Name & CWID ____

PH 253 Exam I

Instructions

1. Solve five of the seven problems below. All problems have equal weight.
2. Do your work on separate sheets.
3. Bring your exam paper with you when you leave - you need it for the next homework.
4. You are allowed 1 sheet of standard 8.5x11 in paper and a calculator.

LeClair
F10

1. A train 1/2 km long (as measured by an observer on the train) is traveling at a speed of 100 km/hr. Two lightning bolts strike the ends of the train simultaneously as determined by an observer on the ground. What is the time separation as measured by an observer on the train?

2. The speed of light in still water is c/n, where n is the index of refraction, approximately $n=4/3$ for water. Fizeau, in 1851, found that the speed (relative to the laboratory) of light in water moving at speed V (relative to the laboratory) could be expressed as

$$u = \frac{c}{n} + kV \tag{1}$$

where the "dragging coefficient" was measured by him to be $k \approx 0.44$. Determine the value of k predicted by the Lorentz velocity transformations. Note $(1+x)^{-1} \approx 1 - x$ for $x \ll 1$.

3. An electron initially moving at constant speed v is brought to rest with uniform deceleration a lasting for a time $t = v/a$. Compare the electromagnetic energy radiated during this deceleration with the electron's initial kinetic energy. Express the ratio in terms of two lengths, the distance light travels in time t and the classical electron radius $r_e = e^2/4\pi\epsilon_0 mc^2$.

4. In an experiment to find the value of h, light at wavelengths 218 and 431 nm were shone on a clean sodium surface. The potentials that stopped the fastest photoelectrons were 5.69 and 0.59 V, respectively. What values of h and W, the sodium work function, are deduced?

5. In Compton scattering, an incident photon of energy E_γ and momentum $p = \hbar k$ scatters off of an electron at rest. The photon emerges at angle θ with reduced energy E'_γ and momentum $p' = \hbar k'$. The electron is ejected with energy E_{e^-} and momentum p_{e^-}.

Show that the exiting photon's energy as a function of its energy and ejection angle θ is

$$E'_\gamma = \frac{mc^2}{(1-\cos\theta) + mc^2/E_\gamma} \tag{2}$$

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6. **(a)** An FM radio transmitter has a power output of 130 kW and operates at a frequency of 98.3 MHz. How many photons per second does the transmitter emit?

(b) A pulsed ruby laser emits light at 694.3 nm. For a 13.6 ps pulse containing 3.40 J of energy, how many photons are in the pulse? 1 ps is 10^{-12}s.

7. A molecule is known to exist in an unstable higher energy configuration for $\Delta t = 10$ nsec, after which it relaxes to its lower energy stable state by emitting a photon.

(a) What uncertainty in the frequency Δf of the emitted photon is implied? **(b)** If this state is being probed with Nuclear Magnetic Resonance (NMR) at a frequency of $f \approx 500$ MHz, what is the relative uncertainty in the measurement, $\Delta f/f$?

Constants:

$$N_A = 6.022\times10^{23}\text{ things/mol}$$
$$k_e \equiv 1/4\pi\epsilon_o = 8.98755\times10^{9}\text{ N}\cdot\text{m}^2\cdot\text{C}^{-2}$$
$$\epsilon_o = 8.85\times10^{-12}\text{ C}^2/\text{N}\cdot\text{m}^2$$
$$\mu_o \equiv 4\pi\times10^{-7}\text{ T}\cdot\text{m/A}$$
$$e = 1.60218\times10^{-19}\text{ C}$$
$$h = 6.6261\times10^{-34}\text{ J}\cdot\text{s} = 4.1357\times10^{-15}\text{ eV}\cdot\text{s}$$
$$\hbar = \frac{h}{2\pi}$$
$$k_B = 1.38065\times10^{-23}\text{ J}\cdot\text{K}^{-1} = 8.6173\times10^{-5}\text{ eV}\cdot\text{K}^{-1}$$
$$c = \frac{1}{\sqrt{\mu_0\epsilon_0}} = 2.99792\times10^{8}\text{ m/s}$$
$$hc = 1240\text{ eV}\cdot\text{nm}$$
$$m_e = 9.10938\times10^{-31}\text{ kg}\qquad m_ec^2 = 510.998\text{ keV}$$
$$m_p = 1.67262\times10^{-27}\text{ kg}\qquad m_pc^2 = 938.272\text{ MeV}$$

Quadratic formula:

$$0 = ax^2 + bx^2 + c \Longrightarrow x = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

Basic Equations:

$$\vec{F}_{net} = \frac{d\vec{p}}{dt} = m\vec{a}\qquad \text{Newton's Second Law}$$
$$\vec{F}_{centr} = -\frac{mv^2}{r}\hat{r}\qquad \text{Centripetal}$$
$$\mathscr{P} = \frac{\Delta E}{\Delta t}\qquad \text{power}$$

E & M

$$\vec{F}_{12} = k_e\frac{q_1q_2}{r_{12}^2}\hat{r}_{12} = q_2\vec{E}_1\qquad \vec{r}_{12} = \vec{r}_1 - \vec{r}_2$$
$$\vec{E}_1 = \vec{F}_{12}/q_2 = k_e\frac{q_1}{r_{12}^2}\hat{r}_{12}$$
$$\vec{F}_B = q\vec{v}\times\vec{B}$$

EM Waves:

$$c = \lambda f = \frac{|\vec{E}|}{|\vec{B}|}$$
$$I = \left[\frac{\text{photons}}{\text{time}}\right]\left[\frac{\text{energy}}{\text{photon}}\right]\left[\frac{1}{\text{Area}}\right]$$
$$I = \frac{\text{energy}}{\text{time}\cdot\text{area}} = \frac{E_{max}B_{max}}{2\mu_0} = \frac{\text{power }(\mathscr{P})}{\text{area}} = \frac{E_{max}^2}{2\mu_0c}$$

Oscillators

$$E = \left(n+\frac{1}{2}\right)hf$$
$$E = \frac{1}{2}kA^2 = \frac{1}{2}\omega^2mA^2 = 2\pi^2mf^2A^2$$
$$\omega = 2\pi f = \sqrt{k/m}$$

Approximations, $x \ll 1$

$$(1+x)^n \approx 1 + nx + \frac{1}{2}n(n+1)x^2\qquad \tan x \approx x + \frac{1}{3}x^3$$
$$e^x \approx 1 + x + \frac{1}{2}x\qquad \sin x \approx x - \frac{1}{6}x^3\qquad \cos x \approx 1 - \frac{1}{2}x^2$$

Radiation

$$P_{rad} = \frac{q^2a^2}{6\pi\epsilon_oc^3}\qquad \text{total emitted power, E and B fields}$$
$$E_{tot} = \sigma T^4\qquad \sigma = 5.672\times10^{-8}\text{ W}\cdot\text{m}^{-2}\cdot\text{K}^{-4}$$
$$T\lambda_{max} = 2.9\times10^{-3}\text{ m}\cdot\text{K}\qquad \text{Wien}$$
$$E_{quantum} = hf$$
$$\langle E_{oscillator}\rangle = hf/\left(e^{hf/k_BT}-1\right)$$
$$I(\lambda,T) = \frac{(\text{const})}{\lambda^5}\left[e^{\frac{hc}{\lambda k_bT}}-1\right]^{-1}$$
$$I(f,t) = (\text{const})\,f^3\left[e^{\frac{hf}{k_bT}}-1\right]^{-1}$$

Relativity

$$\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$$
$$\Delta t'_{moving} = \gamma\Delta t_{stationary} = \gamma\Delta t_p$$
$$L'_{moving} = \frac{L_{stationary}}{\gamma} = \frac{L_p}{\gamma}$$
$$x' = \gamma(x - vt)\qquad x = \gamma(x' + vt')$$
$$t' = \gamma\left(t - \frac{vx}{c^2}\right)\qquad t = \gamma\left(t' + \frac{vx'}{c^2}\right)$$
$$v_{obj} = \frac{v + v'_{obj}}{1+\frac{vv'_{obj}}{c^2}}\qquad v'_{obj} = \frac{v_{obj}-v}{1-\frac{vv_{obj}}{c^2}}$$
$$KE = (\gamma-1)mc^2 = \sqrt{m^2c^4+c^2p^2} - mc^2$$
$$E_{rest} = mc^2$$
$$p = \gamma mv$$
$$E^2 = p^2c^2 + m^2c^4 = \left(\gamma mc^2\right)^2$$

Quantum

$$E = hf = \hbar\omega\qquad p = h/\lambda = \hbar k = E/c\qquad \lambda f = c\qquad \text{photons}$$
$$\lambda_f - \lambda_i = \frac{h}{m_ec}(1-\cos\theta)$$
$$\lambda = \frac{h}{|\vec{p}|} = \frac{h}{\gamma mv} \approx \frac{h}{mv}\qquad \text{de Broglie}$$
$$\Delta x\Delta p \geqslant \frac{h}{4\pi}$$
$$\Delta E\Delta t \geqslant \frac{h}{4\pi}$$
$$eV_{stopping} = KE_{electron} = hf - \varphi = hf - W$$

Calculus of possible utility:

$$\int\frac{1}{x}dx = \ln x + c$$
$$\int u\,dv = uv - \int v\,du$$

Vectors:

$$|\vec{F}| = \sqrt{F_x^2+F_y^2}\quad \text{magnitude}\qquad \theta = \tan^{-1}\left[\frac{F_y}{F_x}\right]\quad \text{direction}$$
$$\hat{r} = \vec{r}/|\vec{r}|\quad \text{construct any unit vector}$$
$$\text{let}\quad \vec{a} = a_x\hat{x} + a_y\hat{y} + a_z\hat{z}\quad \text{and}\quad \vec{b} = b_x\hat{x} + b_y\hat{y} + b_z\hat{z}$$
$$\vec{a}\cdot\vec{b} = a_xb_x + a_yb_y + a_zb_z = \sum_{i=1}^{n}a_ib_i = |\vec{a}||\vec{b}|\cos\theta$$
$$|\vec{a}\times\vec{b}| = |\vec{a}||\vec{b}|\sin\theta$$
$$\vec{a}\times\vec{b} = \begin{vmatrix}\hat{x} & \hat{y} & \hat{z}\\ a_x & a_y & a_z\\ b_x & b_y & b_z\end{vmatrix} = (a_yb_z - a_zb_y)\hat{x} + (a_zb_x - a_xb_z)\hat{y} + (a_xb_y - a_yb_x)\hat{z}$$