Ph301 L28
F18

Non-inertial frames

- usually we avoid them, but sometimes you want to describe how a non-inertial observer will see things
  - e.g. tossing a coin in a car
    accounting for rotation of earth

Acceleration w/out rotation

S non-inertial

S_0 inertial frame

- car moves w/velocity $\vec{V}$ and accel $\vec{A} = \dot{\vec{V}}$ rel to inertial frame S_0
- ball is tossed in car, mass m

- in S_0, Newton's laws hold, $m\ddot{\vec{r}}_0 = \vec{F}$ = net sum of all forces
  measured relative to S_0

- in S, ball's velocity is

ball rel S_0 = $\dot{\vec{r}}_0 = \dot{\vec{r}} + \vec{V}$ = (ball rel car) + (car rel ground)

- differentiating, $\ddot{\vec{r}}_0 = \ddot{\vec{r}} + \vec{A}$ or $\underline{\ddot{\vec{r}} = \ddot{\vec{r}}_0 - A}$

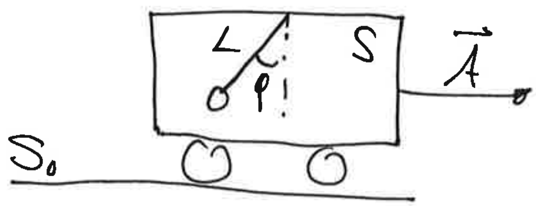
$$\Rightarrow m\ddot{\vec{r}} = m\ddot{\vec{r}}_0 - m\vec{A} = \vec{F} - m\vec{A}$$

- in addition to all forces present, in S there is an extra term $-m\vec{A}$! Can use Newton's law <u>provided</u> we agree to add in this extra force-like term! "<u>inertial force</u>"

$$\vec{F}_{\text{inertial}} = -m\vec{A}$$

- already saw this with central forces - price of using a non-inertial frame
- just like sitting in an accelerating car - sudden braking ($\vec{A}$ backwards) gives $-m\vec{A}$ forwards
- or going around a curve - centrifugal force we already saw

Example: pendulum in an accelerating car

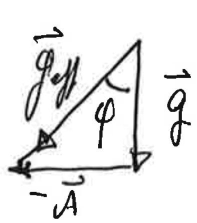


- what is the equilibrium angle?
- in an inertial frame S_0, forces are tension and weight $\vec{F} = \vec{T} + m\vec{g}$

In the non-inertial frame S we have the extra inertial force $-m\vec{A}$, so $m\ddot{\vec{r}} = \vec{T} + m\vec{g} - m\vec{A}$

$$\Rightarrow m\ddot{\vec{r}} = \vec{T} + m(\vec{g} - \vec{A}) = \vec{T} + m\vec{g}_{eff} \quad w/\ \vec{g}_{eff} = \vec{g} - \vec{A}$$

! motion is just like in the inertial frame, except that $\vec{g}$ is replaced by an effective value $\vec{g}_{eff}$



in equilibrium, $\ddot{\vec{r}} = 0$, so $\vec{T} = -m\vec{g}_{eff}$

and $\varphi_{eq} = \tan^{-1}\left(\frac{A}{g}\right)$ (check: $A = 0$, $\varphi_{eq} = 0$ ✓)

since equation of motion is same as inertial case, we know

$$\omega = \sqrt{\frac{g_{eff}}{L}} = \sqrt{\frac{\sqrt{g^2 + A^2}}{L}}$$ ← very hard by other means!

The tides

one wrong model:  moon pulls water and creates a bulge

problem: there would only be 1 tide per day

Correct model is more complicated

- main effect of moon is to give earth (water & all) an acceleration $\vec{A}$ toward the moon
- this is the centripetal accel of the earth as the earth and moon circle around their center of mass
- almost the same as if all mass were concentrated at the center of earth (not quite spherical)
- on side nearer to moon: force is slightly larger than it would be at the center of earth ⇒ bulge
- on side away from moon: force is slightly smaller, as though objects are slightly repelled from moon

forces in play? for mass m on earth surface

(1) gravitational pull $m\vec{g}$ of earth

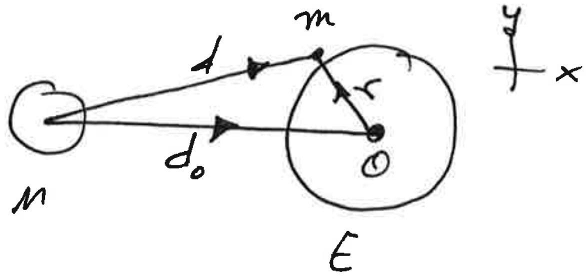
(2) grav. pull of moon $-\frac{GM_m m \hat{d}}{d^2}$

M_m = moon mass

d = dist from object to moon

(3) net non-grav force $\vec{F}_{ng}$ e.g. buoyancy

the acceleration of earth's center is $\vec{A} = -GM_m \frac{\hat{d}_o}{d_o^2}$



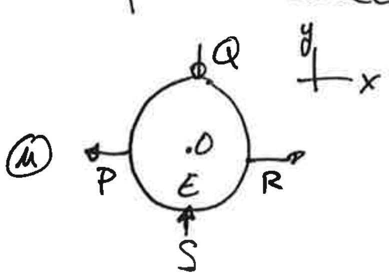
d_o = center to center dist earth-moon

overall, $m\ddot{\vec{r}} = \vec{F} - m\vec{A} = \left(m\vec{g} - GM_m m \frac{\hat{d}}{d^2} + \vec{F}_{ng}\right) + \underbrace{GM_m m \frac{\hat{d}_o}{d_o^2}}_{\text{inertial force}}$

with $\vec{F}_{tidal} = -GM_m m\left(\frac{\hat{d}}{d^2} - \frac{\hat{d}_o}{d_o^2}\right)$ ~ diff in force between earth's center and loc. of m

$\Rightarrow \quad m\ddot{\vec{r}} = m\vec{g} + \vec{F}_{tidal} + \vec{F}_{ng}$

- entire effect of moon on the motion of an object near earth is $\vec{F}_{tidal}$
- point directly facing moon? $\vec{d} = \vec{MP}$ $\vec{d}_o = \vec{MO}$ point in the same direction, but $d < d_o \Rightarrow 1^{st}$ term in $\vec{F}_{tid}$ dominates



- point R directly away from moon?
  $\vec{d} = \vec{MR}$, $\vec{d}_o = \vec{MO}$, and $d > d_o$
  $\Rightarrow 2^{nd}$ term dominates, force is away from moon
- Q, S? two terms cancel almost exactly along x but only d^2 term has a y component
  $\Rightarrow$ force is inward toward O
  $\Rightarrow$ 2 tides per day as observed

Magnitude of the tides

- Surface of the ocean is an equipotential surface
- drop of water? in equilibrium w/3 forces:

  $m\vec{g}$ from earth

  $\vec{F}_{tidal}$ from moon

  $\vec{F}_P$ pressure of surrounding water

- static fluids can't have shear forces, $\vec{F}_P$ must be normal to the surface of the ocean
- because drop is in equ, $\vec{F}_{tidal} + m\vec{g}$ must also be normal
- both $\vec{F}_{tidal}$ and $m\vec{g}$ are conservative, so

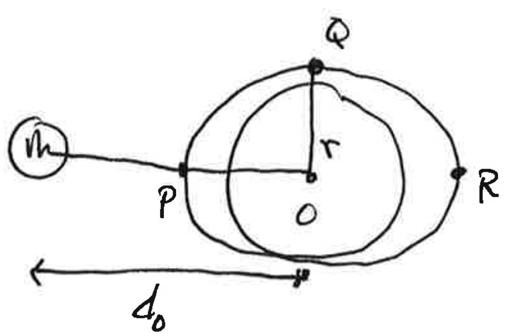
$$m\vec{g} = -\nabla U_{eg} \quad \text{and} \quad \vec{F}_{tid} = -\nabla U_{tid}$$

$$\Rightarrow U_{tidal} = -GM_m m\left(\frac{1}{d} + \frac{x}{d_0^2}\right) \quad \text{after integrating}$$

- if $\vec{F}_{tidal} + m\vec{g}$ is normal to surface, so is $\nabla(U_{eg} + U_{tid})$

  $\Rightarrow U = U_{eg} + U_{tid}$ is constant on the surface

  so ocean's surface is an equipotential



$$\Rightarrow U(P) = U(Q)$$

$$\Rightarrow \underbrace{U_{eg}(P) - U_{eg}(Q)}_{mgh,\ h = \text{height of tide}} = U_{tid}(Q) - U_{tid}(P)$$

for RHS, need U_{tid} at Q, $d=\sqrt{d_o^2+r^2}$ $(r \simeq R_e)$ and $x=0$

$$\Rightarrow U_{tid}(Q) = -GM_m m \frac{1}{\sqrt{d_o^2+R_e^2}} = \frac{-GM_m m}{d_o} \frac{1}{\sqrt{1+(R_e/d_o)^2}}$$

recall for $x \ll 1$, $(1+x)^n \simeq 1+nx$ – $R_e \ll d_o$

$$\Rightarrow U_{tid}(Q) \simeq -\frac{GM_m m}{d_o}\left(1-\frac{R_e^2}{2d_o^2}\right)$$

at P, $d=d_o-R_e$, $x=-R_e$, same procedure,

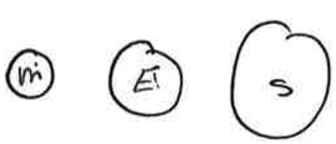
$$U_{tid}(P) \simeq -\frac{GM_m m}{d_o}\left(1+\frac{R_e^2}{d_o^2}\right)$$

$$\Rightarrow U_{tid}(Q) - U_{tid}(P) = \frac{GM_m m}{d_o} \cdot \frac{3R_e^2}{2d_o^2} = mgh$$

noting $g = \frac{GM_e}{R_e^2}$, $h \simeq \frac{3M_m R_e^4}{2M_e d_o^3} \simeq 54\,cm$

- Sun has a similar but smaller effect

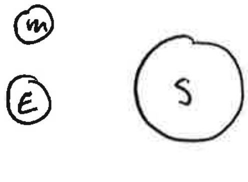
  $M_s \gg M_m$ but d_o much larger $\Rightarrow h_{sun} \simeq 25\,cm$



full moon (new moon if its in the middle)

$\Rightarrow$ 2 tidal forces combine: spring tides

$h_{spring} \simeq 54+25 = 79\,cm$

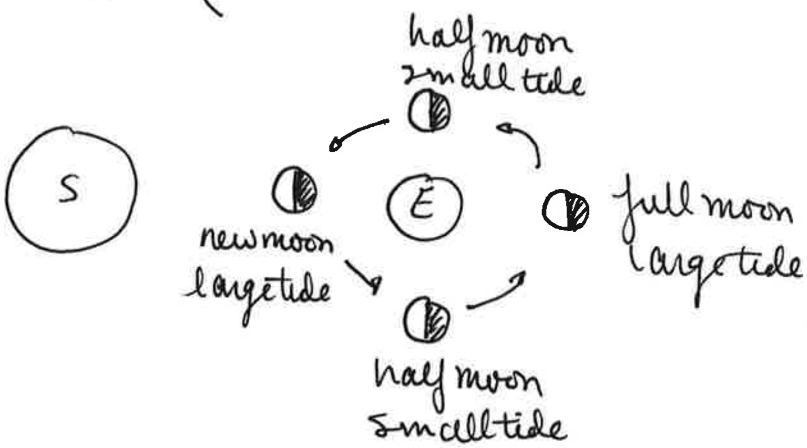


right angle: effects oppose each other – neap tide

$h_{neap} \simeq 54-25 = 29\,cm$

geography still key - small seas shut off from ocean will have much smaller tides; a bay or inlet can cause tides to "build up"

Bay of Fundy: 16m (50')! Mediterranean: few cm
Great Lakes



~ 2 week cycle