magnetic recording: field of the write head

ph587 / 26Jan09

Reminder - Maxwell's equations

$$\vec{\nabla}\cdot\vec{\mathbf{E}} = \rho/\epsilon_0 \tag{1}$$

$$\vec{\nabla}\times\vec{\mathbf{B}} - \mu_0\epsilon_0\frac{\partial\vec{\mathbf{E}}}{\partial t} = \mu_0\vec{\mathbf{j}} \tag{2}$$

$$\vec{\nabla}\cdot\vec{\mathbf{B}} = 0 \tag{3}$$

$$\vec{\nabla}\times\vec{\mathbf{E}} + \frac{\partial\vec{\mathbf{B}}}{\partial t} = 0 \tag{4}$$

$$\vec{\mathbf{B}} = \frac{\mu_0}{4\pi}\iiint\frac{\vec{\mathbf{J}}(\vec{\mathbf{r}}')\times(\vec{\mathbf{r}}-\vec{\mathbf{r}}')}{|\vec{\mathbf{r}}-\vec{\mathbf{r}}'|}d^3\vec{\mathbf{r}} \tag{5}$$

In general this means we have a *scalar* electric potential, and a *vector* magnetic potential.

$$\vec{\mathbf{E}} = -\vec{\nabla}\phi - \frac{\partial\vec{\mathbf{A}}}{\partial t} \tag{6}$$

$$\vec{\mathbf{B}} = \vec{\nabla}\times\vec{\mathbf{A}} \tag{7}$$

Usually we will choose the Coulomb gauge, $\vec{\nabla}\cdot\vec{\mathbf{A}} = 0$, which gives us Poisson's equation (twice):

$$-\nabla^2\varphi = \frac{\rho}{\epsilon_0} \tag{8}$$

$$-\nabla^2\vec{\mathbf{A}} = -\mu\vec{\mathbf{j}} \tag{9}$$

Now we worry only about the *static* case, with **no currents** (but consider real materials):

$$\frac{\partial \vec{\mathbf{E}}}{\partial t} = \frac{\partial \vec{\mathbf{B}}}{\partial t} = \vec{\mathbf{j}} = 0 \tag{10}$$

$$\tag{11}$$

$$\Rightarrow \qquad \vec{\boldsymbol{\nabla}} \cdot \vec{\mathbf{D}} = \rho \tag{12}$$

$$\vec{\boldsymbol{\nabla}} \times \vec{\mathbf{H}} = \vec{\mathbf{j}} = 0 \tag{13}$$

$$\vec{\boldsymbol{\nabla}} \cdot \vec{\mathbf{B}} = 0 \tag{14}$$

$$\vec{\boldsymbol{\nabla}} \times \vec{\mathbf{E}} = 0 \tag{15}$$

or

$$\oint_S \vec{\mathbf{D}} \cdot d\vec{\mathbf{A}} = \int_V \rho \, dV \tag{16}$$

$$\oint_S \vec{\mathbf{B}} \cdot d\vec{\mathbf{A}} = 0 \tag{17}$$

$$\oint_C \vec{\mathbf{H}} \cdot d\vec{\mathbf{l}} = \int_S \vec{\mathbf{j}} \cdot d\vec{\mathbf{A}} = 0 \tag{18}$$

(We allow microscopic dipoles, just presume a moment $\vec{\mathbf{m}}$ coming from nowhere.)

Recall

$$\vec{\mathbf{B}}_{\text{dipole}}(\vec{\mathbf{r}}) = \frac{\mu_0}{4\pi}\left[\frac{3(\vec{\mathbf{m}} \cdot \vec{\mathbf{r}})}{r^5}\vec{\mathbf{r}} - \frac{\vec{\mathbf{m}}}{r^3}\right] \qquad \vec{\mathbf{m}} = IA \qquad \text{(dir by RHR)} \tag{19}$$

Gauss's law & magnetostatics

$$\vec{\nabla} \cdot \vec{D} = \rho \qquad \vec{\nabla} \times \vec{E} = 0 \tag{20}$$

Without time dependence ...

$$\vec{H} = \frac{1}{\mu_0}\vec{B} - \vec{M} \tag{21}$$

$$\vec{\nabla} \cdot \vec{H} = -\frac{1}{\mu_0}\vec{\nabla} \cdot \vec{B} - \vec{\nabla} \cdot \vec{M} = -\vec{\nabla} \cdot \vec{M} \tag{22}$$

$$\vec{\nabla} \times \vec{H} = \vec{j} \tag{23}$$

But we also note $\vec{\nabla} \times \vec{\nabla} F = 0$ for any F:

$$\vec{\nabla} \times \vec{H} = \vec{\nabla} \times (-\vec{\nabla}\Psi) = -\vec{\nabla} \times \vec{\nabla}\Psi = 0 = \vec{j} \tag{24}$$

This means we can choose a Ψ such that $H = -\vec{\nabla}\Psi$

Without time dependence or currents, we have a scalar magnetic potential, and a magnetic analogue of Gauss's law:

$$\vec{H} = -\vec{\nabla}\Psi \tag{25}$$

$$\vec{\nabla} \cdot \vec{H} = = -\nabla^2\Psi = -\vec{\nabla} \cdot \vec{M} \equiv \rho_m \tag{26}$$

where ρ_m is the *magnetic charge density*, or $\rho_m = -\vec{\nabla} \cdot \vec{M}$

Now magnetostatics is just like electrostatics

$$\vec{\nabla} \cdot \vec{D} = \nabla^2 \phi = \rho \tag{27}$$

$$\vec{\nabla} \cdot \vec{H} = \nabla^2 \Psi = \rho_m = -\vec{\nabla} \cdot \vec{M} \tag{28}$$

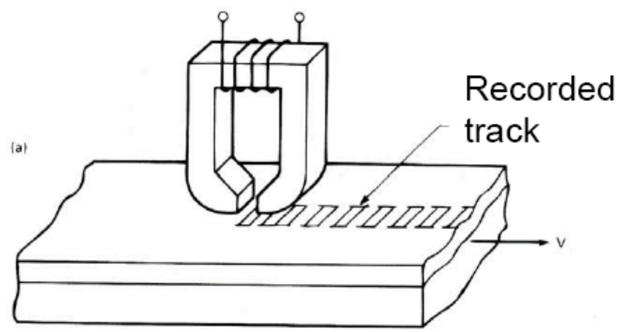
Scalar magnetic potential cannot support sources (requires $\vec{j} = 0$) *except* by applying discontinuities.

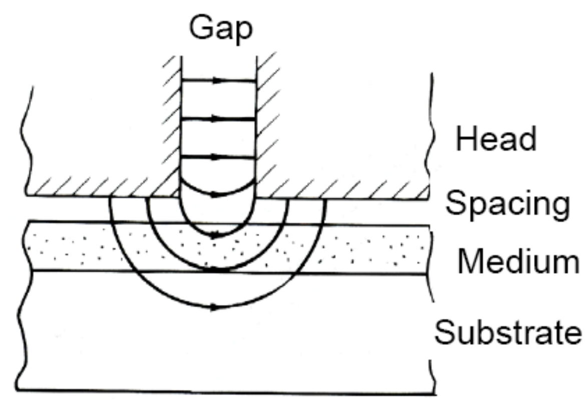
ρ_m = *magnetic poles*

At an interface, the magnetic charge per unit area is:

$$\sigma_m \equiv -\vec{n}_{12} \cdot (\vec{M}_2 - \vec{M}_1) \tag{29}$$

The basic writing scheme:





Read/write configurations:

Various Read/Write Configurations

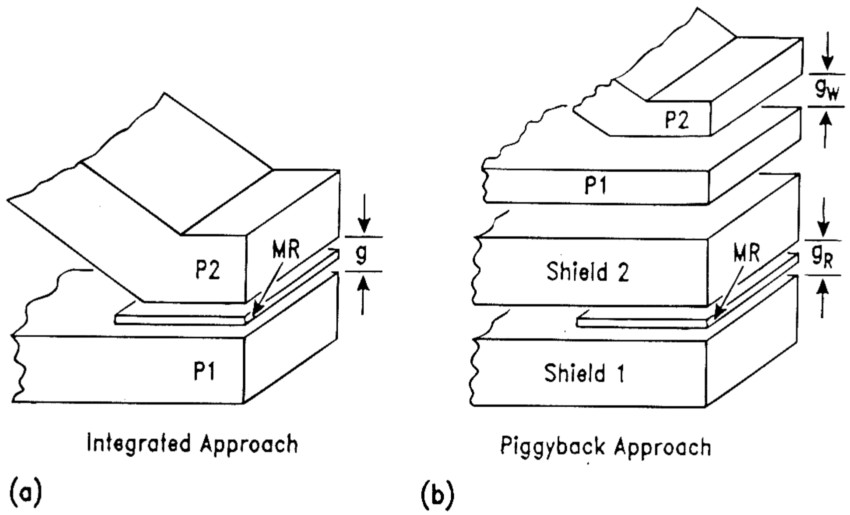
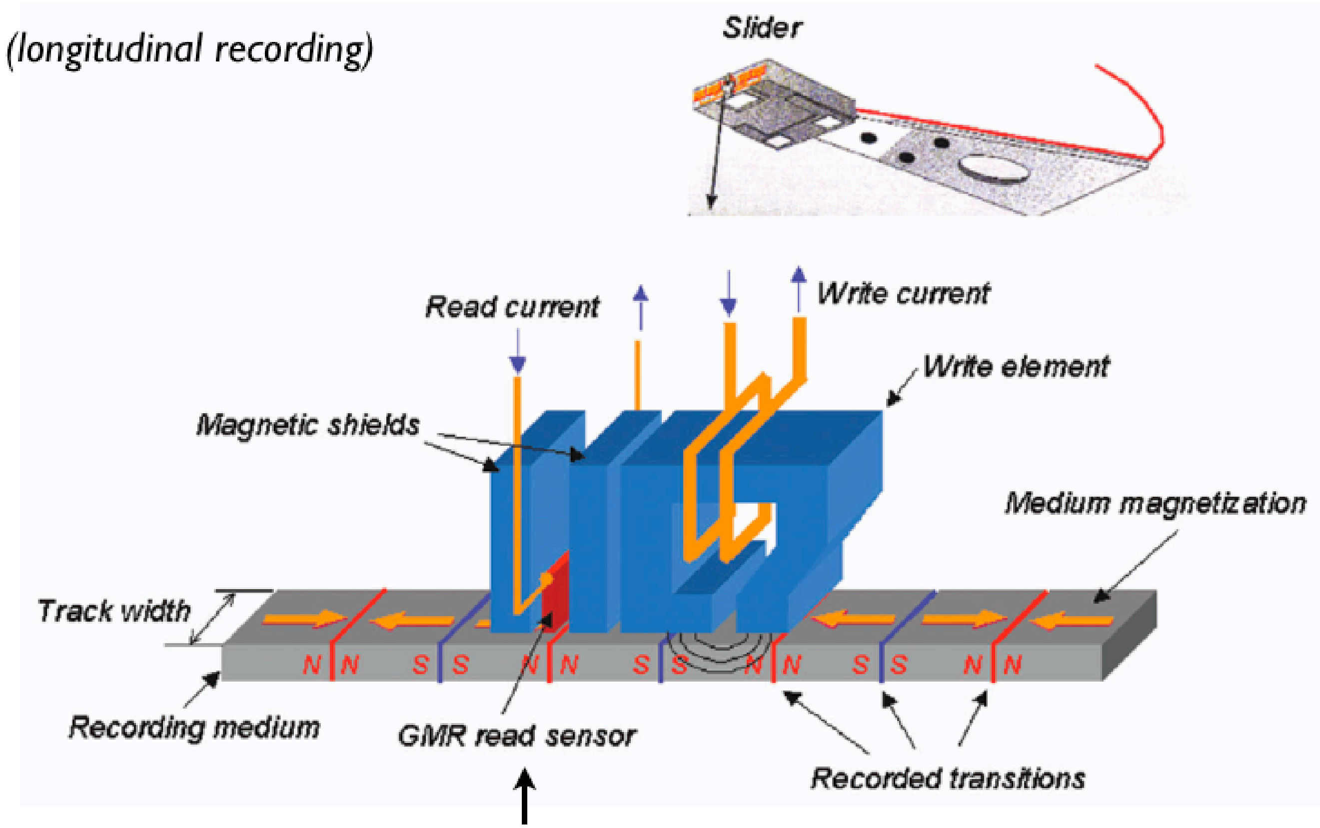


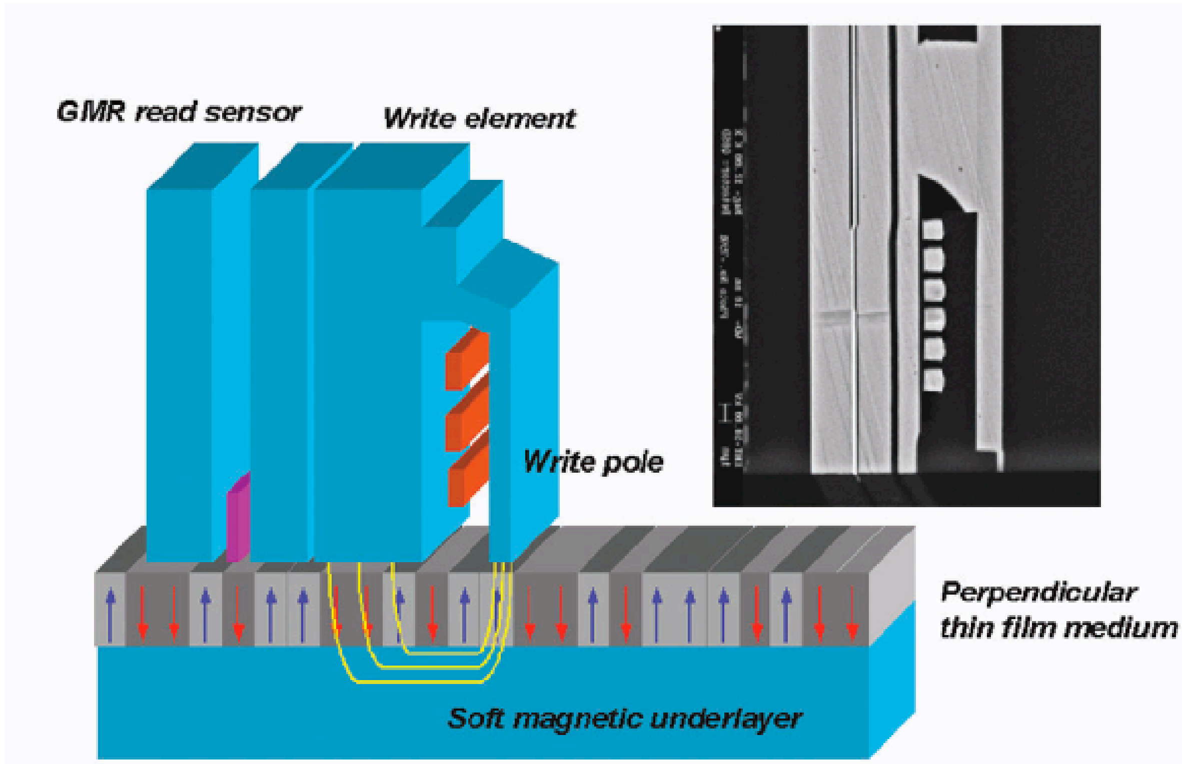
FIGURE 6.34 Various Read/Write Configurations with MR head: (*a*) Integrated head; (*b*) Piggyback head.

Longitudinal recording - ring head:

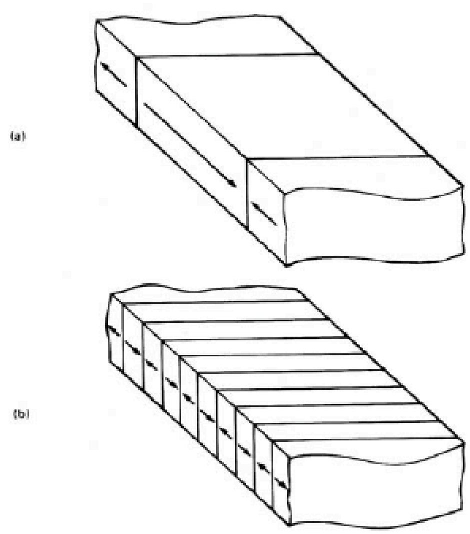


Perpendicular recording - separated pole head:

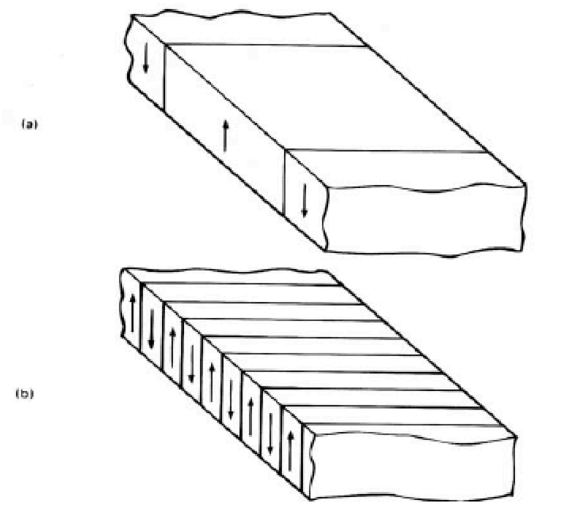
(perpendicular recording)



Longitudinal vs. perpendicular media



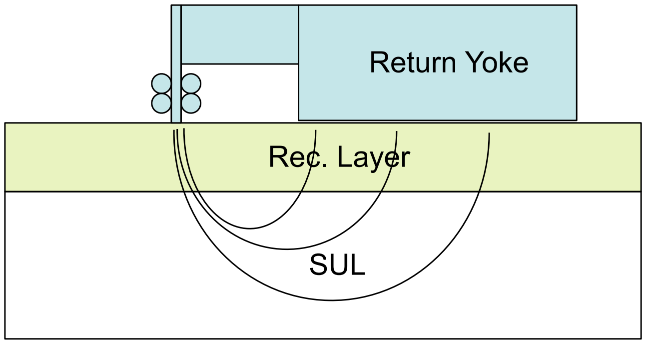
Uniaxial anisotropy
// to track direction



Uniaxial anisotropy
⊥ to plane

Single Pole Heads:

Conceptual Structure of Single Pole Head



What is the spatial and time variation of the field near a magnetic source like?

Fourier transforms are useful.

Temporal: $x = vt$

$$F(f) = \int_{-\infty}^{+\infty} f(t) e^{-2\pi i f t} dt \tag{30}$$

$$f(t) = \int_{-\infty}^{+\infty} F(f) e^{2\pi i f t} df \tag{31}$$

Spatial: $k = 2\pi/\lambda = 2\pi f/v$

$$F(k) = \int_{-\infty}^{+\infty} f(x) e^{-ikx} dx \tag{32}$$

$$f(x) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} F(k) e^{ikx} dk \tag{33}$$

Relation between waveforms:

$$f(x) \equiv f(t)$$

$$F(k) = v \cdot F(f)$$

	Function	Fourier Transform
Differentiation	$\frac{\partial f(x)}{\partial x}$	$ikF(k)$
Translation	$f(x-a)$	$e^{-ika}F(k)$
Modulation	$e^{ik_0x}f(x)$	$F(k-k_0)$
Convolution	$f(x)*g(x)$	$F(k)G(k)$
Multiplicaiton	$f(x)g(x)$	$F(k)*G(k)$

recall the definition of convolution

$$f(x)*g(x) \equiv \int_{-\infty}^{+\infty} f(x')g(x-x')dx' \tag{34}$$

and that Fourier transforms are normalized

Parseval's theorem:

$$\int_{-\infty}^{+\infty} |f(x)|^2 dx = \frac{1}{2\pi}\int_{-\infty}^{+\infty} |F(k)|^2 dk \tag{35}$$

(normalization conditions vary by discipline ...)

Consider the free-space magnetostatic potential in 2D:

$$\nabla^2\Psi = 0 \qquad \text{and} \qquad H = -\vec{\boldsymbol{\nabla}}\Psi \tag{36}$$

$$\Rightarrow \frac{\partial^2\Psi(x,y)}{\partial x^2} + \frac{\partial^2\Psi(x,y)}{\partial y^2} = 0 \tag{37}$$

Now take the Fourier transform of this with respect to x

$$FT_x\left[\frac{\partial^2\Psi(x,y)}{\partial x^2} + \frac{\partial^2\Psi(x,y)}{\partial y^2}\right] = 0 \tag{38}$$

$$(ik)^2\Psi(k,y) + \frac{\partial^2\Psi(k,y)}{\partial y^2} = 0 \tag{39}$$

$$\Longrightarrow \qquad \frac{\partial^2\Psi(k,y)}{\partial y^2} = k^2\Psi(k,y) \tag{40}$$

We know this equation. A general solution for $\Psi(k,y)$ is

$$\Psi(k,y) = A_+(k)e^{ky} + A_-(k)e^{-ky} \tag{41}$$

If we place any sources of magnetic charges at $y < 0$, with

$$\lim_{y\to\infty}\Psi(k,y) = 0 \qquad \Rightarrow \qquad \Psi(k,y) = \Psi(k,0)e^{-|k|y} \qquad (y > 0) \tag{42}$$

In other words ... the Fourier transform of the potential depends only on relative location. If we know it at one place, we know it everywhere.

So what are the fields in real space?

$$\begin{aligned}
\Psi(x,y) &= FT^{-1}\left[\Psi(k,y)\right] = FT^{-1}\left[\Psi(k,0)e^{-|k|y}\right] && (43)\\
&= \Psi(x,0) * FT^{-1}\left[e^{-|k|y}\right] && (44)\\
&= \Psi_s(x) * \frac{y}{\pi\left(x^2+y^2\right)} && (45)
\end{aligned}$$

Here we define $\Psi_s(x) \equiv \Psi(x,0)$ as the surface magnetic potential at $y=0$.

Now we can find the magnetic potential *at any point in terms of the surface potential at* $y=0$:

$$\Psi(x,y) = \frac{y}{\pi}\int_{-\infty}^{+\infty} \frac{\Psi_s(x')}{\left(x-x'\right)^2+y^2}dx' \tag{46}$$

Our "sources" are magnetic surface charges, and knowing their surface potential at $y=0$ is sufficient to determine the magnetic field and potential anywhere else in surrounding free space.

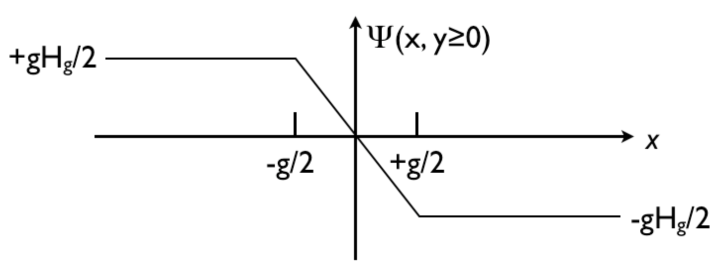
Karlqvist (ring) head equation

Fields from real heads are *ugly*. Karlqvist came up with an idealized inductive head model:

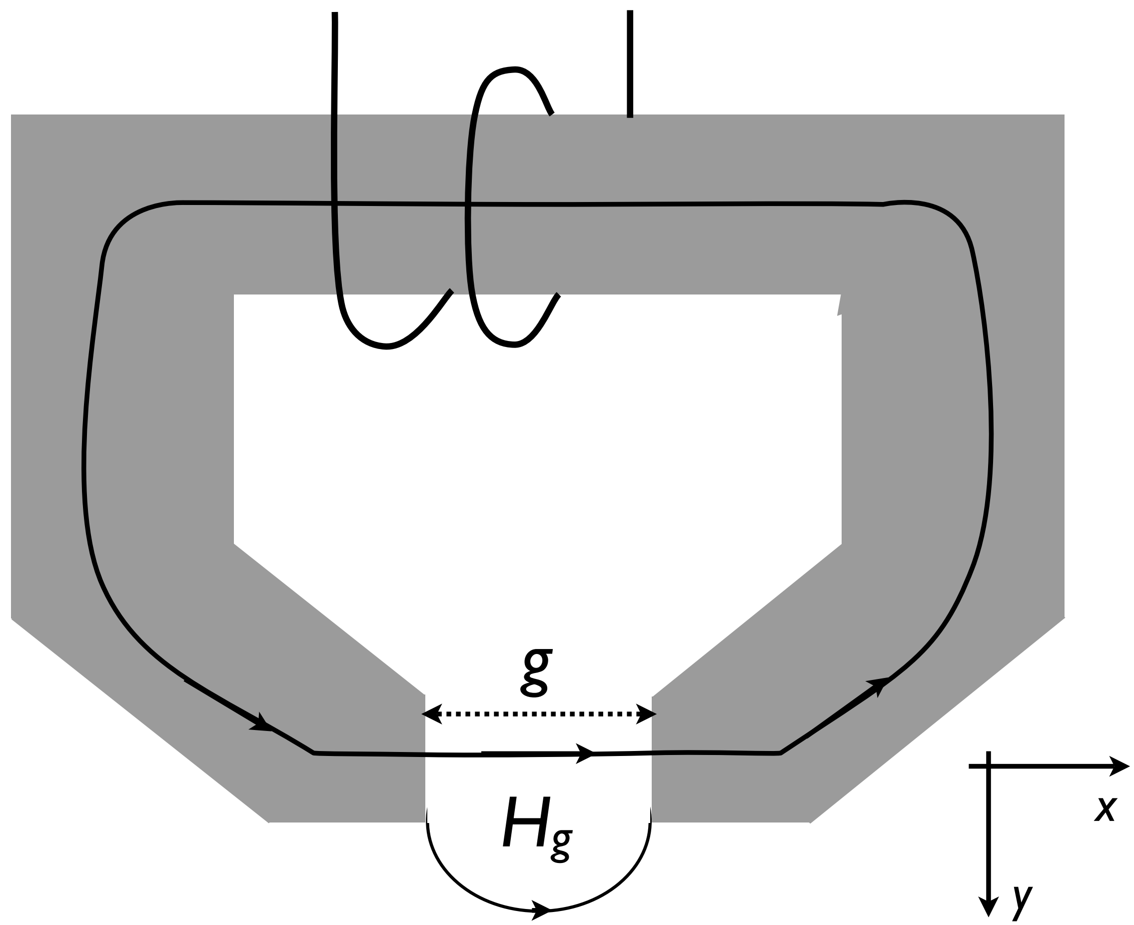
Karlqvist model assumptions:

1. The permeability of the magnetic core or film is infinite
2. The region of interest is small compared with the size of the magnetic core. **Core is wider than gap**, and **head is infinite in the z dir.** *Approximately 2D.*
3. The magnetic potential across the head gap varies linearly with x. Thus the field in the head gap H_g is constant down to $y = 0$, and the magnetic potential in the core is zero since permeability is infinite. The surface magnetic potential at $y = 0$ is the same as deep across the head gap

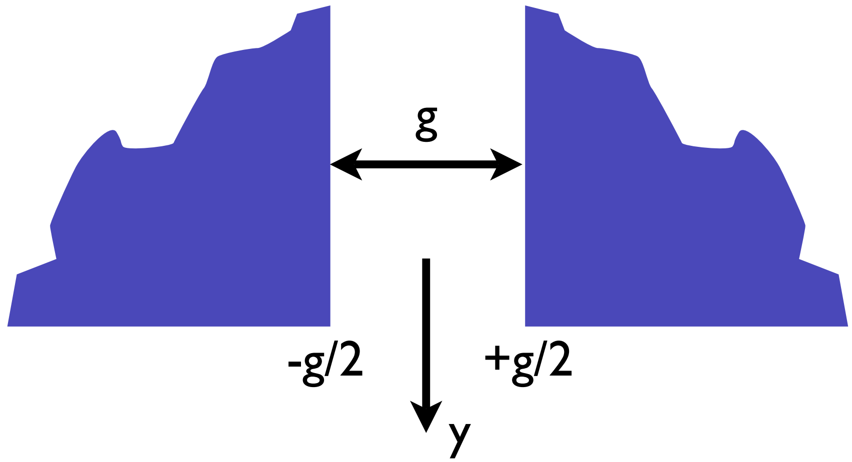
$$\Psi_s(x) \equiv \begin{cases} gH_g/2 & x \leq -g/2 \\ -H_g x & |x| \leq g/2 \\ -gH_g/2 & x \geq g/2 \end{cases} \tag{47}$$

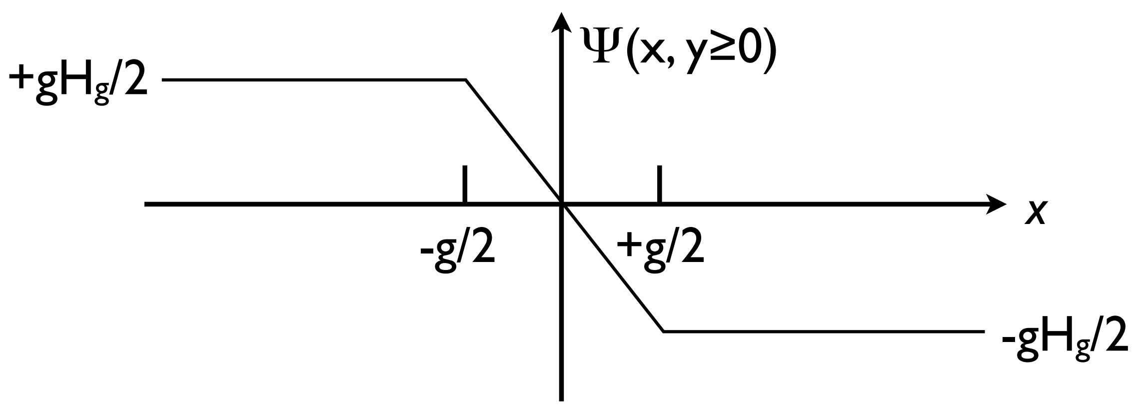


Schematic:



Potential variation:





Read/write configurations:

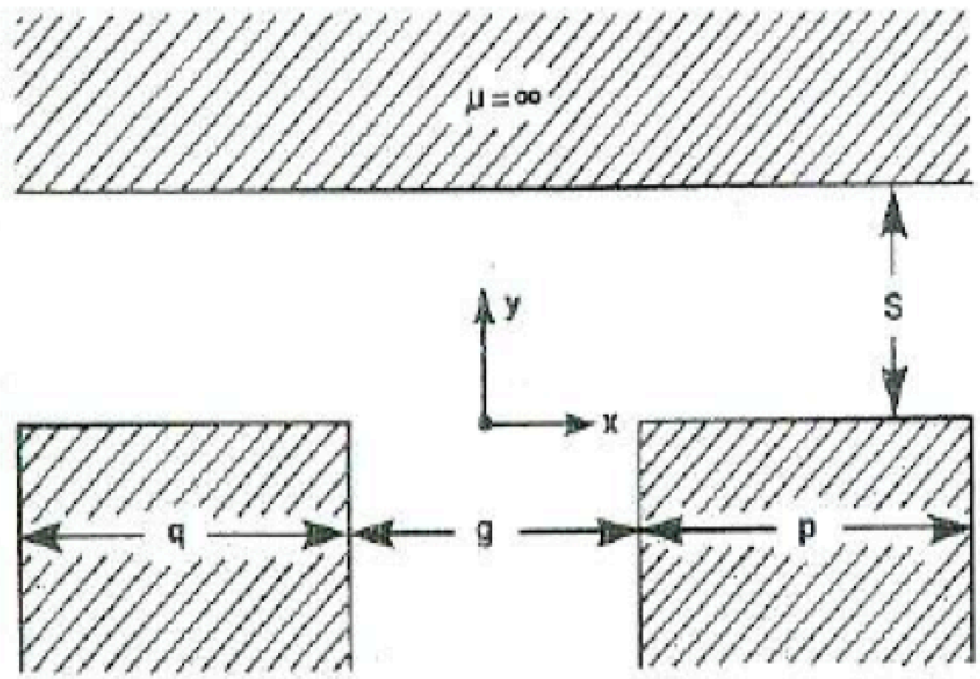
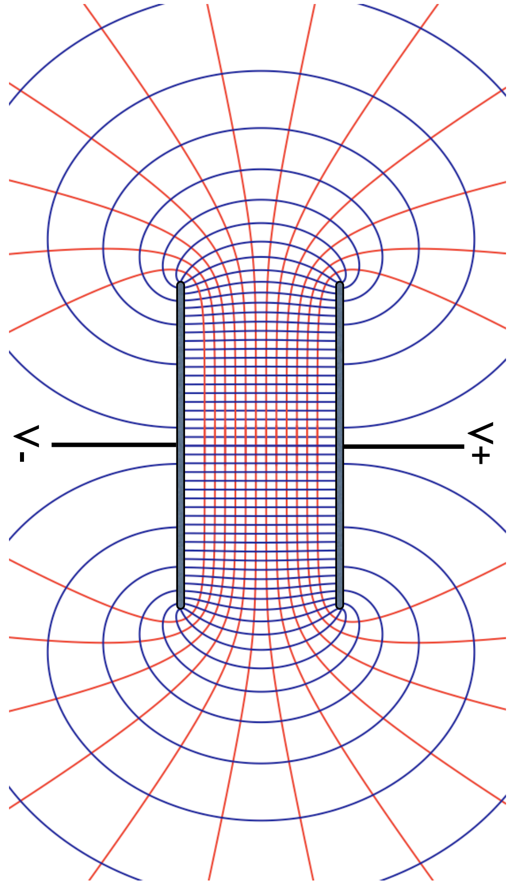


Fig. 1a. Geometry for the general case.

What are the potential and field for this head, for $y > 0$? Fringing fields for parallel plate capacitor ...



$$
\begin{aligned}
\Psi(x,y) &= \frac{y}{\pi}\int_{-\infty}^{+\infty}\frac{\Psi_s(x')}{(x-x')^2+y^2}dx' && (48)\\
&= -\frac{H_g}{\pi}\left[\left(x+\frac{g}{2}\right)\tan^{-1}\left(\frac{x+\frac{g}{2}}{y}\right)-\left(x-\frac{g}{2}\right)\tan^{-1}\left(\frac{x-\frac{g}{2}}{y}\right)-\frac{y}{2}\ln\frac{\left(x+\frac{g}{2}\right)^2+y^2}{\left(x-\frac{g}{2}\right)^2+y^2}\right] && (49)
\end{aligned}
$$

$$
\begin{aligned}
H_x(x,y) &= -\frac{\partial\Psi}{\partial x}=\frac{H_g}{\pi}\tan^{-1}\left[\frac{gy}{x^2+y^2-\left(\frac{g}{2}\right)^2}\right] && (50)\\
H_y(x,y) &= -\frac{\partial\Psi}{\partial y}=-\frac{H_g}{2\pi}\ln\left[\frac{\left(x+\frac{g}{2}\right)^2+y^2}{\left(x-\frac{g}{2}\right)^2+y^2}\right] && (51)
\end{aligned}
$$

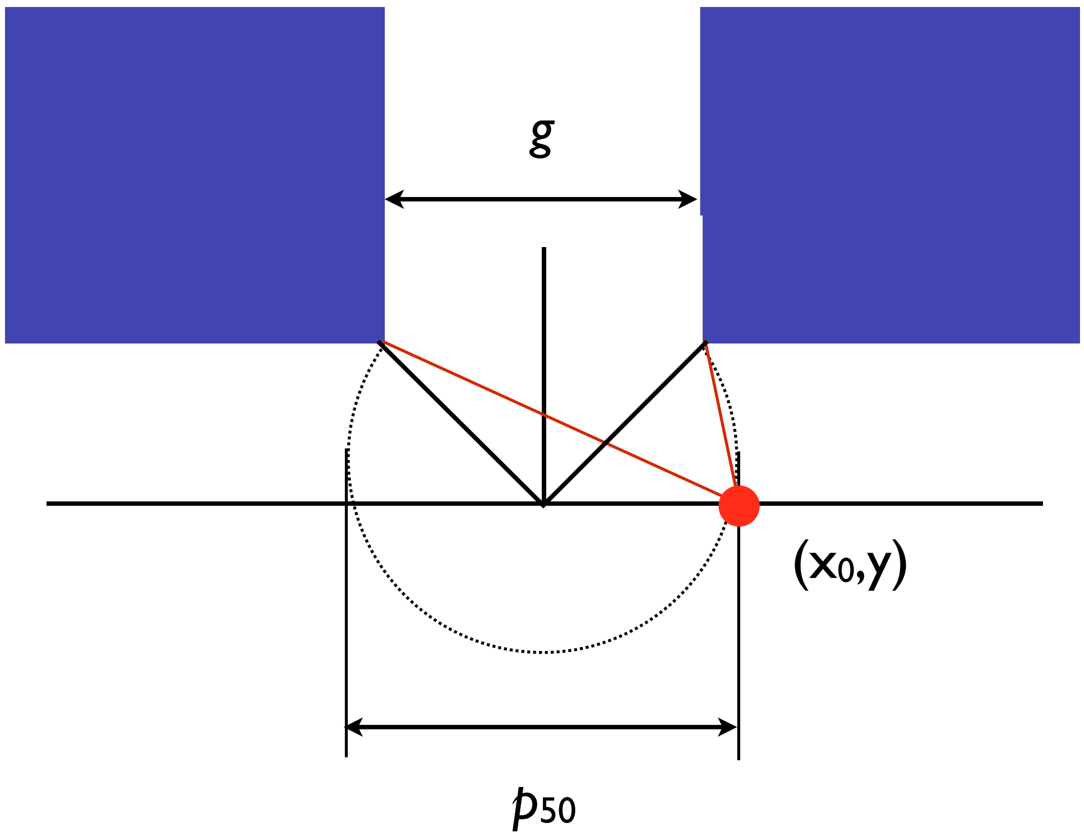
Contours of constant H_x are circles going through the head gap corners.

Small gap limit, $y \gg g$:

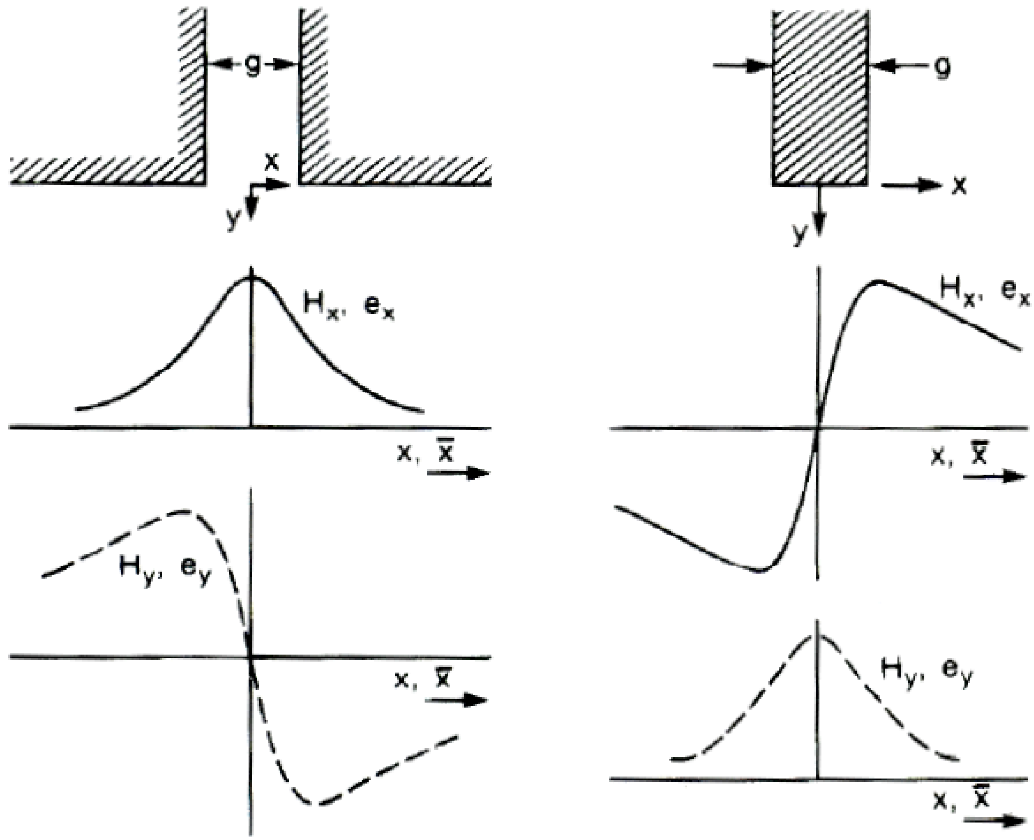
$$
\begin{aligned}
\Rightarrow H_x(x,y) &= +\frac{H_g g}{\pi}\left[\frac{y}{x^2+y^2}\right] && (52)\\
H_y(x,y) &= -\frac{H_g g}{\pi}\left[\frac{x}{x^2+y^2}\right] && (53)
\end{aligned}
$$

This is just the field produced by a line current located at the head gap edge.

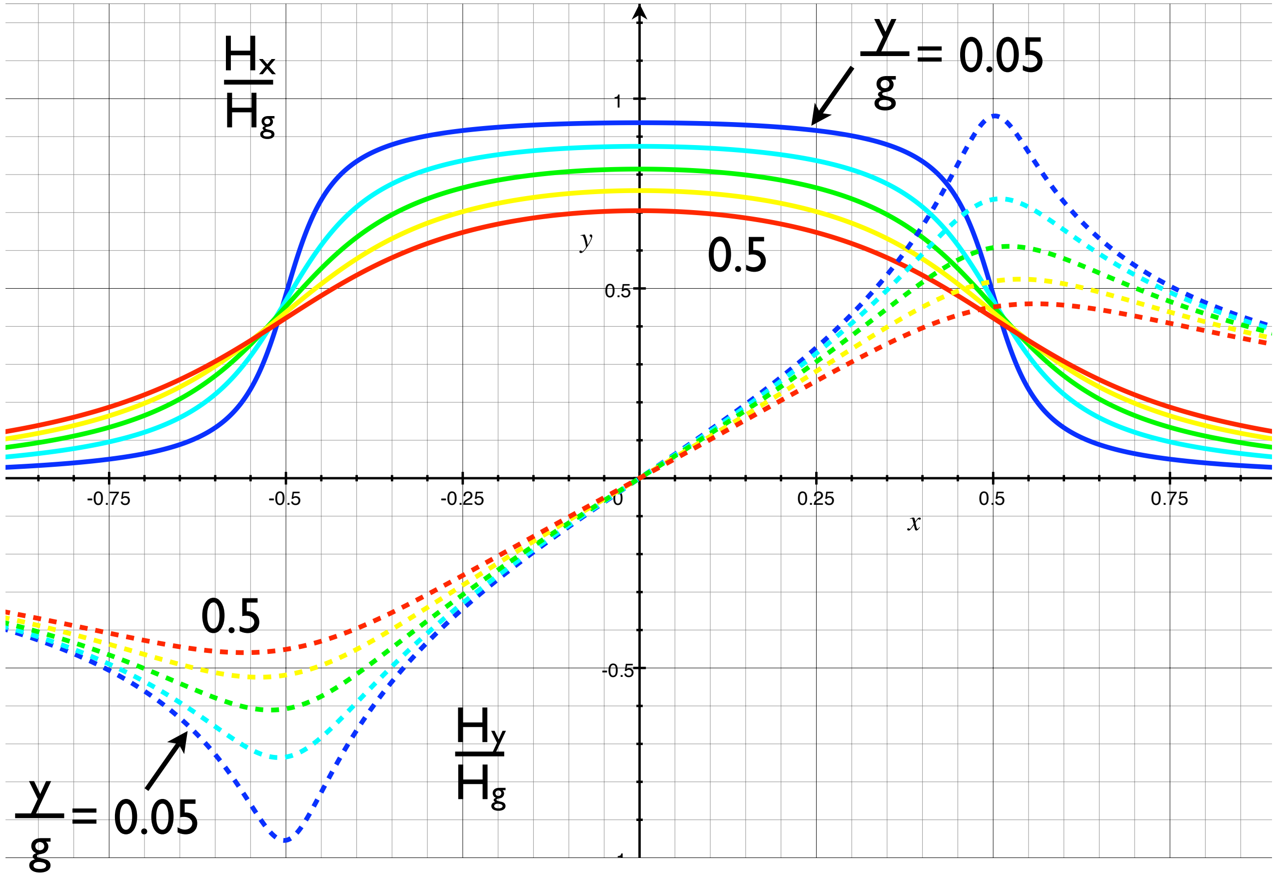
Schematic



Head Fields:

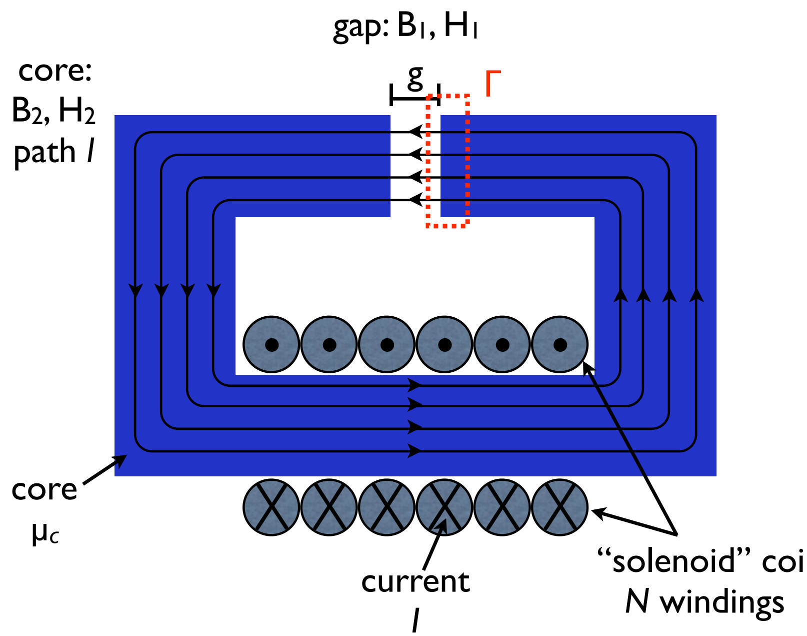


Head Fields:



$\frac{H_x}{H_g}$
$\frac{y}{g} = 0.05$
0.5
$\frac{H_y}{H_g}$
0.5
$\frac{y}{g} = 0.05$
y
x
1
0.5
-0.5
-0.75
-0.5
-0.25
0
0.25
0.5
0.75

Electromagnets



Surface S around pole tip: flux is zero

$$\Phi_S = B_g A_g - B_c A_c = 0 \tag{54}$$

If $A_g = A_c$, then $B_g = B_c$.

Now take a line integral around Γ:

$$\oint_\Gamma \vec{\mathbf{H}} \cdot d\vec{\mathbf{l}} = \mu_0 \int \vec{\mathbf{j}} \cdot \vec{\mathbf{n}}\, da \quad \Rightarrow \quad H_1 g + H_2 l = \mu_0 \left(\frac{IN}{A}\right) A = \mu_0 N I \tag{55}$$

We need to know $B(H)$ for the core ... if we work in the small field regime ("minor loops"), then we can approximate it as linear: $B_2 \sim \mu_c H_2$, where μ_c is the core permeability:

$$\begin{aligned} B_g A_g &= B_c A_c = A_g \mu_0 H_1 = A_c \mu_c H_2 && (56) \\ \Rightarrow H_1 &= \frac{\mu_c}{\mu_0} H_2 \qquad\qquad (\text{if} \quad A_c = A_g) && (57) \end{aligned}$$

So the field in the gap is *enhanced by a factor* μ_c. Using this:

Gap field:

$$\begin{aligned} H_1 g + H_2 l &= H_1 g + \frac{\mu_0}{\mu_c} H_1 l = \mu_0 N I && (58) \\ H_1 \left[\frac{\mu_0}{\mu_c} l + g \right] &= \mu_0 N I && (59) \\ \Rightarrow H_1 &= \frac{NI}{\left(\frac{\mu_0}{\mu_c}\right) l + g} && (60) \end{aligned}$$

Basically: we can relate the gap field to I and the core material properties.

High μ gives higher gap field for a given I. High B_s gives high max gap field. (Why does H_c matter?)

"Head efficiency" = portion of NI across the head gaps compared to coil ...

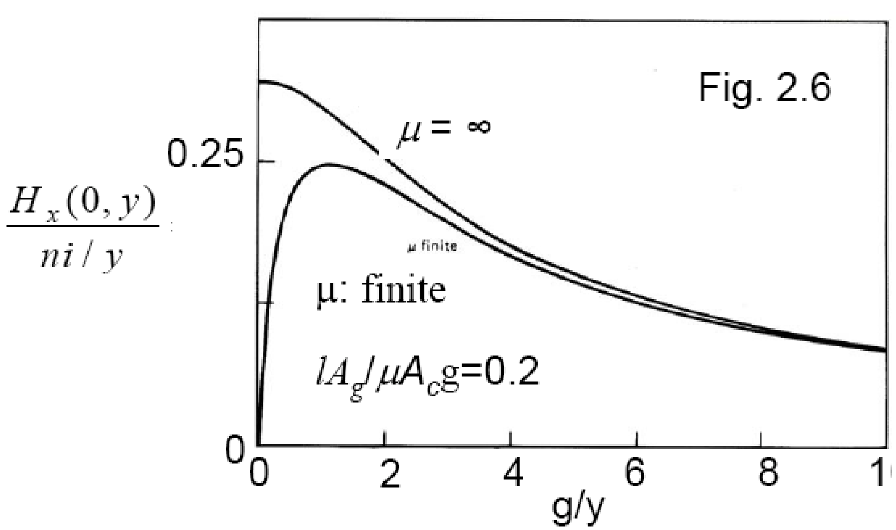
$$E \equiv \frac{H_1 g}{NI} = \frac{H_1 g}{H_2 l + H_1 g} = \left(1 + \frac{H_2}{H_1} \frac{l}{g}\right)^{-1} \leq 1 \qquad (61)$$

Back to the small gap limit, $y \gg g$:

$$H_x(x,y) = +\frac{H_g g}{\pi}\left[\frac{y}{x^2+y^2}\right] \qquad \text{and} \qquad H_y(x,y) = -\frac{H_g g}{\pi}\left[\frac{x}{x^2+y^2}\right] \tag{62}$$

we can now write down the gap field for a real material:

$$H_x(x,y) = +\left(\frac{NI}{\pi + \frac{lA_g\pi}{\mu A_c g}}\right)\left[\frac{y}{x^2+y^2}\right] \qquad \text{and} \qquad H_y(x,y) = -\left(\frac{NI}{\pi + \frac{lA_g\pi}{\mu A_c g}}\right)\left[\frac{x}{x^2+y^2}\right] \tag{63}$$



effect of finite permeability - reduced field at small gap!

$$\frac{H_x(0,y)}{ni/y} = \frac{1}{\pi}\frac{2y}{g + lA_g/\mu A_c}\tan^{-1}\frac{g}{2y} \tag{64}$$

Materials:

Desirable Properties for Head Materials

1. High Saturation Magnetization Bs
2. High Permeability μ
3. Low Coercivity Hc
4. Appropriate Loss Factor α
5. Low Magnetostriction λ
6. High Resistivity ρ
7. Weak Temperature Dependence
8. Small Aging Effect
9. High Wear Resistance
10. High Corrosion Resistance
11. Good Machinability

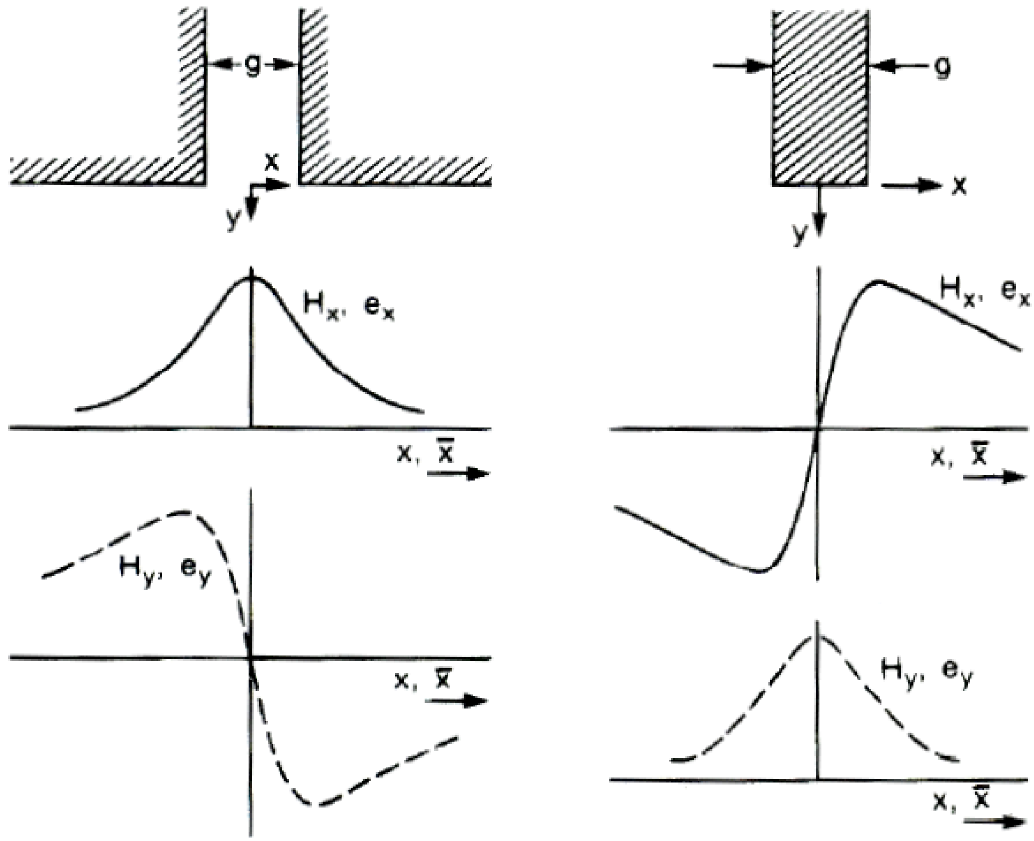
Constraints

Head-medium Interface (Spacing between head and medium must be critically small.)

1. Surfaces: flat, smooth, free of asperities, resistant to wear
2. Flexible media: heads and media are in contact
3. Rigid disks: Heads are mounted on a slider flying above rotating disk (20μm $\rightarrow$ 50nm).
    - Occasional head crash
    - Disk magnetic layers are protected by lubricant and very thin protective overcoat layers (C)

Everything we have done so far is (mostly) valid for a single-pole write head!

Same thing, turned on its side ... (switch x and y, only one pole)!



Finite pole extent:

H_y(x, 0.01): z dependence

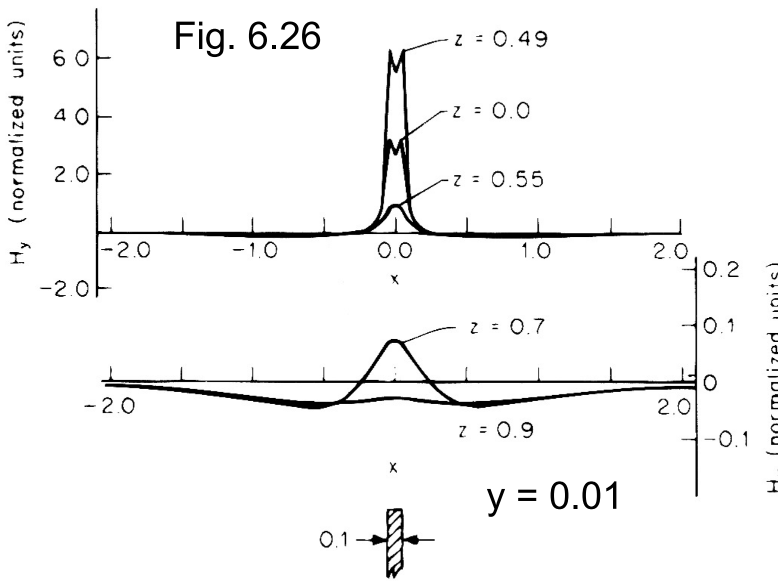
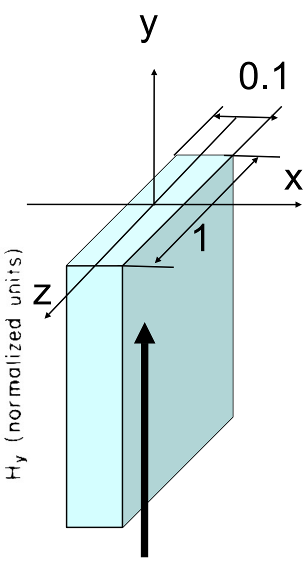


FIGURE 6.26 Perpendicular component of a magnetic field for a single-pole head. Center of the head is at $z = 0$, the geometrical edge is at $z = 0.5$, and the field is shown for $y = 0.01$ (*Luitjens and van Herk, 1982*)



Excited by homogenious field

Soft Underlayer:

Effect of SUL

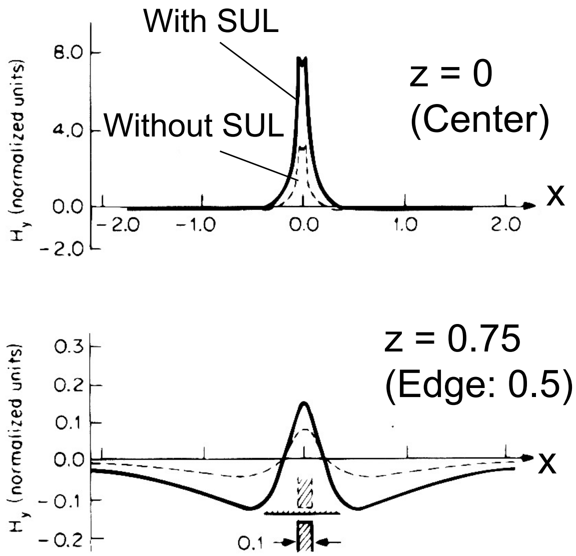


Fig. 6.27

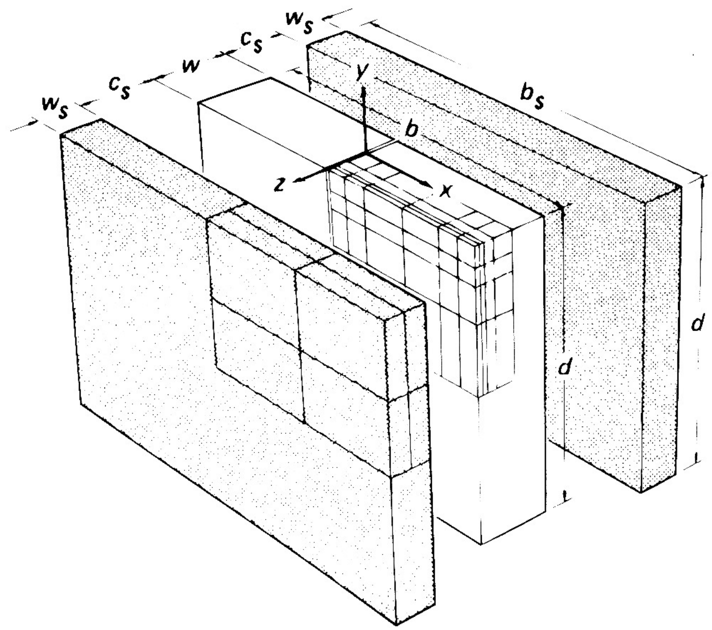
H_y varies less rapidly with y in the presence of SUL!

Side Shield:

Effect of Side Shield (Lindholm, 1980)

Surface-Integration methid

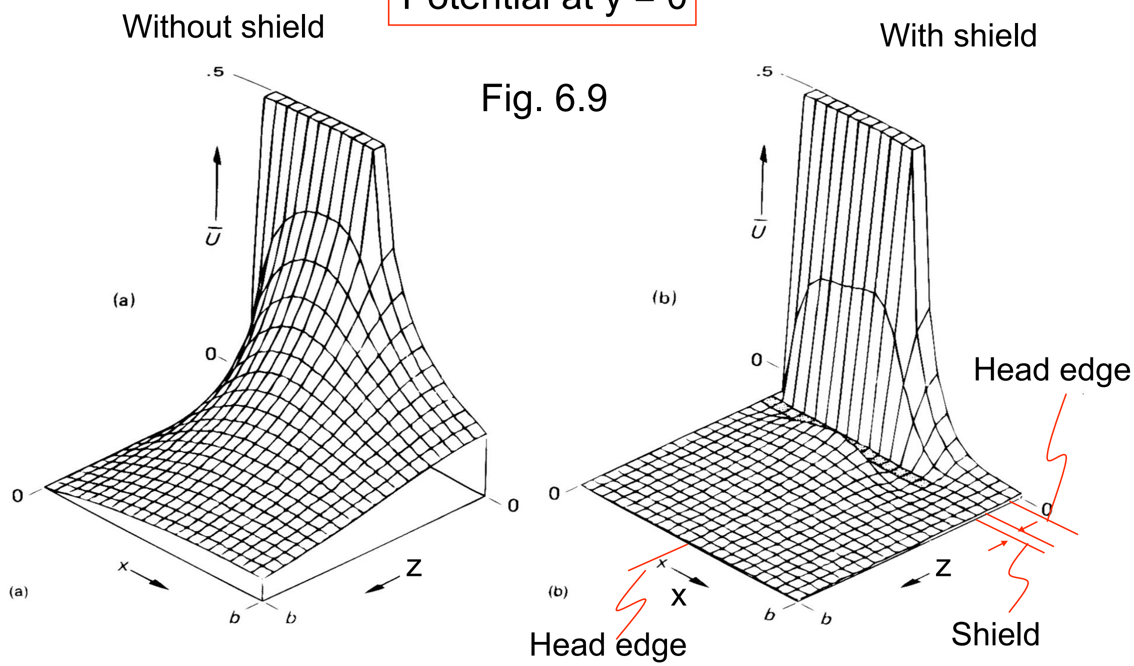
Fig. 6.10



Side Shield:

Effect of Side Shield (continued)

Potential at y = 0



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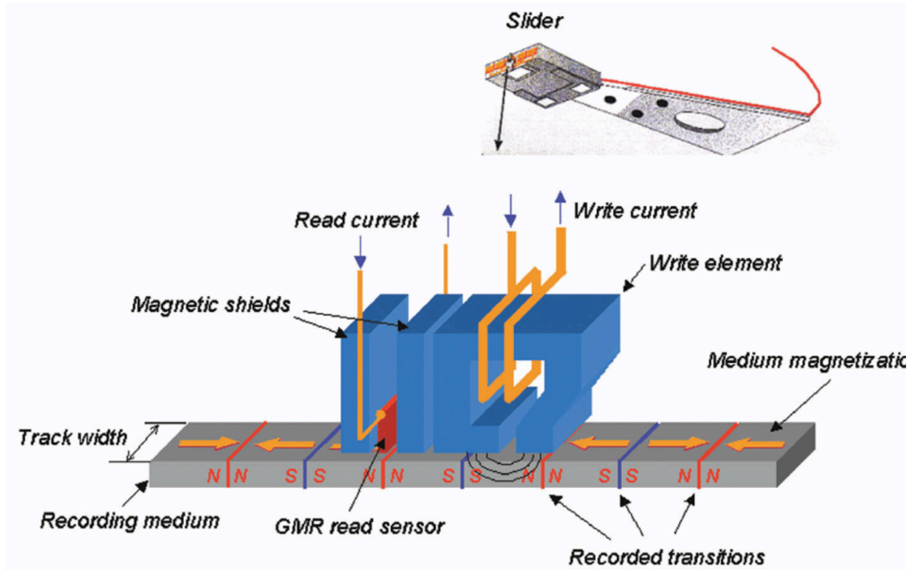
Inductive read process

ph187
2Feb09

What can we sense?



Head moving over transitions = time-varying magnetic flux.

What we will show: inductive writing and reading are the same thing.

More precisely: writing is the effect of head flux on media. Reading is the effect of media flux on the head. *They are simply related through mutual inductance.*

Maxwell's equations and solutions

Maxwell's equations:

$$\vec{\boldsymbol{\nabla}} \cdot \vec{\mathbf{E}} = \frac{\rho}{\epsilon_0} \tag{65}$$

$$\vec{\boldsymbol{\nabla}} \times \vec{\mathbf{E}} = -\frac{\partial \vec{\mathbf{B}}}{\partial t} \tag{66}$$

$$\vec{\boldsymbol{\nabla}} \cdot \vec{\mathbf{B}} = 0 \tag{67}$$

$$c^2 \vec{\boldsymbol{\nabla}} \times \vec{\mathbf{B}} = \frac{\vec{\mathbf{j}}}{\epsilon_0} + \frac{\partial \vec{\mathbf{E}}}{\partial t} \tag{68}$$

Their solutions:

$$\vec{\mathbf{E}} = -\vec{\boldsymbol{\nabla}} \varphi - \frac{\partial \vec{\mathbf{A}}}{\partial t} \tag{69}$$

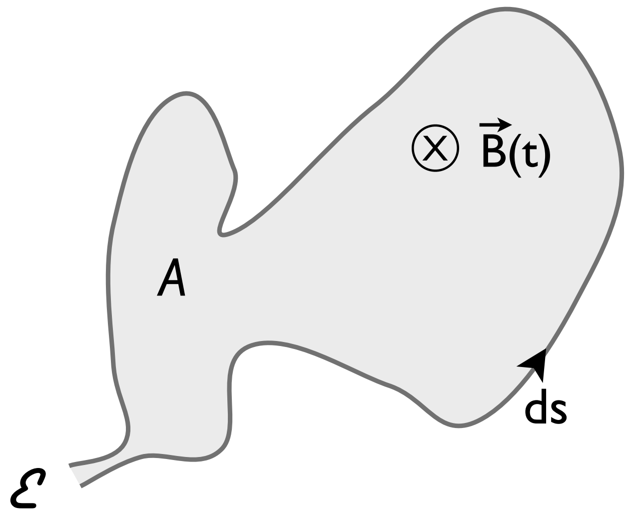
$$\vec{\mathbf{B}} = \vec{\boldsymbol{\nabla}} \times \vec{\mathbf{A}} \tag{70}$$

$$\varphi(1, t) = \int \frac{\rho(2, t - r_{12}/c)}{4\pi\epsilon_0 r_{12}}\, dV_2 \tag{71}$$

$$\vec{\mathbf{A}}(1, t) = \int \frac{\vec{\mathbf{j}}(2, t - r_{12}/c)}{4\pi\epsilon_0 c^2 r_{12}}\, dV_2 \tag{72}$$

Note that $\frac{1}{\epsilon_0 \mu_0} = c^2$.

Faraday's law and induction



Induced EMF in a loop of wire:

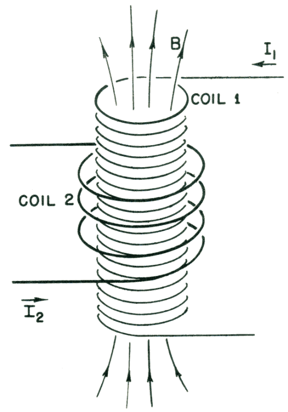
$$\mathcal{E} = -\frac{\mathrm{d}\Phi}{\mathrm{d}t} = -\frac{\mathrm{d}}{\mathrm{d}t}\int \vec{\mathbf{B}}\cdot\vec{\mathbf{n}}\,da = -\frac{\mathrm{d}}{\mathrm{d}t}\int\left(\vec{\boldsymbol{\nabla}}\times\vec{\mathbf{A}}\right)\cdot\vec{\mathbf{n}}\,da = -\frac{\mathrm{d}}{\mathrm{d}t}\oint\vec{\mathbf{A}}\cdot d\vec{\mathbf{s}} \qquad (73)$$

(Faraday + definition of $\vec{\mathbf{A}}$ + Stokes)

If the wire is fixed, and $\vec{\mathbf{B}}$ is the same all over the surface:

$$\mathcal{E} = -\frac{\mathrm{d}}{\mathrm{d}t}\left(B_\perp A\right) = -A\,\frac{\mathrm{d}B_\perp}{\mathrm{d}t} \qquad (74)$$

Mutual Inductance



One coil surrounding another (1 = inner, 2 = outer).
Coil 2 encloses the flux of coil 1 completely.

Field inside coil 1 & 2:

$$B_1 = \mu_0 \frac{N_1 I_1}{l} \tag{75}$$

Assume coils are long, so $B_{\text{outside}} \approx 0$.

Induced voltage in coil 2 due to B_1:

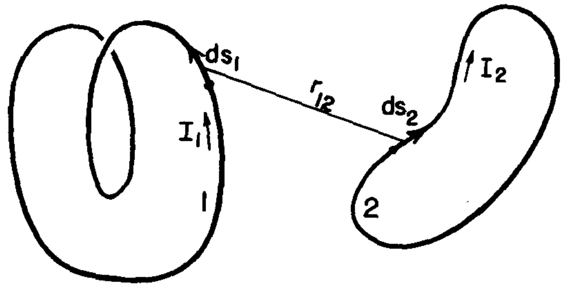
$$\mathcal{E}_2 = N_2 \left(-A_2 \frac{\mathrm{d}B_1}{\mathrm{d}t} \right) = -N_2 A_2 \frac{\mathrm{d}B_1}{\mathrm{d}t} \tag{76}$$

$$\frac{\mathrm{d}B_1}{\mathrm{d}t} = \frac{\mathrm{d}}{\mathrm{d}t} \left[\mu_0 \frac{N_1 I_1}{l} \right] = \mu_0 \frac{N_1}{l} \frac{\mathrm{d}I_1}{\mathrm{d}t} \tag{77}$$

$$\Rightarrow \mathcal{E}_2 = -N_2 A_2 \frac{\mu_0 N_1}{l} \frac{\mathrm{d}I_1}{\mathrm{d}t} = -\frac{\mu_0 N_1 N_2 A_2}{l} \frac{\mathrm{d}I_1}{\mathrm{d}t} \equiv \mathfrak{M}_{12} \frac{\mathrm{d}I_1}{\mathrm{d}t} \tag{78}$$

$\mathfrak{M}_{12} = -\dfrac{\mu_0 N_1 N_2 A_2}{l}$ is the **Mutual inductance** of the pair of coils. (79)

Mutual Inductance



Suppose now we have 2 arbitrary coils.

Induced EMF in coil 1 due to I_2:

$$\mathcal{E}_1 = -\frac{\mathrm{d}}{\mathrm{d}t}\int_{(1)} \vec{\mathbf{B}}_2 \cdot \vec{\mathbf{n}}\, da = -\frac{\mathrm{d}}{\mathrm{d}t}\oint_{(1)} \vec{\mathbf{A}}_1 \cdot d\vec{\mathbf{s}}_1 \quad (80)$$

If we assume $\vec{\mathbf{A}}$ at loop 1 comes only from I_2, we can write it as a line integral around loop 2:

Vector potential around loop 1 due to I_2:

$$\vec{\mathbf{A}}_1 = \frac{\mu_0}{4\pi}\int_{(2)} \frac{\vec{\mathbf{j}}_2}{r_{12}}\, dV_2 = \frac{\mu_0}{4\pi}\oint_{(2)} \frac{\vec{\mathbf{I}}_2}{r_{12}}\, d\vec{\mathbf{s}}_2 \quad (81)$$

Which gives us the EMF in coil 1 as:

$$\mathcal{E}_1 = -\frac{\mu_0}{4\pi}\frac{\mathrm{d}}{\mathrm{d}t}\oint_{(1)}\oint_{(2)} \frac{\vec{\mathbf{I}}_2\, d\vec{\mathbf{s}}_2}{r_{12}} \cdot d\vec{\mathbf{s}}_1 \quad (82)$$

(We relied on the cross-section of the wires being small compared to r_{12} ...)

I_2 does not depend on the coordinates, since we assume stationary circuits.

Therefore we can pull I_2 out of the integrals:

$$\mathcal{E}_1 = -\frac{\mu_0}{4\pi}\frac{\mathrm{d}}{\mathrm{d}t}\oint_{(1)}\oint_{(2)}\frac{\vec{\mathbf{I}}_2\, d\vec{\mathbf{s}}_2}{r_{12}}\cdot d\vec{\mathbf{s}}_1 = \left(\frac{\mathrm{d}\vec{\mathbf{I}}_2}{\mathrm{d}t}\right)\left(-\frac{\mu_0}{4\pi}\oint_{(1)}\oint_{(2)}\frac{d\vec{\mathbf{s}}_2\cdot d\vec{\mathbf{s}}_1}{r_{12}}\right) \tag{83}$$

The integrals are just describing the geometry of the coils. Since the coil geometry is static, they just give a constant, which we again call $\mathfrak{M}_{12}$.

And now we get something familiar:

Mutual inductance of two coils:

$$\mathcal{E}_1 = \mathfrak{M}_{12}\frac{\mathrm{d}\vec{\mathbf{I}}_2}{\mathrm{d}t} \qquad \text{with} \qquad \mathfrak{M}_{12} = -\frac{\mu_0}{4\pi}\oint_{(1)}\oint_{(2)}\frac{d\vec{\mathbf{s}}_2\cdot d\vec{\mathbf{s}}_1}{r_{12}} \tag{84}$$

Now it is easy to see why $\mathfrak{M}_{12} = \mathfrak{M}_{21} \equiv \mathfrak{M}$ ... right? ($\vec{\mathbf{a}}\cdot\vec{\mathbf{b}} = \vec{\mathbf{b}}\cdot\vec{\mathbf{a}}$)

Physically, the integral is something like an average separation of the coils, weighted more for parallel bits.

Self Inductance & "Back EMF"

What have we missed? (1) What if both coils carry current? (2) Field from a coil affecting itself.

Including self-inductance and "back emf"

$$\mathcal{E}_2 = \mathfrak{M}_{21}\frac{\mathrm{d}I_1}{\mathrm{d}t} + \mathfrak{M}_{22}\frac{\mathrm{d}I_2}{\mathrm{d}t} \quad \text{and} \quad \mathcal{E}_1 = \mathfrak{M}_{12}\frac{\mathrm{d}I_2}{\mathrm{d}t} + \mathfrak{M}_{11}\frac{\mathrm{d}I_1}{\mathrm{d}t} \tag{85}$$

Where again $\mathfrak{M}_{21} = \mathfrak{M}_{12} \equiv \mathfrak{M}$. (We could write $\mathcal{E}_i = \sum_j \mathfrak{M}_{ij}\dot{I}_j$ ... but that is a bit much!)

Usually, we say $\mathfrak{M}_{11} = -\mathscr{L}_1$ and $\mathfrak{M}_{22} = -\mathscr{L}_2$.

This is the "normal" inductance we think of, we can calculate it (but NOT from our integral).

In general,

$$\mathfrak{M}_{12} = k\sqrt{\mathscr{L}_1\mathscr{L}_2} \tag{86}$$

where $k < 1$ is the "coefficient of coupling" of the two coils.

Physically, k represents something like the amount of coil 1's flux that coil 2 captures.

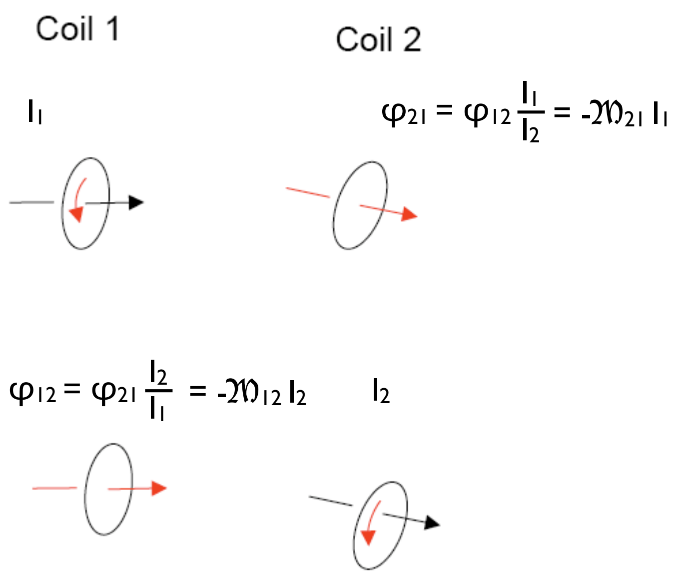
With only one coil, we have *only* the effect of the coil on itself, and get the usual result:

Only one coil: self-inductance

$$\mathcal{E} = \mathscr{L}\frac{\mathrm{d}I}{\mathrm{d}t} = -\mathfrak{M}_{11}\frac{\mathrm{d}I}{\mathrm{d}t} \quad \text{The "F=ma" of circuits.} \tag{87}$$

"Reciprocity"

Now assume that induced currents are small, and "back-emf" can be neglected.



One coil energized:

$$\mathcal{E}_1 = \mathfrak{M}_{12}\frac{\mathrm{d}I_2}{\mathrm{d}t} = -\frac{\mathrm{d}\Phi_{12}}{\mathrm{d}t} \tag{88}$$

integrate the rightmost two parts with respect to t:

$$\int \mathfrak{M}_{12}\frac{\mathrm{d}I_2}{\mathrm{d}t}\,dt = -\int \frac{\mathrm{d}\Phi_{12}}{\mathrm{d}t}\,dt \tag{89}$$

$$\mathfrak{M}_{12}I_2 = -\Phi_{12} \qquad \text{or} \qquad \mathfrak{M}_{12} = -\frac{\Phi_{12}}{I_2} \tag{90}$$

But we know that $\mathfrak{M}_{12} = \mathfrak{M}_{21}$! So we can show:

Reciprocity:

$$-\mathfrak{M}_{12} = \frac{\Phi_{12}}{I_2} = \frac{\Phi_{21}}{I_1} \qquad \text{or} \qquad \Phi_{12}I_1 = \Phi_{21}I_2 \tag{91}$$

We can find one flux from the other, so long as we can find the fictitious current that would exist in the opposite case.

Now think about a small element of magnetic material ...

“Reciprocity”

So far, everything is very general and coordinate-free. Most generally, we know

$$\mathfrak{M}(\dot{I}_1 - \dot{I}_2) = \mathfrak{M}_{11}\dot{I}_1 - \mathfrak{M}_{22}\dot{I}_2 \tag{92}$$

We can only use *flux* when the back EMF is negligible, only one coil is on, or r_{12} is small

Two coils, one I, B constant, no back-emf:

$$\frac{\Phi_{12}}{I_2} = \frac{\Phi_{21}}{I_1} \qquad \text{or} \qquad \Phi_{12}I_1 = \Phi_{21}I_2 \tag{93}$$

For any pair of coils, calculating the induced EMF or current in one due to the other is sufficient.

It doesn’t matter which one actually carries a current, we can calculate the most convenient case.

If we have n_1 and n_2 turns in the coils:

$$\Phi_{12}n_1I_1 = \Phi_{21}n_2I_2 \tag{94}$$

"Reciprocity"

Why bother? **Mutual inductance tells us that:**

flux through pickup coil due to media

$\Updownarrow$

flux through media due to head field

One problem is **much** easier than the other.

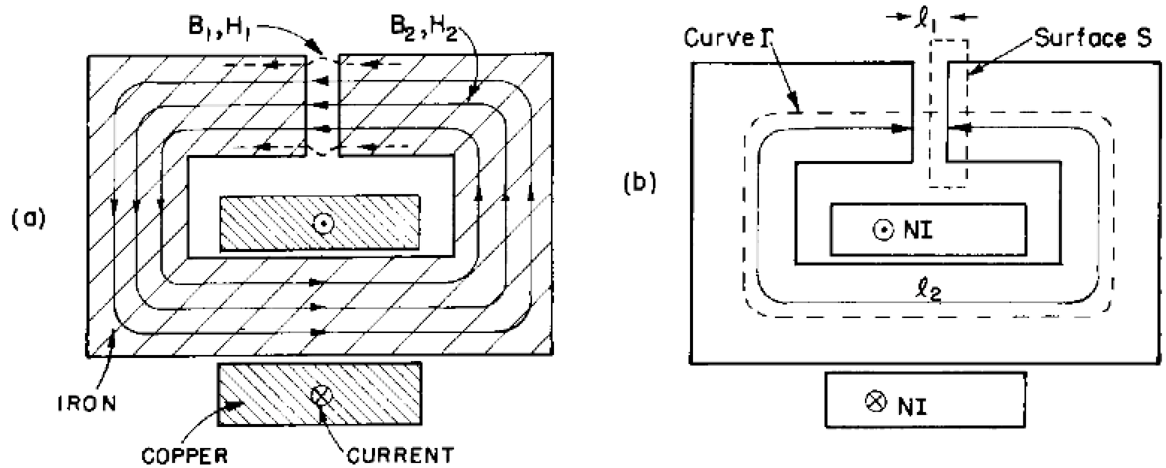


Fig. 36–11. Cross section of an electromagnet.

Surface S around pole tip: flux is zero

$$\Phi_S = B_1 A_1 - B_2 A_2 = 0 \tag{95}$$

If $A_1 = A_2$, then $B_1 = B_2$.

Now take a line integral around Γ:

$$\oint_\Gamma \vec{\mathbf{H}} \cdot d\vec{\mathbf{l}} = \mu_0 \int \vec{\mathbf{j}} \cdot \vec{\mathbf{n}}\, da \quad \Rightarrow \quad H_1 l_1 + H_2 l_2 = \mu_0 \left(\frac{IN}{A} \right) A = \mu_0 NI \tag{96}$$

The loop Γ encloses a current density of $j = NI/A$. Break up the $d\vec{\mathbf{l}}$ integral into a part through the core, and a part through the air gap.

Electromagnets

We need to know $B(H)$ for the core ... if we work in the small field regime ("minor loops"), then we can approximate it as linear - $B_2 \sim \mu H_2$, where μ is the permeability:

$$B_1 A_1 = B_2 A_2 = A_1 \mu_0 H_1 = A_2 \mu H_2 \tag{97}$$

$$\Rightarrow H_1 = \frac{\mu}{\mu_0} H_2 \qquad (\text{if} \quad A_1 = A_2) \tag{98}$$

So the field in the gap is *enhanced by a factor* μ. Using this:

Gap field:

$$H_1 l_1 + H_2 l_2 = H_1 l_1 + \frac{\mu_0}{\mu} H_1 l_2 = \mu_0 N I \tag{99}$$

$$H_1 \left[\frac{\mu_0}{\mu} l_2 + l_1 \right] = \mu_0 N I \tag{100}$$

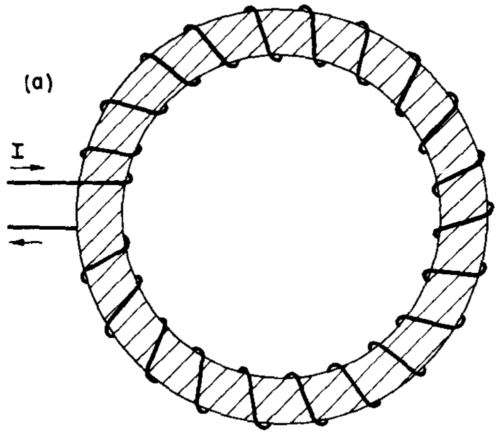
$$\Rightarrow H_1 = \frac{N I}{\frac{\mu_0}{\mu} l_2 + l_1} \tag{101}$$

Basically: we can relate the gap field to I and the core material properties.

High μ gives higher gap field for a given I. High μ is high efficiency.
High B_s gives high max gap field.

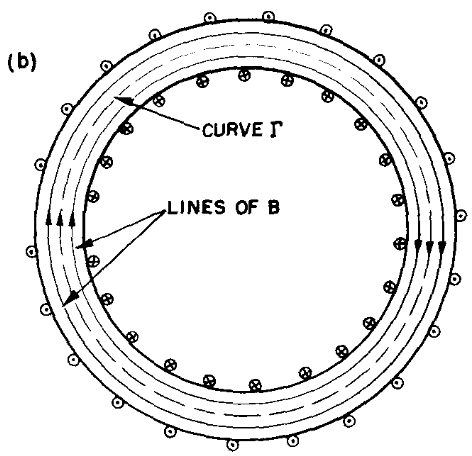
Why does H_c matter?

Iron core inductors



$$\oint_\Gamma \vec{\mathbf{H}} \cdot d\vec{\mathbf{s}} = \mu_0 \int_S \vec{\mathbf{j}} \cdot \vec{\mathbf{n}}\, da$$
$$Hl = \mu_0 N I$$

$$H = \mu_0 \frac{NI}{l} \quad \text{or} \quad I = \frac{Hl}{\mu_0 N}$$



$$\frac{\mathrm{d}U}{\mathrm{d}t} = VI = \left(N\frac{\mathrm{d}B}{\mathrm{d}t}\right)\left(\frac{Hl}{\mu_0 N}\right)$$
$$= \frac{lA}{\mu_0} H \frac{\mathrm{d}B}{\mathrm{d}t}$$

$$U = \frac{lA}{\mu_0} \int H\, dB$$

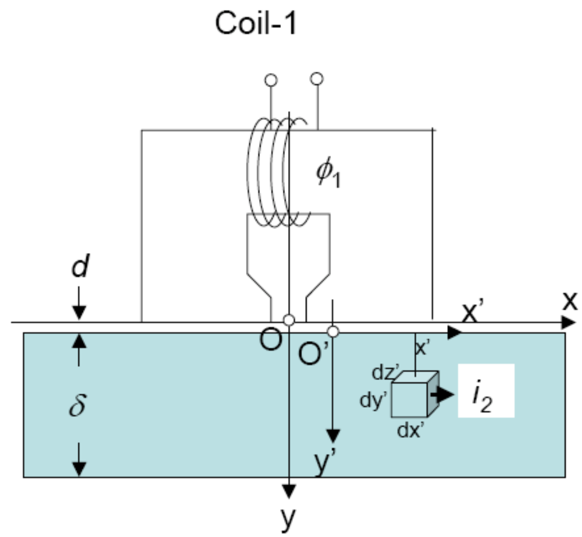
The energy used in driving the core is proportional to the area of the $B - H$ loop!

Head loop narrow, soft materials. Low loss, less heating, less thermal noise.

Further: we want low remanence. *Why?*

Resistivity: eddy current losses. *Ideally, get benefit of M without j.* Ferrite core inductors.

The Head



Since the mutual inductance is a property of a *pair* of loops, we can relate the flux through I_1 to the field of the media *or vice-versa*.

Which is easier? Finding the flux through the *media* from the head. Remember that we can model the bit of media as a dipole, or a **current loop**.

The Head

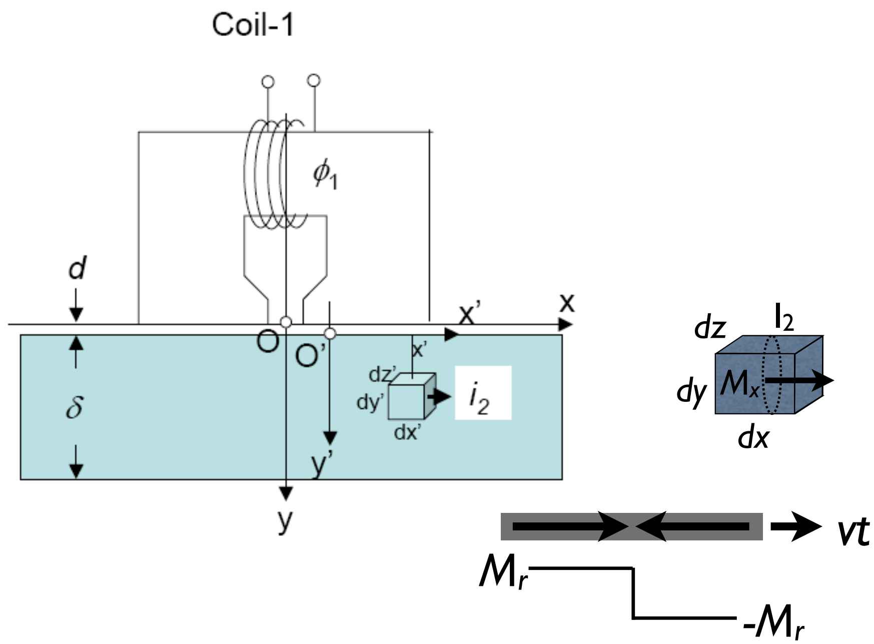
Media is moving at velocity v, head is fixed

$$\Rightarrow \text{media position} = vt \equiv \overline{x}$$

lateral displacement between media region and head center

$$\Rightarrow \text{head-media displacement} = x - \overline{x} = x - vt$$

The Head



If we were *writing*, we could say:

Flux $d\Phi_{21}$ through the element of medium due to a current in the head coil

$$d\Phi_{21} = \vec{\mathbf{B}} \cdot \vec{\mathbf{n}}\, da = \mu_0 \left(\vec{\mathbf{H}}_1 + \vec{\mathbf{M}}_1 \right) \cdot \vec{\mathbf{n}}\, da = \mu_0\, \vec{\mathbf{H}}_1 \cdot \vec{\mathbf{n}}\, da = \mu_0 H_{1x} dy dz \qquad (102)$$

M_1 (due to the head) is zero in the media, and only H_x gives a non-zero contribution.

The Head

What we really want is the opposite for reading.

We want Φ_{12} - the "pickup" flux in the reader coil due to the bit of media $dx\,dy\,dz$. Now we use our earlier result:

Head flux at media ⇔ media flux at head

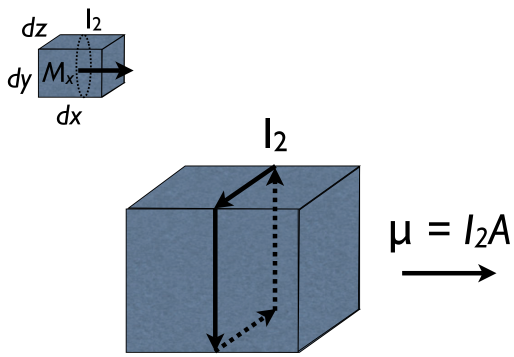
$$\frac{d\Phi_{12}}{I_2} = \frac{d\Phi_{21}}{I_1} \qquad \text{or} \qquad d\Phi_{12} = \frac{I_2}{I_1}\, d\Phi_{21} \tag{103}$$

(flux at head due to media) $=$ (ratio of currents) $\times$ (flux at media due to head)

What we have shown is that reading and writing are basically the same thing! We do not need to solve the problem twice.

From now on, H refers to the head field, and M refers to the media magnetization.

The Media



So what is I_2?

Moment of a current: $\mu = IA$ or $\mu = MV$:

$$\mu = I_2 A = I_2\, dy\, dz \quad (104)$$
$$= M_2\, dV = M_{2x}\, dx\, dy\, dz \quad (105)$$
$$\Rightarrow I_2 = M_{2x}\, dx \quad (106)$$

Remember: the media is moving! So what we really mean by "M_x" is $M_x(x - \overline{x})$.

Using mutual inductance and accounting for moving media:

$$d\Phi_{12} = \frac{I_2}{I_1}\, d\Phi_{21} = \frac{M_x dx}{I_1}\, d\Phi_{21} = \frac{M_x dx}{I_1}\, \mu_0 H_x\, dy\, dz \quad (107)$$
$$= \frac{\mu_0 H_x(x,y,z)}{I_1}\, M_x(x - \overline{x}, y, z)\, dx\, dy\, dz \quad (108)$$

Note that H_x/I_1 is independent of I_1 ... recall the gap field:

$$\frac{H_g}{I_1} = \frac{N}{\frac{\mu_0}{\mu} l_2 + l_1} \quad (109)$$

The Signal

Now we can just integrate $d\Phi_{12}$ over the *media* - infinite in x, $y : d \rightarrow d + \delta$, $z : -W/2 \rightarrow W/2$.

If we assume everything is homogeneous over the track width W, the dz integral gives a factor W.

Also assume that M is uniform along the thickness (y) direction.

$$\Phi_{12} = \iiint\limits_{\text{media}} d\Phi_{12}\, dV \tag{110}$$

$$= \mu_0 \int_{-\infty}^{\infty} dx \int_{d}^{d+\delta} dy \int_{-W/2}^{W/2} dz \left[\frac{H_x(x,y,z)}{I_1}\right] M_x(x-\overline{x},y,z)$$

$$= \mu_0 W \int_{-\infty}^{\infty} dx \int_{d}^{d+\delta} dy \left[\frac{H_x(x,y,z)}{I_1}\right] M_x(x-\overline{x}) \tag{111}$$

Flux is not helpful, we want the *induced voltage*:

Induced V as a function of media position

$$V_x(\overline{x}) = -\frac{\mathrm{d}\Phi_{12}}{\mathrm{d}t} = -\mu_0 W \frac{\mathrm{d}}{\mathrm{d}t} \int_{-\infty}^{\infty} dx \int_{d}^{d+\delta} dy \left[\frac{H_x(x,y,z)}{I_1}\right] M_x(x-\overline{x}) \tag{112}$$

The Signal

Only M_x depends on time, through $\overline{x} = vt$. Chain rule madness:

$$\frac{\mathrm{d}M_x(x-\overline{x})}{\mathrm{d}t} = \frac{\mathrm{d}\overline{x}}{\mathrm{d}t} \cdot \frac{\mathrm{d}M_x(x-\overline{x})}{\mathrm{d}\overline{x}} \qquad \text{and} \qquad \frac{\mathrm{d}\overline{x}}{\mathrm{d}t} = v \tag{113}$$

$$V_x(\overline{x}) = -\mu_0 W v \int_{-\infty}^{\infty} dx \int_{d}^{d+\delta} dy \left[\frac{H_x(x,y,z)}{I_1}\right] \frac{\mathrm{d}M_x(x-\overline{x},y,z)}{\mathrm{d}\overline{x}} \tag{114}$$

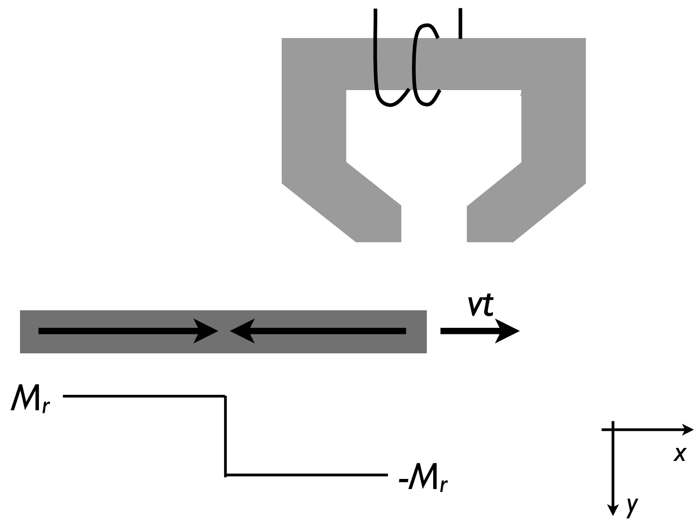
We sense the flux of *moving magnetic charge!* (the inner part looks a bit like $\vec{\nabla} \cdot \vec{\mathrm{M}}$ ...)

If $\vec{\nabla} \cdot \vec{\mathrm{M}} = 0$ or $v = 0$, we read nothing!

ASSUMPTIONS we rely on:

1. The "write current" I_2 is small enough that hysteresis is absent, and $B \sim \mu H$.
2. The "write current" and read excitation are quasi-static.
3. By that we mean flat permeability *vs.* frequency.
4. And by frequency we really mean flat permeability over the bandwidth.

Sharp transition read signal



Moving head maps $M(x) \to M(t)$, so $M(x, y, z) \to V(t)$

Perfectly sharp transition:

$$M(x - \overline{x}) = \begin{cases} -M_r & x < \overline{x} \\ M_r & x > \overline{x} \end{cases} \tag{115}$$

$$\frac{\mathrm{d}M(x-\overline{x})}{\mathrm{d}\overline{x}} = \frac{-\mathrm{d}M_x(x-\overline{x})}{\mathrm{d}x} = -2M_r\delta(x-\overline{x}) \tag{116}$$

$$V_x(\overline{x}) = -\mu_0 W v \int_{-\infty}^{\infty} dx \int_{d}^{d+\delta} dy \frac{H_x(\overline{x}, y)}{I_1} \left[-2M_r\delta(x-\overline{x})\right] \tag{117}$$

$$= 2\mu_o v W M_r \int_{d}^{d+\delta} dy \frac{H_x(\overline{x}, y)}{I_1} \tag{118}$$

Signal from a sharp transition:

$$V_x(\overline{x}) = 2\mu_0 v W M_r \left[\frac{H_x(\overline{x}, d)}{I_1} \cdot \delta\right] \qquad (\delta \ll d, \text{ thin media}) \tag{119}$$

Arctan transition read signal

Arctan transition

$$M_x(x-\overline{x}) = \frac{2M_r}{\pi}\tan^{-1}\left(\frac{x-\overline{x}}{a}\right) \tag{120}$$

$$\frac{\mathrm{d}M_x(x-\overline{x})}{\mathrm{d}\overline{x}} = -\frac{\mathrm{d}M_x(x-\overline{x})}{\mathrm{d}x} = -\frac{2M_r}{\pi}\left[\frac{1}{1+\frac{(x-\overline{x})^2}{a^2}}\right] = -\frac{2M_r}{\pi}\left[\frac{a^2}{a^2+(x-\overline{x})^2}\right] \tag{121}$$

We already know what H_x for a Karlqvist head is ...

$$H_x(x,y) = \frac{H_g}{\pi}\left[\tan^{-1}\left(\frac{x+g/2}{y}\right) - \tan^{-1}\left(\frac{x-g/2}{y}\right)\right] \tag{122}$$

So we can set up $V_x(\overline{x})$:

$$\begin{aligned} V_x(\overline{x}) &= -\mu_0 W v \int_{-\infty}^{\infty} dx \int_{d}^{d+\delta} dy \left[\frac{H_x(x,y,z)}{I_1}\right]\frac{\mathrm{d}M_x(x-\overline{x})}{\mathrm{d}\overline{x}} \\ &= \mu_0 v W \int_{-\infty}^{\infty} dx \int_{d}^{d+\delta} dy\, \frac{H_g}{\pi}\left[\tan^{-1}\left(\frac{x+g/2}{y}\right) - \tan^{-1}\left(\frac{x-g/2}{y}\right)\right]\frac{2M_r a^2}{a^2+(x-\overline{x})^2} \end{aligned} \tag{123}$$

Arctan transition read signal

without proof:

$$\int_{-\infty}^{\infty} \frac{dx}{a^2 + (x-\overline{x})^2}\left[\tan^{-1}\left(\frac{x+c}{y}\right) - \tan^{-1}\left(\frac{x-c}{y}\right)\right] = \frac{\pi}{a}\left[\tan^{-1}\left(\frac{\overline{x}+c}{y+a}\right) - \tan^{-1}\left(\frac{\overline{x}-c}{y+a}\right)\right] \quad (124)$$

$$V_x(\overline{x}) = \frac{2\mu_0 v w H_g M_r}{\pi I_1}\int_{d}^{d+\delta} dy\ \tan^{-1}\left(\frac{\overline{x}+g/2}{y+a}\right) - \tan^{-1}\left(\frac{\overline{x}-g/2}{y+a}\right) \quad (125)$$

!! But

$$\tan^{-1}\left(\frac{\overline{x}+g/2}{y+a}\right) - \tan^{-1}\left(\frac{\overline{x}-g/2}{y+a}\right) = H_x(\overline{x}, y+a) \quad (126)$$

$$\Rightarrow V_x(\overline{x}) = \frac{2\mu_0 v W H_g M_r}{\pi I_1}\int_{d}^{d+\delta} dy\, H_x(\overline{x}, y+a) = \frac{2\mu_0 v W H_g M_r}{\pi I_1}\int_{d+a}^{d+\delta+a} dy\, H_x(\overline{x}, y) \quad (127)$$

An arctan transition at a spacing d looks like a step transition at $d+a$!

Arctan transition read signal

An arctan transition at a spacing d looks like a step transition at $d+a$!

Or, a finite transition width acts as an effective spacing.

Basically transition broadening and increased spacing do the same thing.

For thin media ($\delta \ll d$), things are simpler.

Signal from an arctan transition, thin media:

$$\begin{aligned} V_x(\overline{x}) &= \frac{2\mu_0 v W H_g M_r}{\pi I_1} \int\limits_{d+a}^{d+\delta+a} dy H_x(\overline{x}, y) \approx 2\mu_0 v W M_r \left[\frac{H_x(\overline{x}, d+a)}{I_1} \cdot \delta\right] \\ V_x(\overline{x}) &\approx 2\mu_0 v W M_r \delta \frac{H_x(\overline{x}, d+a)}{I_1} \quad (\delta \ll d, \text{ thin media}) \end{aligned} \tag{128}$$

The voltage readback also scales with $M_r\delta$

But it scales the opposite way compared to media.

A never-ending series of ugly tradeoffs. :-(

arctan + Karlqvist + thin media

Putting it all together: arctan media + Karlqvist head

Karlqvist head field, small gap ($y \gg \delta$)

$$H_x(x,y) = \frac{H_g g}{\pi}\frac{y}{x^2+y^2} \tag{129}$$

Arctan transition, thin media & Karlqvist head

$$V_x(\overline{x}) = 2\mu_0 v W M_r \delta \frac{H_g g}{\pi I_1} \tan^{-1}\left[\frac{g(d+a)}{\overline{x}^2 + (d+a)^2 - (g/2)^2}\right] \tag{130}$$

$$\approx 2\mu_0 v W M_r \delta \frac{H_g g}{\pi I_1}\left[\frac{d+a}{\overline{x}^2 + (d+a)^2}\right] \tag{131}$$

The readback pulse is just a Lorentzian of width $\Gamma = 2(d+a)$.

And .... the homework:

give a (justifiable) order of magnitude estimate of V_x