad \text{charges source E} \tag{2}$$
$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \qquad \text{time varying B, get circulating E} \tag{3}$$
$$c^2 \nabla \times \mathbf{B} = \mathbf{j}/\epsilon_o + \frac{\partial \mathbf{E}}{\partial t} \qquad \text{time-varying E and currents give B} \tag{4}$$
$$\nabla \cdot \mathbf{B} = 0 \qquad \text{no magnetic monopoles} \tag{5}$$
$$\textbf{Conservation of charge} \tag{6}$$
$$\nabla \cdot \mathbf{j} = -\frac{\partial \rho}{\partial t} \tag{7}$$
$$\textbf{Force} \tag{8}$$
$$\mathbf{F} = q\left(\mathbf{E} + \mathbf{v} \times \mathbf{B}\right) \tag{9}$$
$$\textbf{Motion} \tag{10}$$
$$\frac{d\mathbf{p}}{dt} = \mathbf{F} \tag{11}$$
$$\mathbf{p} = \frac{m\mathbf{v}}{\sqrt{1 - v^2/c^2}} \approx m\mathbf{v} \quad (v \ll c) \tag{12}$$
$$\textbf{Gravity} \tag{13}$$
$$\mathbf{F} = -\frac{G m_1 m_2}{r_{12}^2}\,\hat{r}_{12} \tag{14}$$

- The solutions are another matter entirely, *especially* in practical cases.

- Even though superseded by GR, QM, a vast range of everyday problems can be solved completely and to arbitrary accuracy with classical mechanics

- now: focus on mechanics; enable solutions of more complex problems with better use of math

- now: new formulations of force and motion to enable easier solutions (Lagrangian, Hamiltonian) - *optimization* to find a path rather than the brute force newtonian approach

2 Space

Any point in space can be labeled by a position vector $\mathbf{r}$, which points from the origin to that point.

Figure 1: *We define the position of a point P by its position vector $\mathbf{r}$, pointing from the origin O to P. Here we choose to describe $\mathbf{r}$ in cartesian coordinates. From "Classical Mechanics" by Taylor, Fig. 1.1*

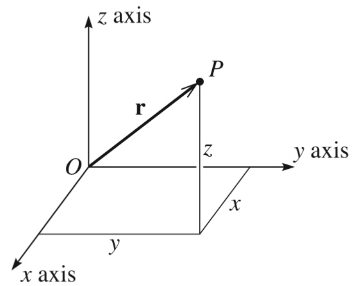

Figure 1.1
Taylor CLASSICAL MECHANICS

- The vector is **coordinate independent**, it is just an arrow, but it has various representations. E.g., xyz, $r\theta\phi$, $r\phi z$.
- Most commonly we'll use xyz for simplicity, but choosing coordinates that match the symmetry of the problem can save a lot of work
- introduce **unit vectors** $\hat{\mathbf{x}}$, $\hat{\mathbf{y}}$, $\hat{\mathbf{z}}$ – walk 1 unit along the given axis. Thus,

$$\mathbf{r} = x\,\hat{\mathbf{x}} + y\,\hat{\mathbf{y}} + z\,\hat{\mathbf{z}} \tag{15}$$

- Some books use $\hat{\boldsymbol{\imath}}$, $\hat{\boldsymbol{\jmath}}$, $\hat{\mathbf{k}}$. Doesn't matter.
- Sometimes we'll write a vector as (x, y, z). Not quite formally OK, but usually unambiguous.
- Sometimes $\hat{e}_1 = \hat{\mathbf{x}}$, $\hat{e}_2 = \hat{\mathbf{y}}$, $\hat{e}_3 = \hat{\mathbf{z}}$ when lots of summations are necessary, e.g. $\mathbf{r} = \sum_{i=1}^{i=3} r_i \mathbf{e}_i$ or $\sum_i \mathbf{e}_i \cdot \mathbf{j}$
- No real *advantage* to last approach, but for complex equations it can help
- We'll use the simplest version that is clear and unambiguous.
- We assume vector operations are understood at this point.

3 Newton's Laws

First law: in the absence of forces, particles move with constant velocity.
Second law: for a particle of mass m, the net force is $\mathbf{F} = m\mathbf{a}$.

NOTATION. We use dots above a variable to represent time derivatives. Two dots, second derivative. Examples:

$$\dot{x} = \frac{dx}{dt} = v \tag{16}$$

$$\ddot{x} = \frac{d^2x}{dt^2} = \frac{dv}{dt} = a \tag{17}$$

WATCH your penmanship - stray ink marks now operate on your equations.

For a particle with *constant* m ($dm/dt = \dot{m} = 0$; no rockets) we can now rephrase:

$$p = mv \tag{18}$$

$$\dot{p} = \frac{dp}{dt} = m\dot{v} + \dot{m}v = m\dot{v} = ma \tag{19}$$

$$\text{or} \qquad F = \dot{p} \tag{20}$$

So things like $\mathbf{F} = m\ddot{\mathbf{r}}$ are differential equations. Some are totally intractable, but computers are awesome now. Some are easy: constant $F = F_o$ for example:

$$\ddot{x} = \frac{F_o}{m} \qquad \Longrightarrow \qquad x = \frac{1}{2}\frac{F_o}{m}t^2 + \ldots \text{ (terms depending on initial conditions)} \tag{21}$$

For constant force, $\ddot{x} = F_o/m$, most common technique is to just integrate

$$\dot{x} = \int \ddot{x}\, dt = v_o + \frac{F_o}{m}t \tag{22}$$

$$x = \int \dot{x}(t)\, dt = x_o + v_o t + \frac{F_o}{2m}t^2 \tag{23}$$

We had a 2nd order differential equation to start with, ($\ddot{x}$ =const), so we require 2 integrations and 2 overall constants (x_o, v_o).

We will not need to use a lot of fancy differential equations techniques, 90% of our problems are solved in one of three ways:

1. trivial as above, just integrate
2. separate variables and integrate both sides
3. force it to look like simple harmonic osc. by approximating or neglecting stuff, $\ddot{x} = -\omega^2 x$.

OTOH, knowing the fancy MA238 techniques will make your life much easier.

4 Intertial Frames

We mean to choose a POV that is not accelerating if at all possible. Otherwise Newton's laws may not hold, and intuition certainly will not.

BUT there are interesting situations where we need to use an accelerating POV, e.g., Earth itself! (Coriolis force.) Small effect, but important consequences. Deal with this in Ch. 9; for now pick inertial frames.

Presume *also* non-relativistic situations so $p = mv$ holds ($v \ll c$). That means that in inertial frames Newton's laws are precisely and uniformly valid.

5 Finally the third law[i]

Two objects interact: $\mathbf{F}_{12} = -\mathbf{F}_{21}$.

Figure 2: *Newton's third law:* $\mathbf{F}_{12} = -\mathbf{F}_{21}$. *From "Classical Mechanics" by Taylor, Fig. 1.5*

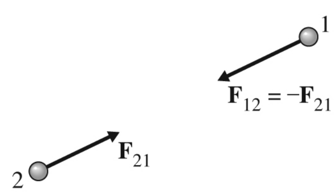

Figure 1.5
Taylor CLASSICAL MECHANICS

As illustrated, central force acts along a line connecting the two particles. Central forces are not *required* for the third law, just a common example.

Net force on particle 1 $\equiv \mathbf{F}_1 = \mathbf{F}_{12} + \mathbf{F}_1^{\text{ext}} = \dot{\mathbf{p}}_1$. This implies $\dot{\mathbf{p}}_2 = \mathbf{F}_{21} + \mathbf{F}_2^{\text{ext}}$.

The total momentum of the two particle system:

$$\dot{\mathbf{P}} = \dot{\mathbf{p}}_1 + \dot{\mathbf{p}}_2 = \mathbf{F}_{12} + \mathbf{F}_{21} + \mathbf{F}_1^{\text{ext}} + \mathbf{F}_2^{\text{ext}} \tag{24}$$

$$\dot{\mathbf{P}} = \mathbf{F}_1^{\text{ext}} + \mathbf{F}_2^{\text{ext}} \equiv \mathbf{F}^{\text{ext}} \tag{25}$$

[i] We skipped most of this in class due to time constraints. It should be familiar.

Figure 3: *Two objects exert forces on each other and may be subject to additional "external" forces from other objects not shown. From "Classical Mechanics" by Taylor, Fig. 1.6*

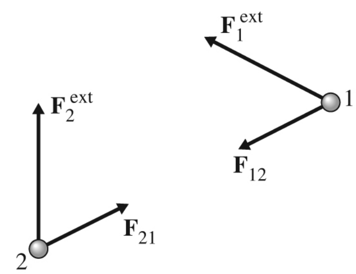

Figure 1.6
Taylor CLASSICAL MECHANICS

So $\dot{\mathbf{P}} = \mathbf{F}^{\text{ext}}$. If $\mathbf{F}^{\text{ext}} = 0$, then

$$\dot{\mathbf{p}}_1 + \dot{\mathbf{p}}_2 = 0 \qquad \Longrightarrow \qquad \dot{\mathbf{p}}_1 = -\dot{\mathbf{p}}_2 \qquad \text{or} \qquad \mathbf{p}_1 + \mathbf{p}_2 = \text{const} \tag{26}$$

Multiparticle systems: you just sum all the forces. All internal interactions cancel because they come in pairs.

$\Longrightarrow$ if $\mathbf{F}^{\text{ext}} = 0$ for an N particle system, the system's total $\mathbf{P}$ is constant - internal forces do not alter total $\mathbf{P}$.

One of the most important results of classical physics - true in relativity and QM as well.

Domain of validity:

- relativity softens the third law: only true at the same time and location for the same observer. One observer may see $F_1(t) = -F_2(t)$ while an observer at another location may not!
- Magnetic interaction between moving charges: "opposing" forces are in different directions! Problem: have to include "field momentum" – $p_1 + p_2 + p_{\text{field}} = \text{const}$ but $p_1 + p_2 \neq \text{const}$
- Non-local interactions, separated observers give trouble
- In both cases, for $v \ll c$ the difference is totally negligible ($F \sim v^2/c^2$ for magnetic case)

We will only do non-relativistic physics, and assume violations of the 3rd law are outside our domain or totally negligible.

6 Newton's 2nd law in cartesian coordinates

$\mathbf{F} = m\ddot{\mathbf{r}}$ noting definitions in cartesian coordinates:

$$\mathbf{F} = F_x\hat{\mathbf{x}} + F_y\hat{\mathbf{y}} + F_z\hat{\mathbf{z}} \tag{27}$$

$$\mathbf{r} = x\hat{\mathbf{x}} + y\hat{\mathbf{y}} + z\hat{\mathbf{z}} \tag{28}$$

Easy to see then $\ddot{\mathbf{r}} = \ddot{x}\hat{\mathbf{x}} + \ddot{y}\hat{\mathbf{y}} + \ddot{z}\hat{\mathbf{z}}$, so

$$F_x\hat{\mathbf{x}} + F_y\hat{\mathbf{y}} + F_z\hat{\mathbf{z}} = \ddot{x}\hat{\mathbf{x}} + \ddot{y}\hat{\mathbf{y}} + \ddot{z}\hat{\mathbf{z}} \tag{29}$$

$$\text{or} \quad \mathbf{F} = m\ddot{\mathbf{r}} \tag{30}$$

Since coordinate axes are orthogonal, the $\hat{\mathbf{x}}$ bits only equate to other $\hat{\mathbf{x}}$ bits, etc., so this implies 3 equations:

$$\mathbf{F} = m\ddot{\mathbf{r}} \qquad \Longleftrightarrow \qquad \begin{cases} F_x &= m\ddot{x} \\ F_y &= m\ddot{y} \\ F_z &= m\ddot{z} \end{cases} \tag{31}$$

7 ENDING

For FRIDAY: "Homework 0" on Blackboard - two warmup problems, PH105 level. Can submit on Blackboard as PDF/PNG/etc, or bring a hard copy to GL206 before 4:45 on Friday.