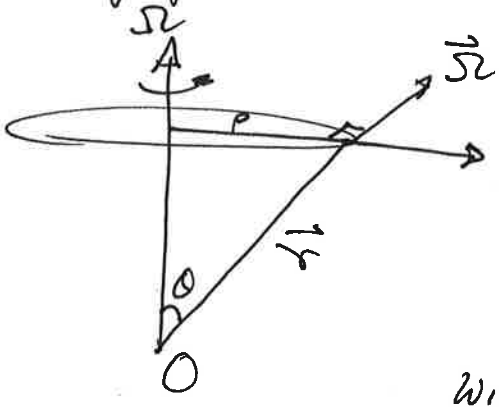
Centrifugal force



$\vec{F}_{cf} = m(\vec{\Omega}\times\vec{r})\times\vec{\Omega}$ radially outward

$|\vec{F}_{cf}| = m\Omega^2\rho$ in $\hat{\rho}$ dir

with $\vec{v}_t = \vec{\Omega}\times\vec{r}$, $v_t = \Omega\rho$ and $F_{cf} = \frac{mv^2}{\rho}$

Free fall acceleration: $\vec{g}$ is surprisingly complicated now

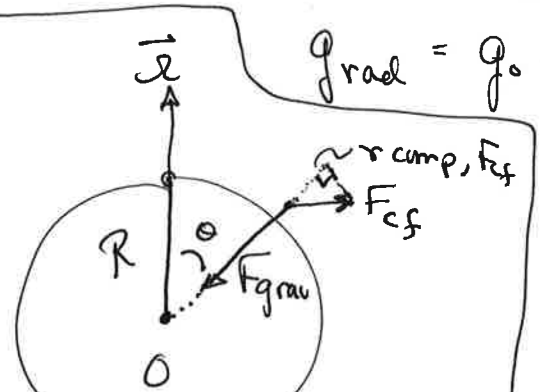
relative to earth, $m\ddot{\vec{r}} = \vec{F}_{grav} + \vec{F}_{cf} = -\frac{GMm}{R^2}\hat{r} + m\Omega^2\rho\hat{\rho}$ hmm... not in same direction!

let $m\vec{g}_0 = -\frac{GMm}{R^2}\hat{r}$ – "true" grav force, what we'd see w/o rotation

effective force is then $m\ddot{\vec{r}} = \vec{F}_{eff} = m\vec{g}_0 + m\Omega^2 R\sin\theta\,\hat{\rho}$ $(\rho = R\sin\theta)$

and accel (effective): $\vec{g} = \vec{g}_0 + \Omega^2 R\sin\theta\,\hat{\rho}$

- the purely radial part projects $\hat{\rho}$ component along $\hat{r}$ (multiply by $\sin\theta$)



$$g_{rad} = g_0 - \underbrace{\Omega^2 R\sin^2\theta}$$

zero at poles $(\theta = 0, \pi)$

max at equator. $\Omega^2 R \sim 0.034\ m/s^2$

(0.3% less @ equ.)

there must also be a tangential part though - the $\cos\theta$ comp

$g_{tan} = \Omega^2 R \sin\theta \cos\theta$ zero @ poles & equator
max at $45°$

$\Rightarrow$ free fall accel is NOT in the direction of grav. force!
there is a sideways component to free-fall accel!

angle btw $\vec{g}, \vec{g}_0 = \alpha \approx \left.\frac{g_{tan}}{g_{rad}}\right|_{\theta=45} = \frac{\Omega^2 R}{2g} \simeq 0.1°$

- actually quite hard to measure - any reasonable definition of "vertical" to define $\vec{g}_0$ winds up being subject to same tangential force
- define vertical = along $\vec{g}$ for practical reasons
  horiz = $\perp \vec{g}$
- usually an irrelevant distinction btw $\vec{g}, \vec{g}_0$

<u>Coriolis Force</u> - we needed a second inertial force in a rot. frame

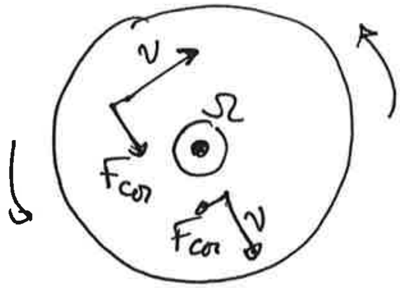
$\vec{F}_{cor} = 2m\dot{\vec{r}} \times \vec{\Omega} = 2m\vec{v} \times \vec{\Omega}$ looks very much like $\vec{F} = q\vec{v} \times \vec{B}$!

- no deep significance, but will help w/ visualization
- can suspect this will curve projectile trajectories

how big? $\Omega \sim 7 \times 10^{-5}\ s^{-1}$ from before
say baseball @ 50 m/s, $a_{max} = \frac{F_{cor}}{m} \sim 2v\Omega \sim 7 \times 10^{-3}\ m/s^2$
negl. compared to g or air resistance

Coriolis will be relevant for very fast, long-range projectiles
- or - when it acts over a long time, e.g. Foucault pendulum

Direction: like magnetic force, $2m\vec{v}\times\vec{\Omega}$ is always $\perp \vec{v}$
and also $\perp$ to $\vec{\Omega}$

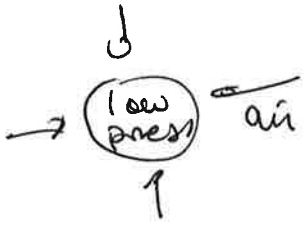


- for counterclockwise rotation, Ω is out of the board, so $\vec{F}_{cor}$ will lie in plane
- for ccw, deflection is always to the right (left for clockwise rot) - like magn force

- this could be northern hemisphere viewed from above
  Coriolis defects bodies to right
  (to left in Southern hemisphere)

- long-range weaponry - have to adjust aim for this

- cyclones! Counterclockwise in northern hemisphere
  clockwise in Southern



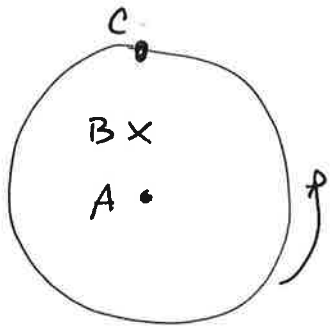
air rushes in, deflected to the right $\Rightarrow$

net rot.

- this is NOT relevant to how your toilets flush!!
- remember these new "forces" are kinematic forces resulting from our using an accelerated frame as POV
- may be easier to analyze in non-inertial frame and "translate" to rot frame (recall: pendulum in elevator)

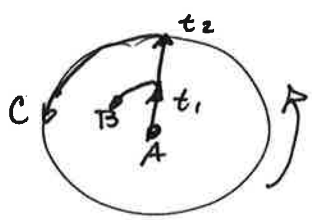
Motion on a turntable

3 observers A, B, C are in line on a turntable

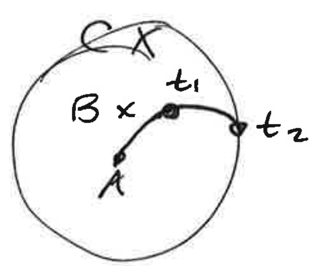


- kick a frictionless puck from center
- A kicks it exactly toward B,
  but by the time it gets there B has moved
  C moves even more

- net force in any inertial frame is zero
- in the inertial frame, puck travels in a straight line
  as B, C, rotate away w/ turntable



- in non inertial frame we have
  - centrifugal - radially outward, incl. to path
  - Coriolis - deflects to right!



⇒ in rotating frame A, B, C remain in line
puck is deflected to right

- much clearer to analyze in ground-based frame

- this 2D example is nice & simple; rarely so w/ F_{cf} and F_{cor}
  usually complicated and messy

Free-Fall & Coriolis

close to earth's surface

Since centrifugal > Coriolis, have to include both

$$m\ddot{\vec{r}} = m\vec{g}_0 + \vec{F}_{cf} + \vec{F}_{cor}$$

$\vec{F}_{cor} = 2m\vec{v}\times\vec{\Omega}$

$m\vec{g}_0$: true force of earth's grav.

$\vec{F}_{cf} = m(\vec{\Omega}\times\vec{r})\times\vec{\Omega}$

can approx $\vec{r}$ by $\vec{R}$

$\vec{r}$ = posn rel center of earth

$\vec{R}$ = point on surface

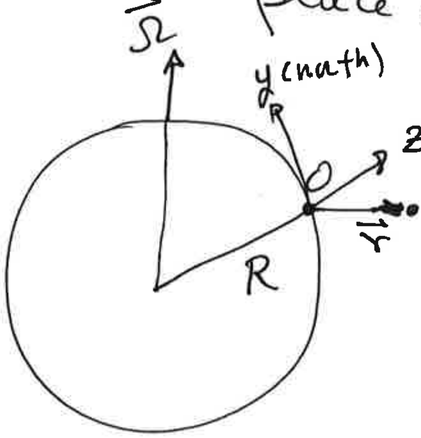
$$\Rightarrow \; F_{cf} = m(\vec{\Omega}\times\vec{R})\times\vec{\Omega}$$

- now recall $m\vec{g}_0 + \vec{F}_{cf} = m\vec{g}$ where $\vec{g}$ is observed free fall accel we can actually see
- that means we can eliminate F_{cf} if we realize we've already incorporated it into our measurement of $\vec{g}$!

$$\Rightarrow \; \ddot{\vec{r}} = \vec{g} + 2\vec{v}\times\vec{\Omega} = \vec{g} + 2\dot{\vec{r}}\times\vec{\Omega}$$

doesn't involve r at all, only $\dot{r}$ and $\ddot{r}$!

$\Rightarrow$ change of origin is irrelevant, so we may as well place origin on earth's surface at $\vec{R}$



y = north, z = up, x = east

now $\dot{\vec{r}} = (\dot{x}, \dot{y}, \dot{z})$

$\vec{\Omega} = (0, \Omega\sin\theta, \Omega\cos\theta)$

$$\Rightarrow \; \dot{\vec{r}}\times\vec{\Omega} = (\dot{y}\Omega\cos\theta - \dot{z}\Omega\sin\theta, \; -\dot{x}\Omega\sin\theta, \; \dot{x}\Omega\sin\theta)$$

$$\Rightarrow \quad \ddot{x} = 2\Omega(\dot{y}\cos\theta - \dot{z}\sin\theta)$$

$$\ddot{y} = -2\Omega\dot{x}\cos\theta \qquad \ddot{z} = -g + 2\Omega\dot{x}\sin\theta$$

need some approximations

- Ω is small. if we ignore it entirely?

$$\ddot{x} = \ddot{y} = 0 \qquad \ddot{z} = -g \quad \Rightarrow \text{intro projectile motion}$$

this is 0^{th} order approx, just ignore (Ω^0 accuracy)

$$\Rightarrow x = 0 = y, \quad z = h - \tfrac{1}{2}gt^2 \text{ dropped from } h$$

- next order approx:
  - use 0^{th} order approx for $\dot{x}, \dot{y}, \dot{z}$

$$\Rightarrow \quad \dot{x} \simeq \dot{y} \simeq 0 \qquad \dot{z} = -gt$$

- using this

$$\ddot{x} \simeq 2\Omega g t \sin\theta \quad \text{— only new one!}$$

$$\ddot{y} \simeq 0$$

$$\ddot{z} \simeq -g$$

- integrate twice, $x = \frac{1}{3}\Omega g t^3 \sin\theta$ ($y = 0$, $z = h - \frac{1}{2}gt^2$ as in 0^{th} order appr.)
  - first order approx, good to Ω^1
  - extra displacement to east we didn't expect before

Lets say we drop something from 100m.

- from z, time to bottom is as always, $t = \sqrt{2h/g}$
- total easterly deflection ($\theta = 90°$, equator; $g \simeq 10\,m/s^2$)

$$x \simeq \tfrac{1}{3}(7\times10^{-5}\,s^{-1})(10\,\tfrac{m}{s^2})(20\,s^2)^{3/2} \simeq 2.2\,cm$$

- Small, but ~~totally~~ observable
- Predicted by Newton, verified by his rival Hooke (late 1600s)
  not properly explained until Coriolis (1835)