University of Alabama
Department of Physics and Astronomy

PH 253 / LeClair — Spring 2010

Exam 1 sample problems

1. De Broglie applied his relativistic treatment of wavelength for photons to particles of nonzero rest mass. The wave nature of particles of nonzero rest mass can also be described consistently with a *nonrelativistic* theory, a fact of vital importance in allowing simple solutions of many quantum-mechanical problems. Carry out the analysis using the equations $E = hf = \hbar\omega$, but using the nonrelativistic expression for a free particle $E = p^2/2m$.

(a) Find $\omega = 2\pi f$ as a function of $k = 2\pi/\lambda$ and compute the phase velocity ω/k and the group velocity $d\omega/dk$.

(b) Which velocity is the same as the particle velocity?

2. *Significance of the Compton wavelength.* Show that the speed of a particle having de Broglie wavelength λ and a Compton wavelength $\lambda_c = h/mc$ is

$$v = \frac{c}{\sqrt{1 + (\lambda/\lambda_c)^2}} \tag{1}$$

Answer: HW4, Spring 2010

3. *How big are atoms?* A simple but sophisticated argument holds that the hydrogen atom has its observed size because this size minimizes the total energy of the system. The argument rests on the assumption that the lowest-energy state corresponds to a physical size comparable to a de Broglie wavelength of the electron. Larger size means larger de Broglie wavelength, hence smaller momentum and kinetic energy. In contrast smaller size means lower potential energy, since the potential well is deepest near the proton. The observed size is a compromise between kinetic and potential energies that minimizes the total energy of the system. Develop the argument explicitly, for example as follows:

(a) Write down the classical expression for the total energy of the hydrogen atom with an electron of momentum p in a circular orbit of radius r. Keep kinetic and potential energies separate.

(b) Failure of classical energy minimization. Use the force law to obtain the total energy as a function of radius. What radius corresponds to the lowest possible energy?

(c) For the lowest-energy state, demand that the orbit circumference be one de Broglie wavelength. Obtain an expression for the total energy as a function of radius. Note how a larger radius decreases the kinetic and increases the potential energy, whereas a smaller radius increases the kinetic and decreases the potential energy.

(d) Take the derivative of the energy versus radius function and find the radius that minimizes the total energy. How large is that radius for a hydrogen atom?

Answer: HW4, Spring 2010

4. *Energy spread of electron beam.* A monochromatic beam of electrons of energy $E = 1\,\text{keV}$ is incident on a shutter that opens for $\Delta t = 1\,\text{ns}$. What is the fractional energy spread $\Delta v/v$ of the electron velocity v after the shutter? (Decide first whether to perform a relativistic or a non-relativistic calculation.)

Answer: 10^{-10}; HW4, Spring 2010

5. Observer O notes that two events are separated in space and time by $600\,\text{m}$ and $8 \times 10^{-7}\,\text{s}$. How fast must observer O' be moving relative to O in order that the events seem simultaneous?

Answer: $v/c = 0.4$

6. An airplane is moving with respect to the earth with a speed of $600\,\text{m/s}$. As determined by earth clocks, how long will it take for the airplane's clock to fall behind by 2 microseconds?

Answer: 11.6 days

7. Pions have a half life of $1.8 \times 10^{-8}\,\text{s}$. A pion leaves an accelerator at a speed of $0.8c$.

(a) Classically, what is the expected distance over which half the pions should decay?
(b) Determine the answer to (a) relativistically.

Answer: 4.32 m; 7.20 m

8. The equation for a spherical pulse of light starting from the origin at $t = t' = 0$ is

$$x^2 + y^2 + z^2 - c^2 t^2 = 0 \tag{2}$$

Show from the Lorentz transformations that an observer in O' moving at relative velocity v along the x axis also measures this pulse to be spherical. *Hint: if it is to be spherical, the equation above must hold with* $x \to x'$*, etc.*

9. The width of a spectral line of mean wavelength 400 nm is measured to be 10^{-5} nm. What is the average time that the atomic system remains in the corresponding energy state? Note that since $E = hc/\lambda$ one can relate the uncertainties $\Delta\lambda$ and ΔE by

$$\frac{\Delta E}{E} = \frac{\Delta\lambda}{\lambda} \tag{3}$$

Answer: 4.24 ns

10. What is the uncertainty in the location of a photon of wavelength 300 nm if this wavelength is known to an accuracy of one part in a million? Note that since $p = h/\lambda$, the uncertainties Δp and $\Delta\lambda$ are related by

$$\frac{\Delta p}{p} = \frac{\Delta\lambda}{\lambda} \tag{4}$$

Answer: 23.9 mm

11. An electron initially moving at constant speed v is brought to rest with uniform deceleration a lasting for a time $t = v/a$. Compare the electromagnetic energy radiated during this deceleration with the electron's initial kinetic energy. Express the ratio in terms of two lengths, the distance light travels in time t and the classical electron radius $r_e = e^2/4\pi\epsilon_o mc^2$.

Answer: see the notes on radiation.